Algebra-Based Physics Students & Vectors : Assessing Physical Understanding in Arrow vs. *ijk*

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Introduction

- Students in a calc-based intro mechanics course have been found to perform better on vector operations in the *ijk* format over the arrow format [1], similar results were found for students in an algebra-based course [2]
- Heckler *et al.* also found evidence that working knowledge of the *ijk* format was necessary to correctly perform vector operations in the arrow format [1]
- We wanted to test if the higher *ijk* scores for the algebra-based students represented better reasoning, or just "plug-and-chug" solving

Methods

- Students enrolled in algebra-based introductory mechanics
- Administered in last week of class, credit for completion, required, opt-out
- Hand-written, free-response, 10 minutes in-class
- Students randomly given Arrow ($N = 130$) or *ijk* ($N = 126$) representation
- All students asked to solve a 2D subtraction question in a physics context, and assess if the direction of the resulting vector made physical sense
- No explicit *ijk* instruction given during the semester

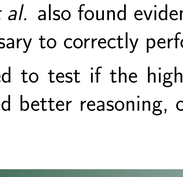
Question Prompt - Arrow (*ijk*)

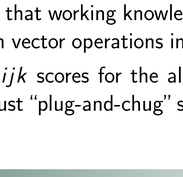
From Newton's Laws we know that **the average net force on an object $\vec{F}_{avg}$ is pointed in the direction of the change in momentum $\Delta\vec{p}$:**

$$\vec{F}_{avg} = \frac{\Delta\vec{p}}{\Delta t} = \frac{\vec{p}_f - \vec{p}_i}{\Delta t}$$

Consider an object with initial momentum $\vec{p}_i$ and final momentum $\vec{p}_f$, as shown below in standard SI units for momentum: one square equals 1 kg m/s. The time interval is $\Delta t = t_f - t_i = 1\text{s}$.

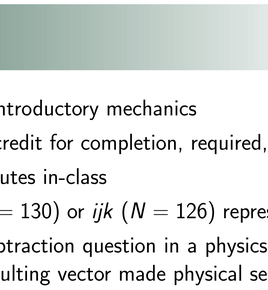
$(\vec{p}_i = 3\hat{i} + 3\hat{j})$





$(\vec{p}_f = -4\hat{i} + 3\hat{j})$

What is the average force on the object during the time Δt? Draw your anaswer on the grid to the right (which is in standard SI units for force: one square is 1 N)



Does the direction of $\vec{F}_{avg}$ make sense physically? Briefly explain

Fig. 1: Arrow (*ijk*) version of prompt. The *ijk* representations were only shown on the *ijk* version of the prompt, with no grids or arrows. They are combined here to save space.

Results & Discussion

	Correct?		Reasoning?	
	Y	N	Y	N
ijk ($N = 126$)	83 (66%)	43 (34%)	55 (44%)	71 (56%)
Arrow ($N = 130$)	32 (25%)	98 (75%)	52 (40%)	78 (60%)

- Scores were significantly higher on *ijk* tasks than arrow tasks [$t(254) = 7.26, p < 1 \times 10^{-11}, d = 0.9$]
- The number of correct responses was lower than the corresponding multiple choice task given in a previous semester (*ijk* : 76%, arrow : 59%)
- The number of students with partially correct reasoning was identical on both arrow and *ijk* [$\chi^2(1) = 0.35, p = 0.55$]

	Reasoning in Correct Subset		Physical Reasoning & Correct Vector
	Y	N	
ijk	($N = 83$) 43 (52%)	40 (48%)	($N = 126$) 43 (34%)
Arrow	($N = 32$) 22 (69%)	10 (31%)	($N = 130$) 22 (17%)

- The number of students who correctly found the answer & used correct reasoning is higher in *ijk* [$\chi^2(1) = 10.00, p = 0.0016$]
- In the subset of students correctly computing the vector, the number using correct physical reasoning is slightly higher, but not statistically significant [$\chi^2(1) = 2.70, p = 0.10, d = 0.34$]
- Heckler found that percent of students able to physically reason about their answer was slightly higher in correct *ijk* responses than correct arrow responses [1], our results show the opposite

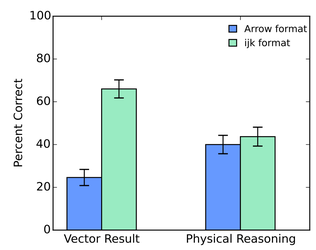


Fig. 2: Scores are higher on *ijk*, physical reasoning used equally. Errors are ± 1 SE.

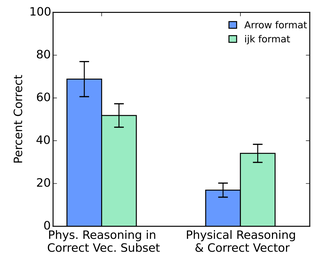


Fig. 3: Correct arrow answers slightly more likely to use reasoning, *ijk* responses more likely to have correct answer & reasoning. Errors are ± 1 SE.

Rep.	$\vec{F}_{avg}$?	Responses counted as at least "partially correct"
ijk	Y	"Yes because the final momentum is in the negative direction"
ijk	Y	"Yes, if the change in momentum is to the left, since the $\vec{F}_{avg}$ is negative. If it was to the right, $\vec{F}_{avg}$ would be positive"
ijk	N	"Yes because the second momentum kills all y-direction force so it only moves in the x-direction"
ijk	N	"No, the object would be going in the opposite direction it started in so it would make sense that the force would be in the negative direction as well."
Arr	Y	"Yes, the two vertical forces cancel and you are left with a negative force in the horiz. direction."
Arr	Y	"Yes, the initial momentum is pointing upward & to the right, in order to cause the momentum to be upward & left, there should be a force acting to the left that is 7 kg m/s."
Arr	N	"It does not, because I would think that it would be a negative force since the vector changes direction?"
Arr	N	"Yes its final momentum was greater in the x direction in the negative than its initial so the object wanted to go back"

- The most common incorrect response in the arrow format was adding the vectors rather than subtracting ($N = 33, 25\%$)
- The most common incorrect response in the *ijk* format was to *ignore* the unit vectors altogether ($N = 12, 10\%$).
- Other common responses included only subtracting the x-component, or computing $(\vec{p}_i - \vec{p}_f)$, possibly since $\vec{p}_i$ is on the left side of the page

Discussion - cont.

- Students in this class were not given explicit *ijk* instruction, leading many with the correct answer to explicitly say they could not interpret it:
- "Unknown, since I am unable to define $\hat{i}$
- "No because I have no idea what the $\hat{i}$ & $\hat{j}$ means, therefore $-7\hat{i}/1\text{s}$ doesn't make any sense as an answer"
- While these responses support the "plug-and-chug" hypothesis, they indicate that the relatively low rate of physical reasoning among *ijk* responses might be improved with *ijk* instruction

Conclusions

- Students in an algebra-based introductory physics class performed better on a 2D vector subtraction problem in a physics context given in the *ijk* format rather than the arrow format
- The rate of correct responses in this free-response question was less than the analogous multiple choice question given in an earlier semester
- The number of students using partially (or fully) correct physical reasoning was the same in each format
- Within the subset of "correct answers" the percent of students using physical reasoning was slightly higher in the arrow format, though the total number of students correctly computing & reasoning was significantly higher in *ijk*
- Many students were explicit about not knowing what $\hat{i}$ & $\hat{j}$ were, suggesting that *ijk* instruction could improve their ability to interpret their answers
- The "plug-and-chug" reflex of students using *ijk* might be used to strengthen student ability in the arrow format: studies testing transfer between the two formats should be done

References

[1] A.F. Heckler and T.M. Scaife, Phys. Rev. ST Phys. Educ. Res. **11**, 010101 (2015).

[2] J.B. Buncher, presented at PERC 2015, College Park, MD, pp. 75-78, doi:10.1119/perc.2015.pr.014

[3] H.D. Young, P.W. Adams and R.J. Chastain, *College Physics, 10th Ed.* (Pearson, 2016); D.C. Giancoli, *Physics: Principles with Applications 7th Ed.*, (Pearson, 2014); R. Knight, B. Jones and S. Field, *College Physics 2nd Ed.* (Pearson, 2010); N. Giordano *College Physics 2nd Ed.*, (Brooks/Cole Cengage, 2013).

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