

Algebra-Based Physics Students & Vectors : Can *ijk* Coaching Help Arrow Subtraction?

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Introduction

- Students in a calc-based intro mechanics course have been found to perform better on vector operations in the *ijk* format than in the arrow format, and evidence was found that working knowledge of the *ijk* format may be necessary to correctly perform vector operations in the arrow format [1]
- Repeated mastery-style practice significantly improved scores on a Vector Skills Assessment for students in an introductory calc-based sequence [2]
- Working knowledge of graphical vector operation is important for mastery of several physics concepts
- *ijk* representation is often not included in algebra-based books
- Broader Question - Can using *ijk* methods in the algebra-based sequence improve performance and understanding on graphical vector tasks?

Methods

- Students enrolled in algebra-based introductory mechanics
- Administered in last week of class, extra credit for completion
- Students randomly given arrow or *ijk* representation
- No explicit *ijk* instruction given during the semester

Intervention Attempt - Arrow vs. *ijk*

- Students were each given the same vector subtraction question (2D, multiple choice, arrows-on-a-grid)
- Students received an “arrow” or “*ijk*” intervention (“*ijk* Transfer Task”, ITT)

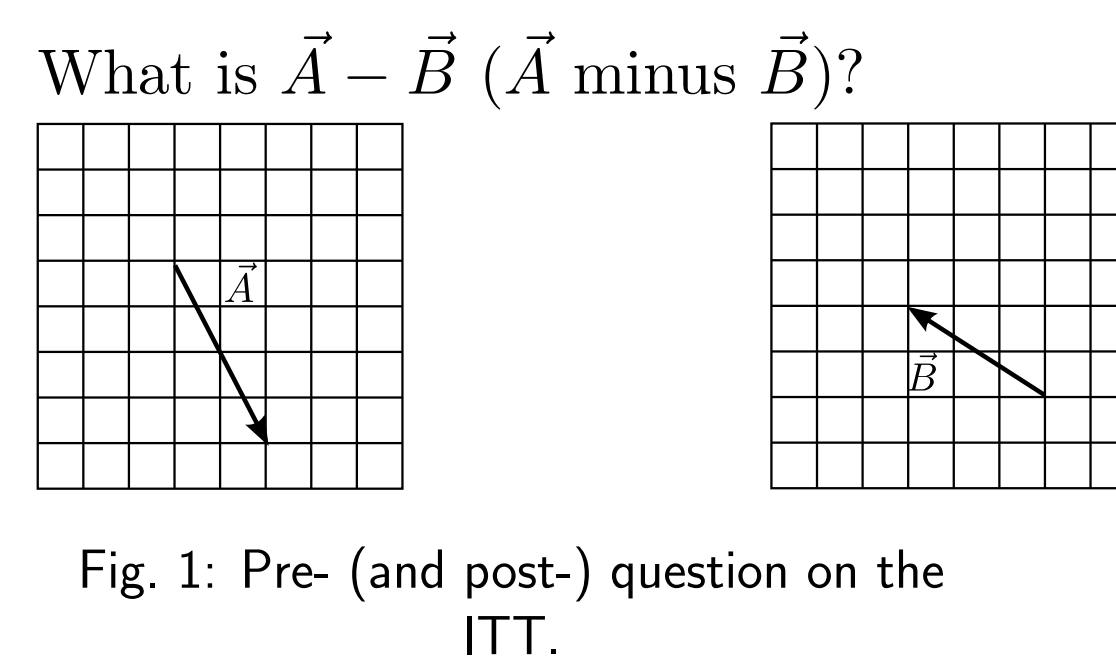


Fig. 1: Pre- (and post-) question on the ITT.

Reminder! To subtract two vectors graphically, such as \vec{P} and \vec{Q} in the figure below, we need to do the following:

- 1.) Since $\vec{P} - \vec{Q}$ is the same as $\vec{P} + (-\vec{Q})$, we need to make the vector $-\vec{Q}$ which has the same size as \vec{Q} but points in the opposite direction.
- 2.) We now add $-\vec{Q}$ to \vec{P} “tip to tail”, putting the tail of $-\vec{Q}$ at the tip of \vec{P} .
- 3.) We draw our new vector from the tail of \vec{P} to the tip of $-\vec{Q}$.
- 4.) We can now remove the original vectors, and are left with $\vec{P} - \vec{Q}$.

One way to subtract two vectors graphically, such as \vec{P} and \vec{Q} in the figure below, is to look at their x- and y- components in “ijk” notation:

- 1.) We first write each vector in ijk notation by putting the x-components in front of the vector \hat{i} , and the y-components in front of the vector \hat{j} , and adding them together. For example, \vec{P} has an x-component of 5 and a y-component of 2, so we write $\vec{P} = 5\hat{i} + 2\hat{j}$. We write a similar expression for \vec{Q} .
- 2.) We now subtract \vec{Q} from \vec{P} by grouping the \hat{i} terms together and the \hat{j} terms.
- 3.) We draw our new vector $\vec{P} - \vec{Q}$ with the x and y components we found from the subtraction. So, we’ll have an x-component of 4 and a y-component of -1.

Fig. 2: Instructional portion of the interventions given in the arrow (left) and *ijk* (right) representations.

Intervention Attempt - Arrow vs. *ijk*

- Students were given 4 “Practice” questions on either identifying the negative of a vector (arrow) or translating a vector into *ijk* (ijk)
- Feedback was given after all 4 questions were completed (correctness and explanation if incorrect)
- The remaining 2 (arrow) or 4 (ijk) question “Synthesis” sequence asked students to perform the practice portion as well as subtraction for the same set of vectors (feedback given after all questions are answered)
- Posttest question (identical to pretest) is given

As a way to practice the above method, choose the correct $-\vec{B}$ for the vector shown below.

As a way to practice the above method, choose the correct way to write out vector \vec{B} in ijk notation.

a.) $4\hat{i} + 3\hat{j}$ b.) $-4\hat{i} + 3\hat{j}$ c.) $-3\hat{i} + -4\hat{j}$
d.) $-4\hat{i} + -3\hat{j}$ e.) $3\hat{i} + -4\hat{j}$ f.) $-3\hat{i} + 4\hat{j}$
g.) $4\hat{i} + -3\hat{j}$ h.) $3\hat{i} + 4\hat{j}$

(other choices not shown to save space)

Fig. 3: Practice questions given in the arrow (left) and *ijk* (right) representations.

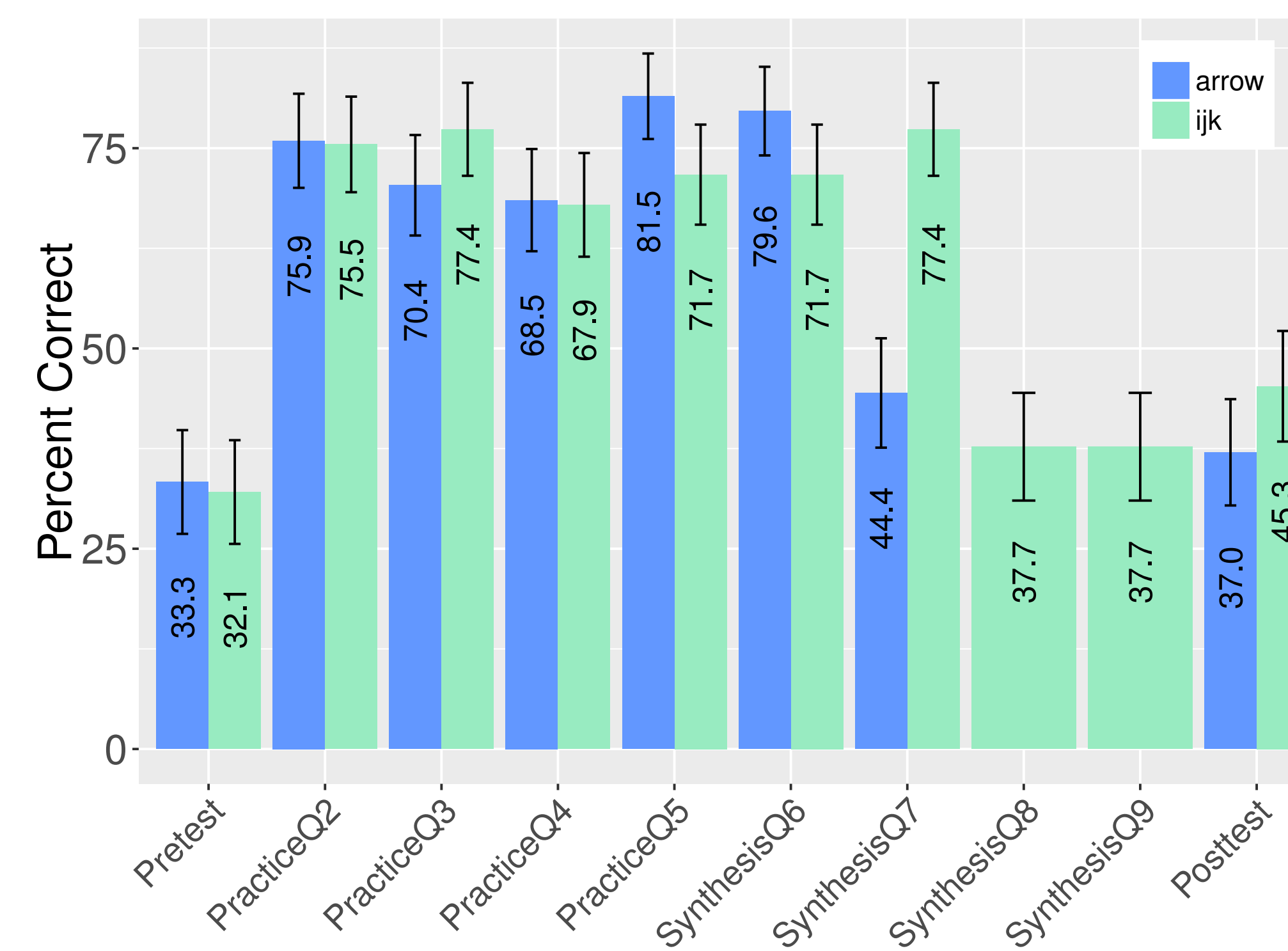


Fig. 4: Percent scores for each question on the intervention. The *ijk* intervention required two more “Synthesis” steps than the arrow, so the arrow intervention does not have an 8th or 9th question before the pretest. Error bars represent ± 1 SE.

	Pre	Post	<i>p</i>
<i>ijk</i> (N = 52)	17 (32.1%)	24 (45.3%)	0.13
Arrow (N = 54)	18 (33.3%)	20 (37.0%)	0.62

- Pretest scores were not different across groups (Wilcoxon rank sum, $W = 1413$, $p = 0.89$)
- Final course grades were not different across groups ($t(105) = -1.28$, $p = 0.20$)
- *ijk* treatment did not show significant gains (Wilcoxon-Pratt signed rank test, $Z = -1.5$, $p = 0.13$)
- Arrow treatment did not show significant gains (Wilcoxon-Pratt signed rank test, $Z = -0.5$, $p = 0.62$)
- In both treatments, the most common distractor chosen on the posttest question is consistent with the students performing vector addition, rather than vector subtraction

Does the Intervention Matter?

- Although the pre/post scores were not different, we can see if performance on the intervention was a predictor of success on the posttest question
- Intervention questions were categorized into those that did not involve subtraction (“Component”) and those that did (“Subtraction”)
- The results of a logistic regression of posttest score against the intervention type, pretest score, “Component” score, and “Intervention” score are shown

Ind. Variable	β	β SE	<i>z</i>	<i>p</i>	OR	OR CI
Intervention (ijk)	0.619	0.469	1.32	0.187	1.85	[0.75, 4.79]
Pretest score	0.819	0.491	1.67	0.096	2.27	[0.86, 6.03]
Component score	1.820	0.817	2.23	0.024*	6.16	[1.39, 36.13]
Subtraction score	1.510	0.493	3.06	0.002*	4.52	[1.75, 12.26]

Table 1: Regression coefficients (β), standard error, *z*-statistic, *p*-value, odds ratio (OR), and 95% confidence interval (CI). The * indicates significance at the $p = 0.05$ level.

- Both “Component” score and “Subtraction” score remain significant predictors of success after controlling for pretest score
- Students who correctly answered all “Component” questions correctly were 6 times more likely to correctly answer the posttest question
- Students who correctly answered all “Subtraction” practice questions correctly were 4.5 times more likely to correctly answer the posttest question

Conclusions & Future Work

- A one-time intervention did not successfully improve arrow subtraction skills
- Although the intervention was not successful, the higher scores on the intervention questions suggest that students are not just “clicking through”
- The logistic regression suggests that further practice on “essential skills” may be key to improving students’ vector subtraction skills [2]
- Additional questions or interviews would help determine why students frequently choose the “addition” distractor

References

- [1] A.F. Heckler and T.M. Scaife, Phys. Rev. ST Phys. Educ. Res. **11**, 010101 (2015)
- [2] B.D. Mikula and A.F. Heckler, Phys. Rev. Phys. Educ. Res. **13**, 010122 (2017)
- [3] J.B. Buncher, 2015 PERC Proceedings, College Park, MD, pp. 75-78, doi:10.1119/perc.2015.pr.014

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