New Ways of Investigating the Canonical Coin Toss Acceleration Problem

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Abstract. Asking students about the acceleration of a tossed object is a well-studied problem in physics education research. Students frequently respond using reasoning that describes the velocity of the object, in particular that acceleration is zero at the top. We created new versions of the canonical multiple-choice Force and Motion Conceptual Evaluation coin-toss questions to investigate what other reasoning students might use. Some students were asked "is the acceleration zero at the top?" Other students were told "the acceleration is not zero" and asked to explain. A third group answered the original multiple-choice version of the question. Our results suggest that some students give answers that they can explain are incorrect. We also find that some students' responses about the acceleration at the turnaround point are affected by question format.

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INTRODUCTION

The coin toss problem first reported on by Clement [1] has been interpreted in multiple ways: students believe a force is required in the direction of motion [1], the force of the hand is used up, at which point the object turns around [2], students fail to recognize the difference between a quantity and a change in that quantity [3], students have not yet differentiated the concept of "motion" into "velocity" and "acceleration" [4], and more. These descriptions are different ways of explaining that students often answer questions about the acceleration of an object thrown into the air as if describing the velocity of that object: on the way up, the acceleration is up, and decreasing; at the turnaround point at the top, the acceleration is zero; and on the way down, the acceleration is down, and increasing.

We investigated student responses to the coin toss problem using three questions, with the middle question being asked in three different ways. This repeated a research design used previously to study understanding of electric circuits [5]. By asking a single large-lecture class different questions (with no student answering more than one question), we expect to develop a better understanding of the spectrum of responses to be found in the class.

In our study, we first developed three versions of the canonical coin toss question which asks about acceleration of the coin at its highest point. One question was the original multiple-choice version of the question (see Figure 1). Another asked students, "Is the acceleration zero at the top?" A third version told students "The acceleration is not zero at the top," and asked them to explain. The three questions were administered to roughly equal sized populations of students taking an introductory physics course. Our analysis first showed that the populations answering different versions of the coin toss problem are similar in the context of this study. We then looked for areas in which their responses differed across question format. Our data also showed a strong effect, most likely due to instructor intervention carried out the day before the questions were assigned to students. We thus compare the data from our three questions to the data from a survey assigned before any instruction.

EXPERIMENTAL DESIGN

The tasks given to students in this study were based on questions 27–29 of the Force and Motion Conceptual Evaluation [6] (see Figure 1). All students were given the multiple-choice version of question 27, one of the three variations of questions 28, and one of the three variations of questions 29. The variations of question 28 were developed based on student's responses to the FMCE given on the first day of class. The development and results of the variations to Question 29 are not discussed in this paper.

The 175 students in this study took part in a calculus-based course for physics majors and engineers.

On the pre-instruction FMCE the most commonly given response pattern for questions 27-29 was G-D-B with 47% of students responding this way. These are the students described previously, answering as if thinking of velocity, or requiring a force in the direction of motion. Though all three answers could be described as if acceleration is proportional to velocity ($a \sim v$), we will, for the sake of description, refer to answer G as $a \sim v$, answer D as $a=0$, and we do not discuss answer B further in this paper.

Also common was the correct answer A-A-A with 13% of students giving this response. While the response pattern of A-D-A is usually the second most common response it was rarely chosen with only 3 students out of 175 choosing this response pattern. For this pattern, students correctly state that acceleration is downward (in the negative direction) and constant, but say that it is zero at the turn-around point.

Results from each question are shown in Table 1.

Questions 27–29 refer to a coin that is tossed straight up into the air. After it is released it moves upward, reaches its highest point and falls back down again. Use one of the following choices (A through G) to indicate the acceleration of the coin during each of the stages of the coin's motion described below. Take up to be the positive direction.
A. The acceleration is in the negative direction and constant (correct for 27,28, and 29)
B. The acceleration is in the negative direction and increasing
C. The acceleration is in the negative direction and decreasing
D. The acceleration is zero.
E. The acceleration is in the positive direction and constant.
F. The acceleration is in the positive direction and increasing.
G. The acceleration is in the positive direction and decreasing.
__27. The coin is moving upward after it is released.
__28. The coin is at its highest point.
__29. The coin is moving downward.

FIGURE 1. FMCE coin toss questions, multiple-choice versions of all three questions.

TABLE 1. FMCE pre-instruction results (N=175). Only responses A, B, D, and G are shown.

Answer	FMCE 27	FMCE 28	FMCE 29
A (correct)	38	26	48
B (neg. increas.)	7	3	108
D (zero)	0	144	0
G (pos. decres.)	103	0	1

Few students answer A (correct a). We found that 96% of students answering G ($a \sim v$) on 27 answered D ($a=0$) on 28. We found that only 58% of the students correctly describing a on 27 did so on 28, as well, while nearly 40% of students correctly describing a on 27 answered $a=0$ on 28.

At the end of the first week of classes, students answered these questions again, this time as part of an online survey. Every student answered the same version of question #1 which was nearly identical to question 27 on the FMCE, though sentences in the description were slightly rearranged to allow for constancy with the following questions. There were three different versions of question #2. In one version, 58 students answered a multiple-choice question nearly identical to FMCE question 28 shown in Figure 1. We call this the “Multiple-Choice” group. In another version, 62 students were given the same description of the problem and asked, “Is the acceleration at the top zero? Explain.” We call this the “Consider One” group, because students were asked to consider only one response. In a third version, 62 students were again given the full description and told, “The acceleration at the top is not zero. Explain.” We call this the “Given Information” group, because students were given the correct response. The reason that we chose to focus on the acceleration being zero at the top was because it is an incorrect answer that is also the most common response on the pre-instruction FMCE. Note that 82% of the students had given the response of acceleration is zero at the top only a few days before, and that the given information response contradicts what many of these students stated previously.

To evaluate whether the three student groups were similar, we used data from question #1, which all students answered in the multiple-choice format. Results are shown in Table 2. We conclude that the populations are similar enough to suggest that differences in our data can be attributed to question format.

TABLE 2. Question 1 results, sorted by Question 2 format.

Student Answer	Multiple Choice N=58	Consider Only One N=62	Given Info N=62
A ($a<0$ and constant)	38%	44%	42%
G ($a \sim v$)	41%	40%	40%

INCONSISTENT RESPONSES

Overall, we wanted to know if students answered question #2 differently depending on the question they were asked. We also investigated how a student’s answer to question #1 related to whether they would state the acceleration is zero at the top on question #2.

We found that students performed better on the online survey than on the pre-instruction FMCE. Where 82% had stated a=0 at the top on the pre-instruction FMCE, only 36% of the Multiple-Choice group and 19% of the Consider One stated the same (see Table 3). The reason for this likely stems from

TABLE 3. Student performance on question #2. For Given Information students, the percentage is of those denying that $a \neq 0$ at the top.

Student Answer	Pre FMCE N=175	MC N=58	Cons. One N=62	Giv Inc. N=62
a=0 or equivalent	82%	36%	19%	5%

instruction that happened on the second day of lecture, with a Peer Instruction [7,8] activity that seems to have strongly affected students' responses.

Our interest in this paper lies not in improvements due to instruction but in how question format might reveal new information about what students in a class are thinking. We note the ratio of correct responses, nearly 2 to 1 depending on the question the students were asked. This suggests that either the Multiple-Choice question is more likely to trigger incorrect thinking in students or that the Consider One and Given Information questions are more likely to trigger correct thinking. To complicate matters, we find that 80% of the Given Information students are able to explain why the acceleration is *not* zero at the top. We could imagine that some of these 80% would have been part of the 36% who answered the Multiple-Choice question incorrectly.

Because answering *a~v* on the way up (answer G on question #1) was so strongly related to saying *a=0* at the top, in our pre-instruction data, we tested to see if the same was true with the other question versions. We found that it was not (see Table 4). Only about 30% of the Consider One and Multiple Choice students stated that *a=0* at the top, as compared to 96% before any instruction.

Students in the Given Information group who answered as if *a~v* (answer G) on question #1 were not as likely to give good answers when explaining that acceleration is not zero at the top. Only 64% gave good explanations, while 92% of the students in the same group who answered correctly on question #1 gave good explanations. Notably, three students in the Given Information group who answered as if *a~v* on question #1 responded to the given information by denying it. This indicates the strength of their convictions in believing that *a=0* at the top (or at least not believing that *a≠0*).

The number of students who correctly described the acceleration in question #1 increased from 20% on the pre-instruction FMCE to 40% on our survey. We were interested in seeing how this answer related to students' correct responses to the different versions of question #2. We also wanted to see how many students changed from the correct to the most common incorrect answer, that *a=0* at the top. Results are shown in Table 5. We observe that students in the Consider One and Given Information groups answer question #2 differently than those in the Multiple-Choice group. The Multiple-Choice group answers similarly to the pre-instruction FMCE scores, while the other two do far better. By way of comparison to results described above, none of the Given Information students who answered correctly on question #1 denied the information given them in question #2.

TABLE 4. Of those who answered as if *a~v* on the way up, how they described acceleration at the top.

Student Answer	Pre FMCE N=103	MC N=23	Cons. One N=25	Given Info[1] N=25
a=0 or equivalent	96%	29%	36%	12%
a≠0 or is negative and constant	3%	50%	52%	64%

TABLE 5. Of those who described acceleration correctly on the way up, how they described acceleration at the top.

Student Answer	Pre FMCE N=38	MC N=22	Cons. One N=27	Given Info[1] N=26
a=0 or equivalent	39%	45%	4%	0%
a≠0 or is negative and constant	58%	41%	93%	92%

DISCUSSION

Asking seemingly identical questions can have profound effects on data that we get about students' understanding of the physics, and can provide us with insights into how students in the class are thinking. In the discussion that follows, and to allow us to make comparisons between the groups, we make the assumption that the results of each group would have been identical had they been given the same questions, based on the similarity of responses to question #1.

Our data indicates that there are possibly students who will answer the Multiple-Choice question incorrectly (as done 36% of the time) and can also explain why that answer is incorrect (as done 80% of the time). For these roughly 15% of students, the correct idea that *a≠0* at the top might not be strongly understood. These students may be likely to be cued to the wrong response by reading the given Multiple-Choice responses. Such an interpretation is strengthened by the idea that only 19% of the Consider One group said yes when asked directly if *a=0* at the top. However, it could also be that students are more likely to be cued to give the correct response when receiving the Consider One and Given Information questions.

We note the effects of instruction on student responses, and that the results were not consistent across question versions. Whereas 96% of students answering as if *a~v* on (the equivalent to) question #1 in the pre-instruction FMCE had then answered *a=0* on (the equivalent of) question #2, only about 30-35% did so after instruction. This is consistent with a sharp drop in answering *a=0* at the top. But, for students who answered correctly on question #1, the version of

[1] For Given Information students, the percentages are of those denying the result and saying a=0 instead, or giving accurate and appropriate explanations for a≠0.

the question they received influenced whether they said *a=0* at the highest point. Of the Multiple-Choice students who answered correctly on question #1, 46% responded that *a=0* at the top. Of the Choose One or Given Information students who answered correctly on question #1, only 4% and 0%, respectively, said *a=0* at the top. The data suggest that instruction assisted those who started out with a more incorrect idea but it is unclear if it helped the students who were answering A-A-A or A-D-A before instruction. This interplay between an instructional intervention and question variations is interesting and may suggest that some variations may be more sensitive to ideas students have just been taught than other versions.

CONCLUSION

In our study, we asked a large group of students one of three questions to compare how the question itself might affect the answers that they gave. We found several examples where performance on questions depended on the question format. Students were twice as likely to answer the Consider One version of the question correctly than the Multiple-Choice version (see Table 3). They were also more likely to stick with a correct answer pattern (namely, answering correctly on question #2 after answering correctly on question #1) when answering the Consider One version of the question than the Multiple-Choice version (see Table 5). But, this difference was not observed when looking at how students performed after answering as if *a~v* on question #1 (see Table 4). Surprisingly, we found that students were more likely to give incorrect answers to the Multiple-Choice version of question #2 when they answer correctly on question #1 than when they answered as if *a~v* on question #1. This suggests that, for many, the correct answer is only partially believed or understood, and the offered response of a=0 at the top can easily distract them. We suggest that the question format, by offering appealing distractors, cues some students' thinking in a way that they would not have otherwise arrived at themselves.

We found other examples of students holding very firmly to their beliefs. Though the numbers are small, we note that three students (out of 62) explicitly denied that the Given Information response (that *a≠0* at the top) was correct, and insisted that it was. This suggests that the 80% of students in the Given Information group who gave good explanations might not believe their own explanations but gave them only because they had to invent an explanation to an unexpected statement. Again, we find it plausible that the question format affects how students answer the question, and therefore, what we as teachers and researchers can learn from student responses.

In sum, we find that asking different versions of a single question within a single population leads to interesting insights that were not available to us when asking only one question. These results tell us about a student's belief in the correctness of an answer and also suggest ways in which students' responses are flexible and changeable

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