Excerpts from an exploratory survey of units/dimensional analysis in introductory physics

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Abstract: Proficiency in units/dimensional analysis is a useful skill in the sciences and engineering, and STEM instructors often presume competence from their students in this area. However, unit analysis techniques and unit systems are not always formally taught, and even with explicit emphasis during instruction, many students lack the repeated exposure necessary to master them. To assess student understanding of units and unit systems, we administered a brief, itemized survey to $N = 53$ calculus-based introductory university physics students. The survey was intended to uncover possible misconceptions and identify previously unknown obstacles to student success within this topic. Our results suggest that as many as half of surveyed participants may not recognize the adaptability of symbolic physical equations to different unit systems. Furthermore, a similar proportion failed to eliminate non-viable answer choices involving analytic functions with units inside their arguments. Consequently, we believe units/dimensional analysis is a topic ripe for further investigation.

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I. INTRODUCTION

Introductory physics students often derive much of their prior experience with mathematical symbols and operations from mathematics courses. This can complicate their early encounters with physical quantities represented as mathematical symbols, because physics and mathematics famously use symbols differently [1]. The “metadata” (e.g., units/dimensional analysis) associated with symbols in a physics course is virtually automatic to experts in that field, but may not even be observed, let alone understood, by novices.

Existing research of student understanding and utilization of units/dimensional analysis in the context of introductory physics is surprisingly novel given its near universal applicability. Evidence of units understanding from engineering suggests few students can confidently demonstrate a practical understanding of dimensional analysis, such as the requirement that units must match across both sides of an equation, or that units of an unknown constant must cooperate with the units of other associated quantities in a mathematical expression [2]. Computer-based training with feedback has been shown to improve performance for some of these skills (including recognition of units within analytic functions), although results vary across individual competencies [3]. An exploratory study designed specifically to gauge proficiency and understanding of units and dimensional analysis among introductory physics found that few students could correctly identify the units of an unknown physical symbol in an equation when the units of all other symbols were provided [4]. Furthermore, most students could not identify units of an unknown symbol solely through dimensional consistency. Students rarely demonstrated an ability to use dimensional analysis techniques to better understand physical equations.

Building on these results, we discuss our efforts to better understand student conceptualization of physical units at the introductory physics level, where students are frequently expected to recognize the utility of unit analysis techniques and implement them for the first time. In particular, we present findings of several questions taken from an itemized exploratory survey of dimensional analysis in the context of calculus-based introductory physics. This survey was designed to elicit and identify previously unexamined difficulties and obstacles to student success in this field.

II. METHODS

A total of $N = 53$ students participated in this exploratory study. All students were enrolled in the first semester of the same calculus-based, introductory physics course at Florida Gulf Coast University, a medium-sized public education and research institution. The study took place in late November, at the conclusion of the semester, to provide ample time for students to become more familiar with their physics curriculum. Demographic data obtained during the research study revealed student participants as 74% engineering majors; 100% of participants had taken or were enrolled in Calculus I, and 67% had completed or were enrolled in Calculus II. Non-engineering majors consisted of various STEM fields. Approximately 70% of participants self-identified as male, and 83% self-identified as native English speakers.

All student participants received course credit for their participation in this task during a regularly scheduled lecture session, but they were given the option to decline involvement in the study if they preferred. Over 98% of students in the class elected to participate. Each student participant was instructed to complete the itemized units survey on paper in a quiet testing room for no more than 30 minutes. The itemized units survey was identical for all student participants. It featured ten questions of differing formats (multiple choice, free response, etc.), all of which were designed by the authors to better understand student thinking and/or reasoning about various aspects of units, unit systems, and dimensional analysis. This study represented the first time student participants engaged with this survey.

In this manuscript, we will be restricting our focus to two of these ten questions. These two questions will be referred to as “Unit Systems” and “Units in Analytic Functions.” The question text is illustrated below in Fig. 1 and Fig. 2 exactly as received by the participants in this exploratory study.

Physicists use units and unit systems as external constructs and/or frameworks to translate universal physical relationships into measurable values. The investigatory purpose of item 1, the Unit Systems question shown in Fig. 1, was to ascertain students’ comfort and competence with the alteration of unit systems in the context of a symbolic worked solution to a typical first semester introductory physics problem. Specifically, our study of this item aimed to determine if introductory physics students were aware and could demonstrate an understanding that symbolic algebra involving physical quantities is universally correct when written correctly.

Students were expected to select steps of the worked solution that would require adjustment due to a change in unit system. Although only the final two steps of the solution would have to change to accommodate U.S. Customary

1. **Check all steps in this conservation of energy problem that would be different if US customary units were used instead of SI units for the values in this problem.**

___ $\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mg\Delta y = 0$

___ $\frac{1}{2}v_f^2 = -g\Delta y$

___ $v_f = \sqrt{-2g\Delta y}$

___ $v_f = \sqrt{-2\left(9.8\,\frac{m}{s^2}\right)(-3\mathrm{m})}$

___ $v_f = 7.7\,\frac{m}{s}$

FIG. 1. Unit Systems question from the itemized units survey.

9. Four students are solving for a distance, x, in a physics problem, but all four found different answers. Check all answers that could possibly be correct, if any, based on the units of the answer.

___ $x = 23m\sin\left(11\frac{m}{s} - 8\frac{m}{s}\right)$

___ $x = 23m\sin\left(11\frac{m}{s} + 8\frac{m}{s}\right)$

___ $x = 23m\sin\left(\frac{11\frac{m}{s}}{8\frac{m}{s}}\right)$

___ $x = 23m\sin\left(11\frac{m}{s} \cdot 8\frac{m}{s}\right)$

If any weren't possibly correct, briefly explain why not.

FIG. 2. Units in Analytic Functions question from the itemized units survey.

units, any or all steps could have been selected by our participants. If students do not view symbolic equations in physics as being valid in other unit systems, they may select more steps in the worked solution than the last two alone.

Item 9, known here as the Units in Analytic Functions question, provided participants with four choices for possible answers to a hypothetical physics problem, all ostensibly representing "distances" measured in meters (see Fig. 2). This survey item sought to gauge students' recognition that only analytic functions (specifically, functions such as trigonometric or exponential functions) with unitless arguments could represent the correct answer. The arguments within each function differed only by the operation used to relate the two values contained therein. The third choice correctly features a plausible solution with a ratio of speeds in its argument, resulting in an interior cancellation of dimensions/units. If students are not aware that unitless arguments are required of certain physical analytic functions in order to have meaning, they may see no fault in any of the similarly styled answer choices.

To score these items simply and efficiently as "correct" or "incorrect," we awarded no partial credit. Students who selected only the correct steps and/or answers and nothing else were deemed "correct." All other combinations of responses were coded as wrong. This scoring method formed the basis for our results discussed in the following section.

III. RESULTS & DISCUSSION

A. Unit systems

The Unit Systems question assessed students' understanding of the implementation of differing unit systems within physical equations. Strikingly, our results show that only 45% (24/53) of respondents were able to demonstrate clear understanding of this question. The most common incorrect answer, given by 21% (11/53) of participants was to mark all steps as requiring adjustment to be valid in a different unit system. Symbolic expressions of physical relationships are true as written regardless of the unit system to be employed, provided a single unit system is used consistently ($e.g., KE = \frac{1}{2}mv^2$ is universally correct, irrespective of the preferred system of measurements). However, these students gave responses that imply that the methods for solving problems are specific to certain systems of units. In other words, at least some changes would be required to a symbolic equation to account for translation to a different unit system. This observation may be related to the tendency for some students to feel discomfort with the use of symbolic representations, insisting on replacing variables with numbers as quickly as possible in the process of solving a problem. If students immediately resort to "plugging in numbers," they may argue that every step in a solution "must change" to accommodate different unit systems.

Other notable patterns of incorrect responses included the complete reversal of correct versus incorrect selections. Students in this subset incorrectly selected all steps of the worked solution that required *no* adjustment to account for a different unit system, while leaving unchecked precisely those steps that did. This was observed in 13% (7/53) of all participants. This could be an indication that they knowingly checked off all the steps that would *not* have required alteration to a new unit system. In short, it is plausible that these students were conceptually strong, but simply misread (and reversed the directions of) the instructional prompt.

B. Units in analytic functions

The Units in Analytic Functions question tested students' understanding of the requirement that trigonometric expressions have unitless arguments. We observed that only 36% (19/53) of respondents answered this question correctly. Students who answered this question incorrectly often commented that the units *within* a trigonometric function's argument would change the units of the overall expression - outside the function. For example, one student believed that all the options were incorrect, stating, "*Since, in the parentheses* [referring to the expression within the sine function]*, the 'm/s' cannot be cancelled out, the distance* [cannot] *be solved for correctly. One would be stuck with meters and seconds.*" We commonly observed written responses consistent with the idea that the units of the expression must be in meters, but also congruent with the belief that the units within a trigonometric function would affect the units of the expression overall.

Alternatively, some students gave the correct answer, but provided incorrect explanations. One such student wrote, "*For option 3* [the correct option]*, m/s divided by itself removes the unit title, which is appropriate as the 'm' from 23 meters makes it in meters. The rest do NOT stay in meters, as 1-2 are in* m^2/s *and 4 is in* m^3/s^2." Once again, this response aligns with the notion that the units *inside* the trigonometric function affect the units of the final answer.

C. Relationship to final course grade

Of particular interest in assessing participants' performance on our units/dimensional analysis survey was the question of potential correlations to their final grade in their introductory physics course. Final course grade (from 0 to 100) serves as one measure of overall student success in physics.

To assess this effect, as described in Section II, we rated student responses to the Unit Systems and Units in Analytic Functions items with a binary "0" or "1" score representing incorrect or correct, respectively. Final course grade information obtained from the instructor revealed a mean of 72% with a standard deviation of 13% ($N = 52$). Our final course grade correlation results are summarized in Table I.

TABLE I. Correlations with associated significance between individual survey questions and physics final course grade.

Survey Item	Correlation with Physics Final Course Grade	Significance
Unit Systems	$R = 0.02$	$p = 0.90$
Units in Analytic Functions	$R = 0.31^{*}$	$p = 0.02$

Our survey item assessing participants' ability to recognize appropriate changes in unit systems showed effectively *no* relationship with their final course grade outcomes. Specifically, with a significance level of $p = 0.9$, our data suggest that there is a 90% chance of obtaining a correlation as strong as observed by random chance alone. We believe that this is a skill with which instructors may expect students to already be familiar, but one that is also very rarely tested in a standard introductory physics course. Persistent use of SI units may leave students unaware of how to transport their learning to alternative unit systems. How often are non-SI unit systems even incorporated into traditional physics pedagogy? It may be sufficiently rare as to make the recognition of when and how to translate physical symbols into numerical values with non-SI units seem an unnecessary ability. Students may simply not know - and may never need to know - based on the priorities set by their physics instructor. Regardless of whether or not this is a skill that some students will need (*e.g.*, in future engineering courses that employ Imperial or U.S. Customary unit systems), our results show no connection to their ability to succeed in the undergraduate introductory physics course in which they were enrolled.

As measured by our survey, recognition that trigonometric functions must have unitless arguments shows small to medium correlation to final course grade. We observed $R = 0.31$, which was statistically significant at the $\alpha = 0.05$ level with $N = 52$ participants. In particular, a

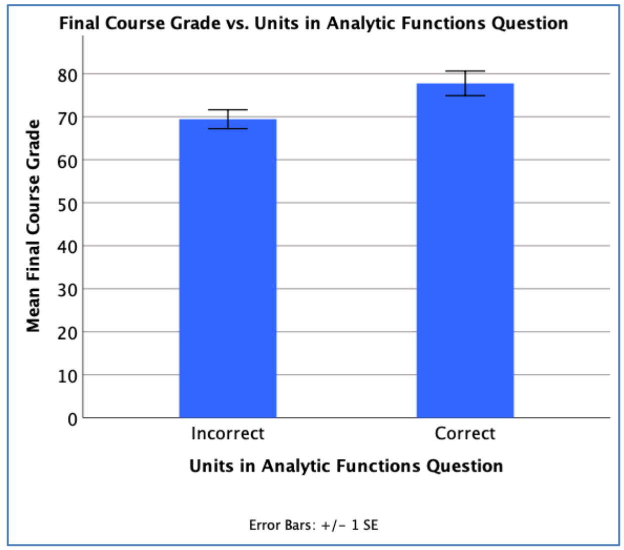

FIG. 3. Final Course Grade versus Units in Analytic Functions survey question response. Error bars are ±1 SE.

one-way ANOVA found a significant effect of Units in Analytic Functions on final course grade [$F(1,51) = 5.250$, $p = 0.026$]. As shown in Fig. 3, students who correctly identified the analytic function featuring an argument without units (while neglecting those functions whose units did not cancel inside the argument) had significantly higher final course grades on average than their peers who did not correctly answer this survey item. The size of this effect, measured by Cohen's d, was found to be $d = 0.7$ – a medium to large effect. Furthermore, we observed that students who answered this question correctly were significantly more likely to obtain a final course grade above the median than students who did not (15 of 19 vs. 11 of 34, respectively; $\chi^2(1,53) = 10.6, p = 0.001$; see Fig. 4).

Unlike alternative unit systems and their appropriate implementations in equations featuring physical symbols,

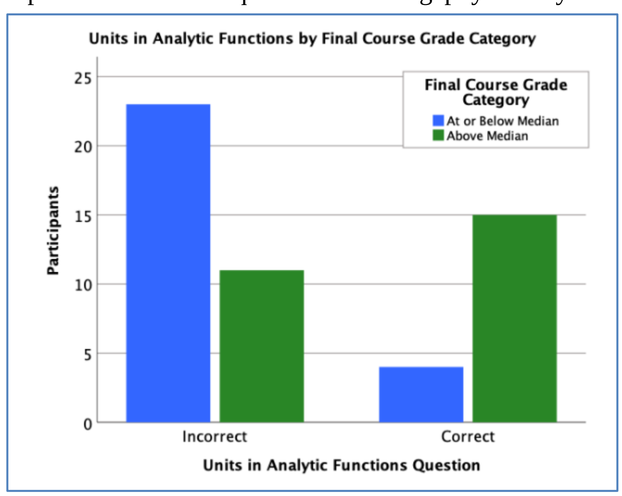

FIG. 4. Participants responses on the Units in Analytic Functions survey question, split by the median final course grade.

understanding the dimensions of an analytic function and its argument may be much more applicable to a standard introductory physics curriculum. The utility of this comprehension extends beyond simply recognizing algebraic mistakes in action; it can also offer powerful insight into physical relationships [5]. Students who possess this key may unlock conceptual doors that remain sealed to others, which in turn, could yield better overall performance in their physics courses.

IV. CONCLUSION

In mathematics instruction during high school and early college, abstraction is often emphasized [1]. As a result, STEM students frequently encounter their first heavy dose of units and dimensional analysis in introductory physics. Our exploratory suggests that a better understanding of units and dimensional analysis may offer qualified benefits for students in undergraduate introductory physics. Physics instructors may help bridge conceptual gaps in problem solving by clarifying how units are to be viewed within an expression, and especially within analytic functions. Given the observed lack of student understanding of unit analysis in general, and among analytic functions in particular, future investigations should also continue to seek more detailed examination of student understanding of units in exponentials, as this becomes more vital in upper-division STEM courses.

Due to its inherently exploratory design, this particular study stops short of claiming that improving students' understanding of unitless arguments in analytic functions directly benefits their overall physics course grade, although a reasonable hypothesis supports further inquiry. Are students trained in dimensional analysis, specifically pertaining to analytic functions such as trigonometric or exponential functions, better equipped to succeed in a physics course than those who aren't? Alternatively, high performing physics students may simply be more likely to already employ dimensional analysis techniques in problem solving. Regardless, we believe that an apparent dearth of existing literature more closely examining this subject deserves attention.

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