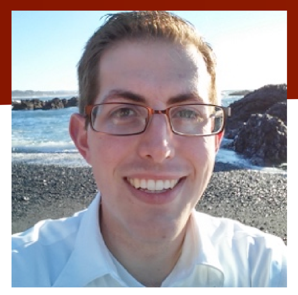
How Students' Understanding of Normalization Changes After Taking Quantum Mechanics



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Introduction

Normalization of kets, wave functions, and vectors that mathematically represent quantum states is particularly important due to the probabilistic nature of quantum mechanics. The goal of my research is to examine students' understanding of normalization as they enter and leave a quantum mechanics course.

Research Questions

1. What understandings of normalization do students have at the beginning of a quantum mechanics course?
2. How do these initial understandings of normalization differ from the understandings of students at the end of studying Quantum Mechanical Spin?

Development of the Framework

- A *conceptual analysis* (von Glasersfeld, 1995) or "a detailed description of what is involved in knowing a particular (mathematical) concept" (Lockwood, 2013, p. 252) was used in developing the framework.

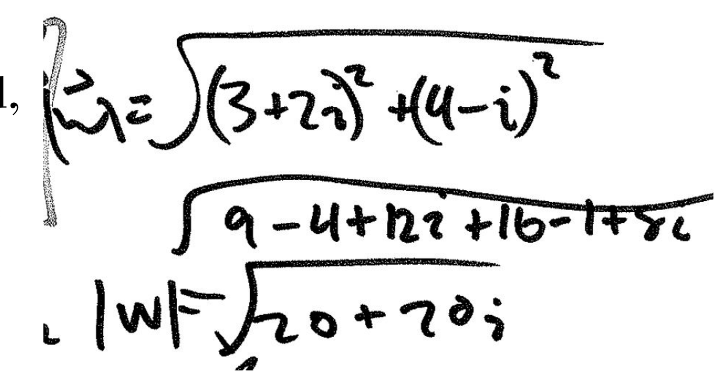
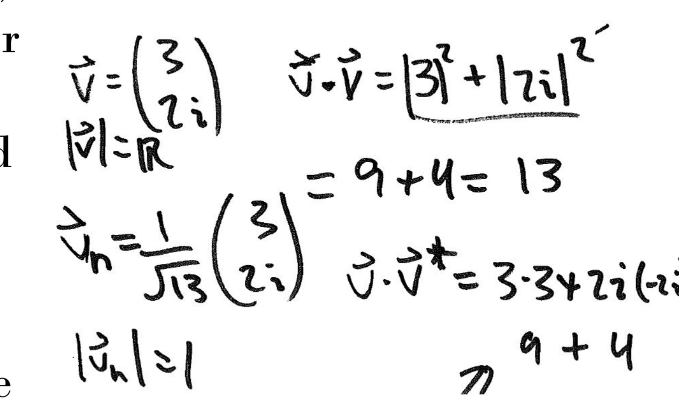
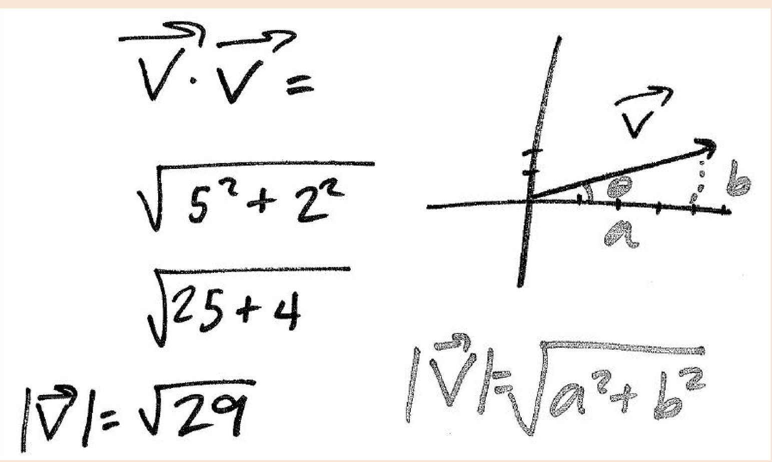
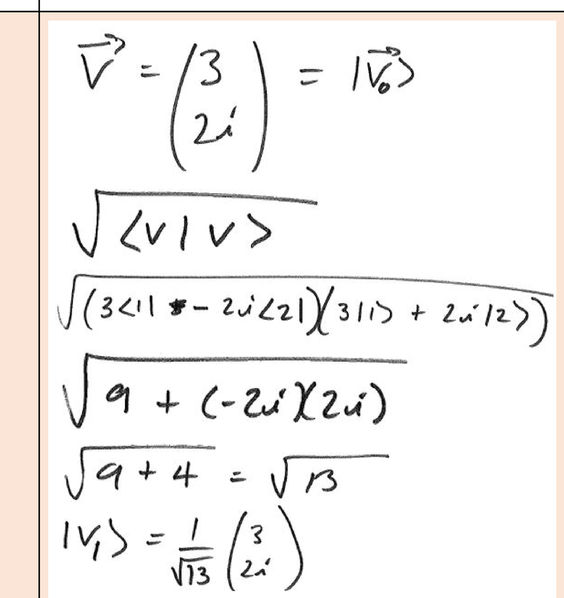
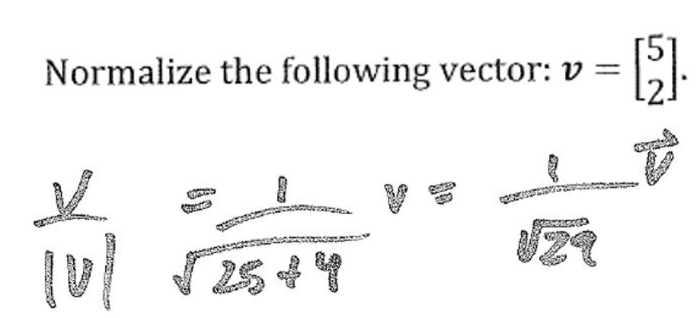
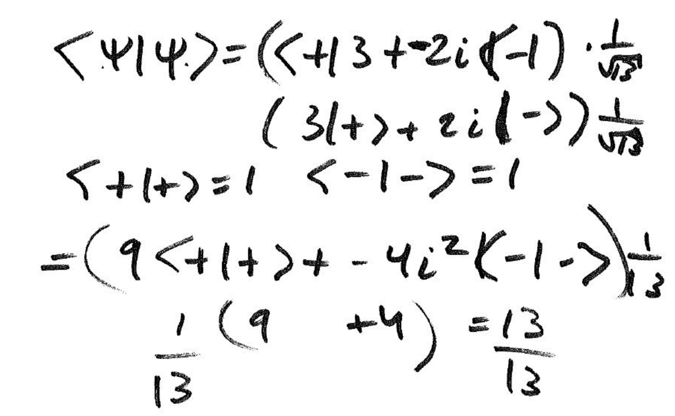
		Students' Understanding
Vectors	Vector Space What is a vector space? Examples of vector spaces	
	Vector Representations	
Norm	Norms: What is a norm? Examples of norms	
	Procedure(s) to Find Norm	
Normalizing	Metaphors for Normalizing	
	Procedure(s) to Normalize	
Normalized Vectors	Properties of Normalized Vectors	
	Reasons for Normalized Vectors	

Methods

- Hour-long, video-recorded, semi-structured interviews with physics students from a university in the northwestern United States
  - Nine were interviewed at the beginning of a junior-level quantum mechanics course
  - Eight were interviewed at the end of a three-week unit on Spin (of which six had participated in the earlier interview)
  - Asked questions about several linear algebra concepts relevant to quantum mechanics. This research only focuses on questions where students were asked to normalize vectors.
- Each students' understanding was summarized by filling out the framework pictured above.
- Patterns were found by looking across students.

Result Highlights

Student Examples

Result Highlights	Beginning of Quantum	After Studying Quantum Spin
Familiarity with Complex Vectors • In the pre-quantum interviews, only two of the nine students were able to normalize a complex vector, and several explicitly mentioned never having seen complex vectors before. • After studying quantum spin, six of the eight students were able to correctly normalize a complex vector.	**Doug**: So, you do three plus two-i squared, uh, plus four minus i squared, square root of that. That's going to equal the w magnitude. … . <u>I haven't dealt with any complex vectors.</u> It's the same arithmetic though. 	**Doug**: So, v squared, like, v dot v dot v, basically for me is … <u>because now, we know how to do that</u>, and that's 9 plus 4 and that's going to be 13. So, you're going to have v normalized, is going to be 1 over root 13, 3 2i. 
Representation of Vectors • In the pre-quantum interviews, students mainly chose to represent vectors in three ways: matrix notation, algebraic vector notation (i.e. $\vec{v}$), and graphically as a directed arrow from the origin. • In the post-quantum spin interviews, some students explicitly chose Dirac Notation for their calculations, and even why they might prefer it.	**Danielle** 	 **Danielle**: So, when [we] were first taught to do this, you're just like, well just leave the i behind [speaking about alternatively calculating $\sqrt{3^2+2^2}$] … I like this way a little bit better just because it treats an i as a number, and it comes along, right, you- you don't just leave it.
Confusion Between Norms and Inner Products • In the pre-quantum interviews, all nine students knew at least one way to find the norm of a real vector, most often using the square root of the sum of the squares of the components. • In the post-quantum spin interviews, four of the eight students showed some possible conflation between the norm of a vector and the inner product of a vector with itself.	**David:** So, when I think of normalization, I think of, um, taking it's magnitude to be equal to one. So, I'm just going to divide v by the magnitude of v …  And, I just squared both components, and then took the square root because that's how you get the magnitude of something. <u>Really, it's v over v-dot-v.</u> Um, but yeah. So, that would be root 29, and vector-v. And that would have magnitude of one.	**David**: … <u>Which is 13. And that's the magnitude of the vector.</u> So, if we want to get rid of that, then, we want to divide by root 13. Because, we have two vectors in here.  So, you can imagine, if we had put a root 13 over here and a root 13 over here, that's going to equal to a one over 13 out front of the entire thing. And then you get 13 over 13.
Reasons for Normalization • Most students in the pre-quantum interviews explained that normalization makes it so you have a vector with a length or magnitude of one, and several mentioned that normalization "gets rid of" the magnitude, leaving you only with direction. • This understanding of normalization was still prevalent in the post-quantum spin interviews; however, two students did explain normalization's importance to quantum.	**Doug**: … So, you're shrinking it down to that to get just the direction. … **Damian**: … I guess, basically what we're doing is we're just cutting down the length to a unit of one. Um, so all we're worrying about here is the direction. … **Drake**: … Which is useful to, like, for, like, you, if you only want the direction of the vector, not the magnitude. …	**David**: And that's the magnitude of the vector. <u>So, if we want to get rid of that</u>, then, we want to divide by root thirteen. **Doug**: … you need to normalize so that way when you're summing up other things, you're getting a probability of 1, otherwise the probability is not going to make any sense. **Doran**: OH YEAH! I think we are reserving [quantum] states to be unit vectors.

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