"So it's the same equation…": A blending analysis of student reasoning in kinematics

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The mathematical function is commonly used in all areas of physics. We analyze student use of function in introductory kinematics through the cognitive blending theory. Specifically, we look at the difference in approaches between a student using a multi-scope blend and a student using a single-scope blend who are working on the same problem.

How are students blending concepts from math and physics mental spaces to make use of the mathematical function in introductory physics courses?

Cognitive Blending

- A theoretical framework of knowledge developed by Fauconnier and Turner [1]
- Knowledge is grouped into resources that are activated together when needed
- New knowledge is constructed by *blending* existing resource groups, or *mental spaces*
- The blended space is supported by the *organizing framework*
- *Single-scope* blends use one of the input mental spaces as their organizing frame
- *Multi-scope* blends have organizing frames that are a combination of the input spaces

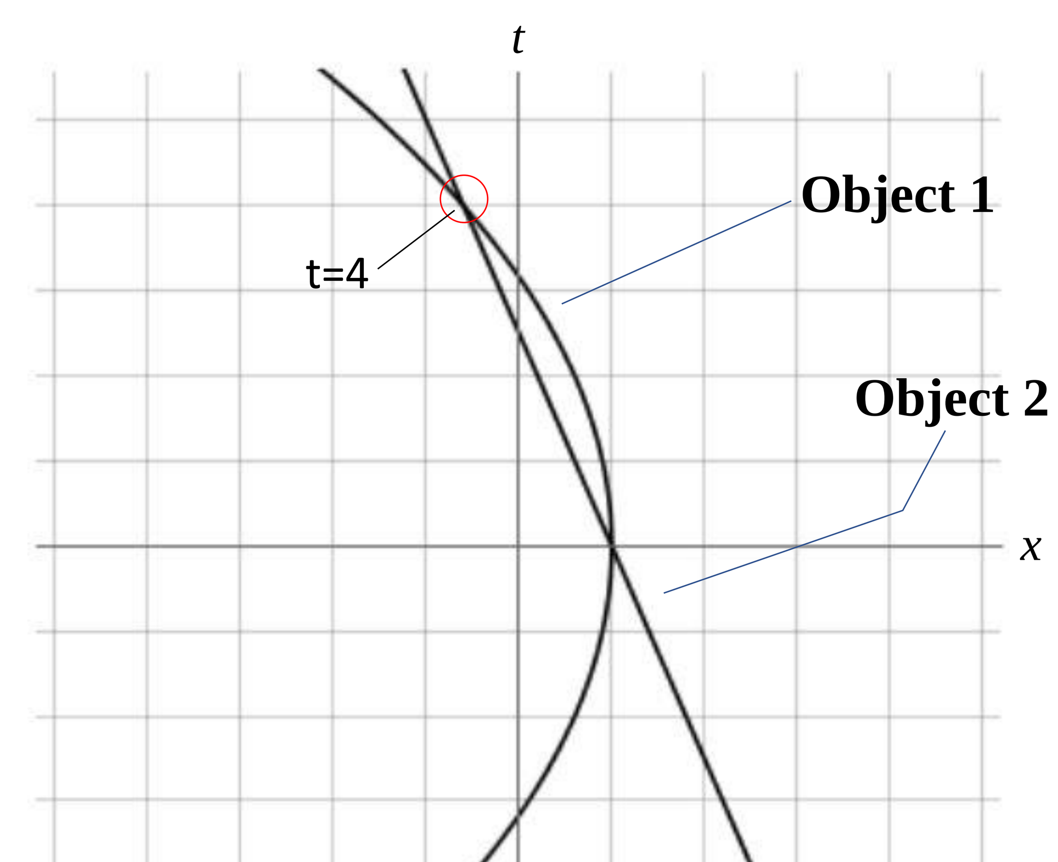
We envision that students would construct an expert blend while taking introductory physics courses. Our ideal expert blend inputs are shown to the right. Each number in the mental spaces corresponds with an analogous element in the other spaces which are blended when problem-solving.

Graphical
1. Height of graph
2. Slope of graph
3. Slope at zero
4. Vertical line test

Symbolic
1. $f(x)$
2. $f'(x)$
3. $f'(0)$
4. One-to-one correspondence

Physical
1. Position $x(t)$
2. Instantaneous velocity $v(t)$
3. Initial velocity
4. One place at a time

Blended Space
1. The position of the object is the height of the graph of the function $f(x)$
2. The velocity is the derivative of the function which is the slope of the graph
3. The initial velocity is the slope of $f(x)$ at $x = 0$
4. A vertical line test shows a position vs. time function has one output per input and the object is not two places at a time



The Interview Tasks

1. Math tasks adapted from a Research in Undergrad Math Education (RUME) paper probing understanding of a graphical representation of a function and its derivative [2]

2. Physics tasks adapted from a kinematics exam consisting of finding quantities from a graph of a position vs. time function

3. Same set of tasks as part 2, but using spacetime convention which was chosen because of previous RUME research [3]

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Student Reasoning Examples

These next examples show the color-coding method used to identify mental spaces.

The Horizontal Line Test

The question: Can Object 1 (shown right) be described using a position vs. time function?

Correct answer: Yes. Both objects can be correctly represented using a function x(t).

S1: No. Because it breaks a vertical line test… Where function is one to one… one x for one y… It would fail because the x value would have 2 y's… The t values would have 2 x's… The object would be at 2 places at the same time.

S2: It can't be a function if it doesn't pass the vertical line test… if it's f of x, the f would have to be up here and then the x would be here. So if it's x of t, then the x would have to be up here, the t would have to be here… If you can't move the axises or like rotate it or anything, then... Let me think. I think you can. ...you would draw like the vertical line tests this way [draws horizontal lines], *and then it would pass it...like the vertical line test just shows...there's two inputs for the same output.*

Comparison of Object Speeds

The question: How do the speeds of Object 1 and Object 2 (shown right) compare at t=4?

Correct answer: Object 1 is moving faster as it has a more gradual slope and is travelling more meters in the same amount of time based off of the graphical information.

S3:Object 2 is going faster than object 1… Cuz the slope is steeper.. Yeah, the linear line is going faster than object 1.

S2:Umm… I think that object 1's speed would be faster… Because it looks like the slope is steeper.

Preliminary Findings

- Students with single-scope blends struggled more with spacetime convention
- Evidence that transition between $y(x)$ and $x(t)$ are not trivial and cannot be taken for granted
- Evidence of a fourth input space–a pictorial space–that students utilize instead of graphical space

Future Work

- Revise interview protocol to examine whether question ordering primed student thinking
- Conduct more interviews to check consistency with current data and consistency with single-scope and double-scope classifications of students
- Investigate further into pictorial input space as part of student blends

Analysis

Students S1 and S3 were classified as single-scope blends based on the totality of their interview responses, while S2 was classified as multi-scope. When presented with the change in axes in the spacetime question, students with single-scope blends using the graphical space as the organizing frame were more prone to disequilibrium than multi-scope blend students.

The Horizontal Line Test

In previous interview tasks, *vertical line test* was analogous to *one-to one*, but for spacetime convention, it's incorrect

S1:	• Misinterprets graph as implying that *t* values have multiple *x* values • Incorrectly blends *vertical line test*, *one-to-one*, and *2 places at a time*
S2:	• Initially, she is against having the axes switched • Changes direction of *vertical line test* to be *horizontal line test* • Correctly blends *vertical line test* with *one output for 2 separate inputs*

Comparison of Object Speeds

In previous interview tasks, a blend between *steeper slope* and *greater speed* was correct, but for the spacetime convention, it's incorrect

S3:	• Incorrectly blends *steeper slope* and *faster*, ending up with an incorrect answer
S2:	• States that *steeper* is analogous to *faster*, just as in previous interview tasks, but concludes that object 1 is moving faster rather than object 2 • Changes her definition of *steeper* to mean *moving more distance per second* even though in literal terms, the actual slope is more gradual

Resources

[1] G. Fauconnier and M. Turner, *The Way We Think: Conceptual Blending and the Mind's Hidden Complexities*, (Perseus Books Group, New York, 2002) Vol. 1.
[2] M. Asiala, J. Cottrill, E. Dubinsky and K. E. Schwingendorf, *The Development of Students' Graphical Understanding of the Derivative* J Math Beh **16**, 4 (1997) 399-431.
[3] K. C. Moore and T. Paoletti, in *Proc XVIII Ann Conf on RUME, 2015*, (West Virginia University, Pittsburgh, 2015), 774-781.
[4] R. Beichner, *Testing student interpretation of kinematics graphs* Am J Phys, **62**, 750 (1994) 750-762.
[5] T. J. Bing and E. F. Redish, "The Cognitive Blending of Mathematics and Physics Knowledge," in *2007 Phys. Educ. Res. Conf. Proc.*, (AIP Conference Proceedings 951, 2007) 26-29.