

Student Difficulties with Trigonometric Vector Components Persist in Multiple Populations



**Physics Education
Research Group**

Brendon D. Mikula & Andrew F. Heckler
Department of Physics,
The Ohio State University, Columbus, OH, 43210



Introduction

An integral step in many physics problems is the application of trigonometric functions to analyze vectors in terms of perpendicular components. The skill itself is fundamental and essential knowledge for introductory physics. As such, it is imperative that students are as close to 100% accuracy as possible at trigonometric vector decomposition. However, as we will show in this study, a significant number of students have specific difficulties even with the simplest of trigonometric vector problems.

Existing Research

Previous studies on student understanding of vector components and decomposition have found significant student difficulties. For example, Van Deventer and Wittmann found that only 54% of students in a large public U.S. university were able to select the correct magnitude of a trigonometric vector component given multiple-choice options to choose from.¹

In a related study about components of vectors on a grid, as opposed to finding components from trig functions, Zavalia and Barniol showed that only 67% and 80% of introductory physics students at a private Mexican university were able to correctly choose the x and y components of a vector presented on a grid.²

Aguirre and Rankin found that only about 50% of the students understood the vector nature on velocity in terms of orthogonal components.³ Finally, Knight found that less than half of students could calculate the magnitude and direction (i.e. angle from x -axis) of a vector presented on a grid or in i - j format.⁴

Participants and Design

Data was collected after relevant instruction over three terms from four introductory physics student populations:

- Algebra-based mechanics (N = 636)
- Algebra-based E&M (N = 168)
- Calculus-based mechanics (N = 229)
- Calculus-based E&M (N = 188)

Data is presented for all four populations.

In order to study the effects of angle placement on student aptitude with trigonometric vector components, we constructed a task consisting of three problems (Fig. 1). Participants were assigned to one of four conditions, each of which consisted of two "simple" problems and one inclined plane problem. The specific questions in each condition are shown in Table 1.

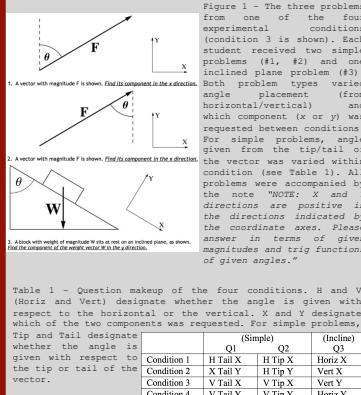


Table 1 - Question makeup of the four conditions. H (Horiz and Vert) designate whether the angle is given with respect to the horizontal or vertical axis, and the direction of which the two components were requested. For simple problems, Tip and Tail designate whether the angle is given with respect to the tip or tail of the vector. For inclined plane problems, Q1 and Q2 designate whether the angle indicated was correct ($\chi^2 = 24.3, 20.0; p < 0.001, 0.001; d = 0.36, 0.47$, respectively). This provides evidence that the act of marking up the angles on the provided question diagram improves accuracy.

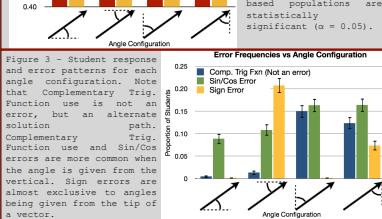
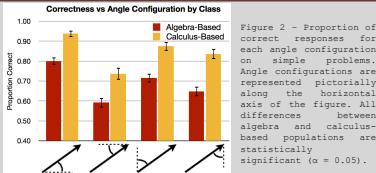
Coding of Student Responses

Each problem was coded for correctness as well as for presence of specific student response patterns and errors. A list of descriptions for each coding category follows:

- **Correct:** Student response is identical or mathematically equivalent to the correct answer.
- **Complementary Trig Function:** The trigonometric function included in the student response is correct, even if other errors—such as a stray negative sign—were present.
- **Complementary Trig Function:** Complementary trig function is used, e.g. $F \cos(90^\circ - \theta)$ instead of $F \sin(\theta)$.
- **Sin/Cos Error:** Student switches sine and cosine in their response, effectively providing the answer for the wrong component.
- **Sign Error:** Student response differs from the correct response in terms of a plus or minus sign.
- **Wrong Triangle Error** (only for inclined plane): Student answers based on a triangle in which the weight vector is not the hypotenuse. This error yields answers containing $\tan(\theta)$, $\cot(\theta)$, $\sec(\theta)$, and $\csc(\theta)$.
- **Wrong Angle Error** (only for inclined plane): When students generated angles on the question diagram, the geometric correctness of the angle marked was noted. Not all students indicated an angle on the question diagram, so results for this error are presented separately.

Students submitting a gradable response to any one of the three questions were graded for all three questions, regardless of completion. Blank and ungradable responses recorded as incorrect, but are not included in the response and error pattern analysis.

When students submitted an incomplete solution (e.g., $\cos(\theta) = y/F$), the lack of completion was noted and the response was carried to its solution (e.g., $y = F \cos(\theta)$) before coding. Problem completion data is not included.



Results (Simple Problems)

No differences were found between x and y components for the first two "simple" problems, so this category was collapsed during analysis.

Figure 2 presents the score for the four angle configurations for the simple problems. Students performed best (80–90% correct) when the angle was given from the tail of the vector with respect to the horizontal. Accuracy was lowest (60–70%) when the angle was given with respect to the horizontal from the tip of the vector and intermediate when the angle was given with respect to the vertical. For each angle configuration, students in the calculus-based sequence outperformed their algebra-based counterparts. It is worth noting that these all represent post-instruction results. Given the fundamental nature of this simple task, the distances from ceiling are troubling.

Specific student response and error patterns also showed dependence on angle configuration (Fig. 3). The primary errors were sine/cosine confusion and a sign error. Students used complementary trig functions more frequently when the angle was given from vertical, suggesting that students are most familiar with angles given from horizontal and attempt to transform them into this familiar form.

A 2-way ANOVA (horizontal-vertical vs. tip-tail) revealed that Sin/Cos errors occurred more frequently for the vertical angle ($F(1) = 6.914, p < 0.001$). This suggests that students may not be familiar with both angle configurations and may need more practice on problems that draw attention to the contrast. No other interaction or main effects were significant for the Sin/Cos error.

Sign errors were committed almost exclusively when the angle was given from the tip of the vector ($F(1) = 193.1, p < 0.001$). This suggests that the angle configurations may be the more salient cue for vector component direction with some students than the direction of the arrow itself. Paraphrasing, when given an angle from the tip of the vector, students seem to be treating the vector as though it were in the third quadrant.

Results (Inclined Plane Problems)

For inclined plane problems, students performed best when the x component was requested, regardless of whether the angle was given with respect to the horizontal or the vertical (Fig. 4). However, even the best scores in the highest-performing populations are barely above 50%; student responses to y component problems are about 10% correct or worse.

Figure 5 presents specific student response patterns. Sign errors occurred frequently and almost exclusively when the y component was requested, accounting for almost all of the score difference between the two components. This error could be aggravated by the fact that sign errors on x component problems represented the erroneous inclusion of a negative sign, where sign errors on y component problems represented a missing negative sign that should have been included.

Students found the correct trig. Function approximately the same proportion of the time, regardless of which component was requested. However, a 2-way ANOVA (x -y vs. angle placement) revealed that the correct use of trig. Functions significantly depended on angle placements ($F(1) = 5.234, p = 0.022$), but there was no significant difference between asking for the x or y component.

A similar 2-way ANOVA on Sin/Cos errors also showed significant dependence on angle placement ($F(1) = 6.914, p = 0.009$) with more errors in the vertical placement. This suggests that $\approx 70\%$ of students are actually working through—as opposed to memorizing—the application of the trig functions to these problems. However, a significant proportion of students appear to be confusing sine and cosine or are simply using a memorized algorithm based on the common horizontal placement of the angle. Part of the Sin/Cos error signal could come from students labeling an incorrect angle on the question diagram; this will be discussed in depth below.

Wrong triangle errors, where students answer based on a triangle that contains the weight vector as a leg instead of the hypotenuse, showed no significant dependence on angle placement or component requested. This suggests that a small, but consistent, proportion of students fail to correctly follow the most basic rules of setting up triangles to break down vectors to their components.

Some students indicated given angles on the diagram presented with the question. These were coded for geometrical correctness and consistency with the trig. function in the student's solution. As shown in Table 2, 65% (50%) of calculus (algebra) students drew an angle on the diagram, and 81% (73%) of these students indicated a geometrically correct angle. Correspondingly, 83% (79%) of these students submitted a response consistent with the angle marked, suggesting that students may be slightly better with trigonometry than with geometry.

Notably calculus (algebra) students who indicated and angle on their diagram were about 22% (17%) more likely to choose the correct trig. function that those who indicated no angle, regardless of whether the angle indicated was correct ($\chi^2 = 24.3, 20.0; p < 0.001, 0.001; d = 0.36, 0.47$, respectively). This provides evidence that the act of marking up the angles on the provided question diagram improves accuracy.

Drew an Angle	Algebra (%)	Calculus (%)
Angle Correct (given angle drawn)	50%	65%
Angle Correct (given angle drawn)	73%	81%
Angle/Answer Consistent (given angle drawn)	79%	83%
Correct Trig Function (given NO angle drawn)	44%	49%
Correct Trig Function (given angle drawn)	61%	71%

Table 2 - Summary of student-drawn angles and responses on inclined plane problems split into algebra and calculus-based populations. Asterisks indicate statistical significance at the $\alpha = 0.05$ level.

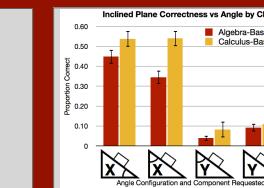


Figure 4 – Proportion of correct responses for each angle configuration on inclined plane problems. Angle configurations are represented pictorially along the horizontal axis of the figure. The only significant difference between algebra- and calculus-based students was that the angle was given from the vertical and the x component was requested ($\alpha = 0.05$).

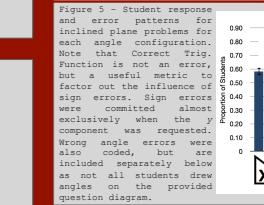


Figure 5 – Student response and error patterns for inclined plane problems for simple angle placement. Note that "Correct Trig. Fxn" is not an error, but a useful metric to filter out the influence of sign errors. Sign errors were committed almost exclusively when the y component was requested. Wrong angle errors were also coded, but are included separately below as not all students drew angles on the provided question diagram.

Conclusions and Implications for Instruction

We found that, even post instruction, student performance is nowhere near the 90–100% accuracy needed for the essential skill of applying trigonometric functions to vector decomposition. Students often confuse sine with cosine when the angle is given from vertical and commit sign errors when the angle is given from the tip of the vector. On inclined plane problems, students very frequently omit necessary signs, confuse sine and cosine, and answer based on incorrectly drawn triangles and incorrectly placed angles. Overall, the results suggest that students often produce answers based on the common, canonical configurations seen in examples (e.g. angles taken from the horizontal axis), regardless of where the reference angle is placed.

While the fact that students struggle with the basic application of trigonometric functions may be familiar to many instructors, this study documents the extent of the problem and carefully characterizes the nature of the difficulties in order to highlight the need for, and suggest avenues to pursue, improving this essential skill. Clearly students need more practice on a variety of angle configurations.

Of special concern are wrong triangle errors, which represent a fundamental failure to understand the concepts of geometry and vector components. Instructional time and student practice spent on vector components—especially for inclined plane problems—should emphasize the correct way to draw triangles to aid in trigonometric vector breakdown.

Lastly, simply attempting to draw an angle on the question diagram—regardless of the geometrical correctness of the angle—is correlated with higher scores. This consequence of marking up the question statement with relevant information could help convince the students of the utility of such practice.

Acknowledgements

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