Models of Math Use in Non-academic Workplace Settings

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It is important to develop models about how mathematics is used in professional physics settings. Existing models of math use focus on mathematical modeling for problem solving. However, workplace problems often include design problems, troubleshooting, and more. To study workplace mathematics, we conducted hour-long, semi-structured interviews with employees at photonics and optics companies in Rochester, NY. We applied an emergent coding process to classify instances of math in the workplace, and present two models of mathematics use within workplace tasks. We describe a four-phase engineer task consisting of defining the problem, designing a product, testing the product, and communicating results. A common technician task replaces the design phase with manufacturing the product. Workplace math is embedded in these phases through various representations such as simulations, schematics, and machining codes. Educators should consider using diverse problem types since they require additional mathematical representations and techniques to be brought to the forefront.

I. INTRODUCTION

Previous research on frameworks that describe math within physics has tended to focus on either mathematical modeling or math use for problem solving at the undergraduate level [1–4]. For example, the ACER framework proposes that when solving a physics problem, students activate a mathematical tool, construct a mathematical model, execute the mathematics, and then reflect on the results [5]. Another modeling framework proposes that a physical system is mapped into a mathematical model, mathematical operations are performed, a conclusion is made based on the calculations, and the conclusion is evaluated to see if it adequately describes the physical world [6]. However, typical problems encountered by researchers and engineers are much more diverse than undergraduate physics problems, upon which previous frameworks have been based [7]. Therefore, it is important to understand how math is used in research and industry so that instruction in introductory physics courses, which tend to be taken by engineers and other science majors, better reflects the diversity of mathematical thinking that students may encounter in their careers. In this paper, we will analyze typical tasks involving mathematics performed by employees of various optics and photonics companies and articulate tentative frameworks to describe the role of mathematics within those tasks.

II. METHODOLOGY

In order to determine examples of typical tasks involving mathematics performed in the workplace, we conducted semi-structured, hour-long interviews with 22 employees (19 male, 3 female) at optics companies in Rochester, NY. The employees volunteered to participate in our study and were compensated with $50, though not all of them accepted the compensation. The interviewees were asked about general skills, communication skills, mathematical skills, and physics skills needed for success in their workplace as part of a larger study. In addition, the interviewees were asked about their experience with each of the skills they viewed as essential for success in their workplace before they were hired and what opportunities existed for them to develop or improve the skills they viewed as essential. Of the 22 employees, 5 were entry-level engineers, 7 were management-level engineers, 4 were entry-level technicians, 4 were management-level technicians, and 2 were HR managers; these titles resulted from how an employer classified their employees and not from our interpretation of the data. Since engineers and technicians work closely together, employers tend to distinguish the two roles through the educational experience required and the typical job responsibilities. In general, engineers require at least a four-year degree and are responsible for designing a product while technicians usually require a two-year degree and are responsible for building and testing a product. As an example, an engineer would be responsible for designing a lens system while the technician would be responsible for constructing the lens system.

To begin analyzing the data, we marked all instances of math being discussed in the interviews using NVivo qualitative data analysis software. This resulted in 487 instances of math being marked. Next, an emergent coding scheme was applied to the data based on the type of math used or the mathematical tools used. By mathematical tool used, we mean both physical tools such as pencil and paper and specialized software or methods such as Microsoft Excel or finite element analysis. Another researcher independently labeled the interviews and inter-rater reliability was conducted. In cases where the researchers disagreed, the researchers discussed why they had or had not chosen to apply a code to a section of the interview and revised the codes as needed so that a statement could clearly fall within or outside of a code. The researchers agreed upon four types of math (arithmetic, algebra, calculus, and trigonometry and geometry), and five main tools (Microsoft Excel, finite element analysis software, MATLAB, calculators, and paper and pencil). A second round of emergent coding was then used to classify...
how the employee talked about the type of math used and the mathematical tools used (such as using trigonometry to find the size of an angle). Throughout the coding process, the codes were not exclusive, meaning that a single statement by an employee could have multiple codes applied to it. After the second round of coding, we went back and focused on frequently occurring and important uses of workplace mathematics in order to construct preliminary themes. We then synthesized these preliminary themes (such as using Excel to perform calculations and writing macros and scripts to simplify calculations) in order to describe the goal-driven tasks of technicians and engineers, similar to how current math in physics frameworks describe the steps students take to solve a physics problem [5, 6].

III. RESULTS

From the specific uses of workplace mathematics we coded, we selected two typical and representative workplace tasks on which to focus our analysis: an engineer designing a lens or lens system and a technician manufacturing a lens or other optical component. Our analysis of math use in each task is a synthesis of responses from our interviews with engineers and technicians rather than an analysis on a single employee’s math use. This is because none of the interviews contained all the essential parts, likely because the interviewee was not directly asked about all the steps in the tasks that they do. In addition, engineers and technicians often worked together and were able to give additional insights into typical tasks performed by each other. As a result, the technician example is constructed from the interviews of ten subjects (4 technicians, 4 technician managers, 1 HR manager, and 1 engineer) while the engineer example is constructed from the interviews of thirteen subjects (4 engineers, 6 engineer managers, 1 technician manager, and 2 HR managers).

A. Engineer Example

Figure 1 overviews a four-stage design process common among engineers. When designing a lens system for various applications (e.g., projectors, cameras, laser systems), an engineer will receive (often through interaction with a client) a set of specifications that the final lens system must satisfy, such as the magnification range and the image size. From these specifications, the engineer can begin to design the lens system.

During the design phase, the engineer often needs to make initial calculations from the specifications such as the necessary focal lengths and distances between lenses. Additionally, an engineer will use trigonometry and geometry to find angles and the distance between a lens and a projection surface or location of an object. These calculations can vary in complexity based on what type of lens system must be created. For example, many of the engineers referred to the calculations as "basic math" when working with flat surfaces, but did not use such language when talking about projecting an image onto a curved surface like a planetarium wall. Due to the possible complexity of the calculations, many engineers used programs such as Excel to perform their initial calculations. The engineers would plug the lens equations into Excel and then have Excel solve for the desired quantity. Others would use Excel to plot equations and solve graphically such as when finding the intersection of a line and a circle. Additionally, some engineers said they used Excel to write macros to simplify complicated or tedious math such as multiplying matrices. Regardless of the method used to perform the calculations, the engineers were expected to be evaluating their results to make sure that they were reasonable.

After constructing an initial model, the engineer constructs a computerized model of the lens system using optical design programs, which allow the user to create a ray-tracing diagram of the proposed design based on the parameters of the lens. The engineer can then use the computerized model to optimize the design of the lens system. In this step, the engineer uses their initial calculations to define the system, then selects which parameters should be optimized, and finally enters any constraints on the parameters, such as maximum and minimum lengths of the entire lens system. Since determining and calculating these constraints inside the program can be difficult, some engineers choose to write macros to simplify the math. After the engineer believes the lens system is sufficiently optimized, they will then use the schematic to create a full 3D model using computer-aided design software. Depending on the complexity of the lens system being constructed, the engineer may need to first construct a mathematical model of the surface of the components of the lens system and use that mathematical model to create the 3D model.

Once the computerized model has been developed, the engineer will test the lens system to make sure it conforms to the specifications by creating a simulation or building a pro-
totype. When testing using a simulation, the engineer would again use industry-specific software (e.g., Zemax or Code V) to simulate how the lens system would perform and could use the results of the simulations to construct a revised version of their lens system. Alternatively, the engineer would actually build a prototype of their lens system when creating and running a simulation would be too complex. In both cases, the engineer collects meaningful data related to the lens system’s performance, and then conducts an analysis to see if it is meeting the specifications. If the lens system does not meet specifications, the engineer returns to the designing stage, modifying their device before testing it again.

Throughout the testing, the engineer needs to be cognizant of sources of error and how the errors will propagate through the manufacturing process. The engineer’s task is to figure out what uncertainty is allowable and how uncertainties in each component of the lens system will affect the overall performance, which can be accomplished by running Monte Carlo simulations. The engineer then computes means and standard deviations on the overall performance and determines if further revisions to the design are needed.

Upon the completion of testing, the engineer begins the last step, which is communicating their results. First, the engineer generates instructions and blueprints for the technicians so that the lens system can be manufactured. Second, the engineer often needs to share the blueprints and 3D models with a mechanical engineer who is responsible for making the parts to house the lens system. This step is not trivial in that the customer often requires the lens system to fit in a specific space and these housing constraints may require changes in the design of the lens system. Once a final design for the lens system and its housing has been created, the engineer typically creates software to manufacture the lens system or program spreadsheets to assist technicians when manufacturing the lens system by entering algebraic equations into the cells. Finally, the engineer will communicate with the sales team or directly with the customer about the design of the lens system before it goes to manufacturing. Depending on the person’s background, the engineer may be required to develop a less technical description of the lens system to show that the proposed design does conform to specifications and performance standards.

B. Technician Example

Figure 2 overviews a four-stage fabrication and testing process common among technicians. Once a design of the lens system has been finalized, the technician receives a blueprint from an engineer laying out how the lens system should look. Depending on the company, the technician may only receive 2D pictures of the final product and have to visualize how the final product will look when they are manufacturing it. The blueprint may also specify sizes and angles in metric units, which the technicians need to convert to US customary units, or give parameters that need to be converted to a different parameter such as radius to diameter.

After interpreting the blueprints, the technician begins to manufacture the lens system. Depending on the component of the lens system being constructed, the technician may use a computer numerical control (CNC) machine, a manually controlled machine, or a combination of the two. A CNC machine controls the movements of tools through a programming language, such as G-code, which the technician is expected to know how to interpret and write. To understand and write G-Code, the technician needs to understand 3D coordinate geometry since the tools are positioned in either a 3D left- or right-handed Cartesian coordinate system. The technician then needs to translate the desired tool motion into G-Code, which can involve representing translations and rotations in the coordinate plane. In addition, the technicians are expected to use trigonometry and geometry to check that the CNC is performing correctly and if not, to know how to make the proper adjustments.

In addition to programming a CNC machine, a technician often needs to do preliminary arithmetic or trigonometry calculations. First, the technician needs to determine which tools will be used and calculate how long each tool will be used. The technician generally uses Excel for this step, which helps them choose the proper tools and helps them determine how much of a part should be removed with each tool. Second, the technician may need to relate how much material is removed to the distance the tool should move and calculate a run time for the machine, both of which often incorporates proportional reasoning and arithmetic. Finally, the technician will use basic arithmetic to calibrate the machine and estimate how many parts can be produced in a set period before starting to manufacture the component of the lens system.

Once the lens system has been manufactured, the technician needs to test it to make sure it performs as expected. Technicians commonly test the shape of a lens using an interferometer, which requires that they calculate the correct
angles to create an interference pattern. In addition, setting up the interferometer requires the technician to use Excel to calculate which reference sphere should be used. The technician then uses the visually presented interference pattern to infer deviations between the lens and the specified shape. The technician can then find defects and incorrect parameters such as the radius of the lens.

When testing other components of the system, the technician will use a mix of computer programs and basic math. First, the technician will make sure that dimensions of the lens system are the correct size and fall within the required tolerances. Next, the technician will test the parts and record the data in an Excel spreadsheet. The Excel spreadsheet will perform any calculations on the data and tell the technician of any adjustments that need to be made to get the required performance. Throughout the testing process, the technician is expected to evaluate their results to ensure that the results are reasonable. After the testing process is completed, the technician will need to count the number of parts made and calculate the rejection rate, which is what percent of the parts manufactured did not perform correctly or meet specifications.

Finally, the technician must communicate their results with their supervisor and possibly the customer. The technician will use Excel or a similar program to convert their data into tables and graphs. The technician will then prepare a report or presentation detailing the testing they performed, where the results are displayed in a table or graph.

IV. CONCLUSIONS

In this article, we examined how mathematics is used in the workplace. Our first significant finding is that not all workplace activities involving math, such as data analysis, programming, and troubleshooting, can be neatly aligned with mathematical modeling. Yet, similar to a mathematical modeling process, we see an interplay between the real-world and abstract mathematical representations; however, new representations emerge as important (e.g., schematics, G-code to control a CNC machine, data visualization). Additionally, whereas modeling begins with a physical phenomenon and then develops an abstract model, both of the processes shown in Figs. 1 and 2 begin with abstract representations of a thing that has not yet been constructed. Finally, in other frameworks [5, 6], the final step of evaluating the result or reflecting on the results is done abstractly, such as by asking are there calculation errors, are the units correct, or is the answer reasonable. As depicted in Figs. 1 and 2, evaluating the final result in the workplace is done physically since an actual object is tested to determine if the problem has been solved.

Our second significant finding is how mathematics is incorporated into the well-known engineering design cycle, which is a multi-step design process consisting of identifying the problem, researching the problem, developing possible solutions, selecting the best solution, constructing a prototype, testing and evaluating the solution, communicating the solution, and redesigning [8]. We did not begin our analysis with the engineering design cycle in mind, but instead found it to be emergent from the data. Thus, our results serve as a window into how mathematics is used in various stages of the design and construction processes.

Our final significant finding is that the math used in the workplace differs from that in the undergraduate curriculum in several ways. First, math in the workplace is typically done with the aid of computer programs such as Excel and MATLAB while typical undergraduate physics courses prioritize pencil and paper math. Second, the types of problems and representations presented in the undergraduate curriculum also vary from those in the workplace. In a typical back-of-the-book problem, a student is given an initial state and asked to determine some number of quantities, which have a single correct answer based on the given information. In the workplace however, engineers and technicians are given the final state in the form of specifications from a customer and asked to design a product that satisfies the specifications, which allows for multiple solutions. Finally, undergraduate physics problem solutions are rarely put into practice, whereas in the workplace, solutions are used to construct physical objects or to create simulations. Incorporating more design problems into physics courses would be a natural way to expand math problem solving in a way relevant for physics-intensive careers. Additionally, future research on math in physics should examine additional problem types, such as these design problems.

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