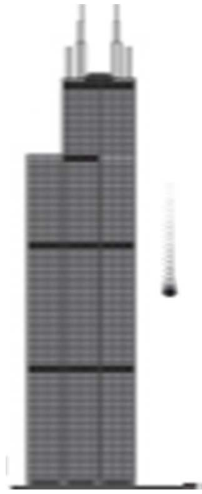
A Falling Sphere without Air Resistance



Consider the problem of a baseball falling from rest from a height *h*.

1. Try the iterative prediction of motion for this problem (by hand) by filing the table below. That is, go through the calculation loop and write down the values for time, momentum and position for each iteration.

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# Parameters and Initial Conditions   (note that comments begin with '#')

ball.mass = 0.2                  # mass (kg)
ball.pos = vector(0,100,0)       # initial position (m), note that h = 100 m
ball.p = vector(0,0,0)           # initial momentum (kg*m/s)

# Time Conditions

t = 0                            # start time (s)
dt = 0.1                         # time step (s)
tmax = 0.3                       # total time to run simulation (s)

# Calculation Loop

while t <= tmax:                 # tabbed lines that  follow repeat until t <= tmax
   t = t + dt                    # update time
   Fg = 9.81*ball.mass*vector(0,-1,0)           # forces acting on the system
   Fnet = Fg

   ball.p = ball.p + Fnet*dt                    # update the momentum

   ball.pos = ball.pos + (ball.p/ball.mass)*dt  # update the position
```

	Iteration #1	Iteration #2	Iteration #3
Time (t)			
Momentum (ball.p)			
Position (ball.pos)			

b) The GlowScript VPython code is online at https://trinket.io/glowscript/8fffadc105
Run the code with the same parameters as above. Compare with your results for each iteration. Comment:

c) Change *tmax* to 2 seconds, and run the code again. What is the height of the baseball after 2 seconds?

d) Change the time interval *dt* to 0.01 seconds, and run the code again. Compare your result to that of the previous question. Comment:

Falling Sphere with Air Resistance[1]

One approximation to the magnitude of the resistive force is given by

$$F_R = \frac{D\rho A}{2} v^2$$

where D is a constant called the drag coefficient of the projectile, ρ is the density of air, and A is the frontal area, the projected 2D area as seen from the direction in which the mass is moving. Here we will assume that D is a constant; but, D, depending on the physical situation, is most often a complicated function of the instantaneous velocity.

2. Consider the problem of a baseball falling from rest from a height h, this time including air resistance.

a) Draw a free body diagram for the baseball including the resistive force.

b) Write an equation for the net force in the y-direction.

c) Modify the GlowScript code to use the iterative method with the above net force. You will need to add the following parameters to the code:

$D = 0.5$	*# drag coefficient of a baseball*
$\rho = 1.29$	*# air density,* $\left(\frac{\text{kg}}{\text{m}^3}\right)$
$A = 0.0043$	*# frontal area,* (m^2)

As well as add a line that calculates F_R, and modify F_{net}.

Note: The *pow(a,n)* function raises '*a*' to the n^{th} power. For the v^2 term you will need to pick out the y-component of velocity: *ball.p.y/ball.mass* and then square it: *pow(ball.p.y/ball.mass,2)*

[1] Adapted from K. Roos, "Falling Sphere with Air Resistance Proportional to v^2," Published in the PICUP Collection, July 2016.

d) Run the code using the following parameters:

$dt = 0.1$ # (s)

$tmax = 10$ # (s)

$ball.mass = 0.145$ # (kg)

$ball.pos = vector(0{,}300{,}0)$ # (m)

$ball.p = vector(0{,}0{,}0)$ # (kg m/s)

Sketch *y(t)* vs. *t*:

Sketch *v(t)* vs. *t*:

e) At what time (approximately) does the value of the velocity become constant? Why is the velocity constant after this time? This velocity is called terminal velocity.

f) How far does the baseball have to fall in order to reach terminal velocity?

3. Finally, consider a skydiver on free fall (i.e. after jumping from the plane but before the parachute opens). Make reasonable assumptions for mass and frontal area.

a) What parameters did you use? List them below.

b) What was the terminal velocity in this case?

c) How far does the skydiver fall before reaching terminal velocity?

d) What is the net force on the skydiver after reaching terminal velocity? Solve Newton's second law for terminal velocity (you can user your answer to question 2(b)).

e) Compare your result from the previous question to the terminal velocity you obtained from the simulation and comment.