

What Is Entropy? Advanced Undergraduate Performance Comparing Ideal Gas Processes

Brandon R. Bucy,¹ John R. Thompson,^{1,2} and Donald B. Mountcastle¹

¹*Department of Physics and Astronomy and* ²*Center for Science and Mathematics Education Research*
The University of Maine, Orono, ME

Abstract. We report data on upper-level student understanding of entropy and the Second Law of Thermodynamics when comparing the isothermal and free expansions of an ideal gas. Data from pre- and post-instruction written questions are presented, and several noteworthy features of student performance are identified and discussed. These features include ways students think about these topics prior to instruction as well as specific difficulties and other interesting aspects of student thought that persist after instruction. Implications for future research are also addressed.

Keywords: Physics education research, thermodynamics, thermal physics, second law of thermodynamics.

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INTRODUCTION

At present, there has been relatively little research conducted on student learning of thermal physics concepts in university physics courses, particularly beyond the introductory level. The research that has been done clearly indicates that university students exhibit significant difficulties when learning thermal physics topics, including conceptual difficulties with heat, temperature, the Ideal Gas Law and the First Law of Thermodynamics [1-4]. Research on student understanding of the more advanced concept of entropy has been conducted with high school students [5] and non-physics majors [6]. At the University of Maine (UMaine), we are currently engaged in a collaborative research project with colleagues at Iowa State University (ISU) to explore student understanding of thermal physics concepts for the purposes of improving instruction. As part of our investigation of student learning of thermal physics, we designed and administered questions to probe what students in an upper-level thermodynamics course know and learn about the concepts of entropy and the Second Law of Thermodynamics.

Rather than teach a single thermal physics course that attempts to cover both classical thermodynamics and statistical mechanics in one semester, these topics are taught in two separate semester-long courses at UMaine. In this paper, we present results from an exploratory survey of the UMaine *Physical*

Thermodynamics course, taught in Fall 2004 (by DBM). The course is designed to cover the first 11 chapters of Carter's textbook [7] while including supplemental material from other sources. Instruction included lecture, demonstrations, and class discussions; homework assignments included standard problems and instructor-designed conceptual questions. Homework was graded and returned with comments and a detailed answer key. We have data from seven of the eight students taking the course. These students had little prior exposure to thermal concepts beyond introductory chemistry; introductory physics courses at UMaine do not include these topics. However, one student had completed our *Statistical Mechanics* course and another had previously taken a thermodynamics course in our College of Engineering. Data include written responses on pretests, homework and exams, online conference postings, and informal class observations. We focus here on student responses to written questions dealing with entropy and the 2nd Law, in the context of ideal gases.

THE QUESTION

As part of our exploratory investigation, we developed a question focusing on two different ideal gas processes: a reversible isothermal expansion and a free expansion. In the basic question scenario, shown in Fig. 1, two identical samples of a monatomic ideal gas undergo these processes. Students were told that

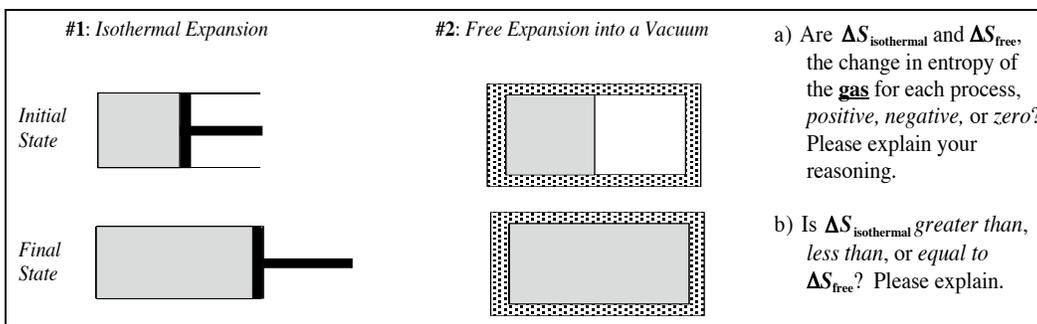


FIGURE 1. The two processes and questions used in diagnostic a) sign and b) comparison tasks.

both processes begin at the same temperatures, volumes, and pressures, and that the final volumes are also identical. On the pretest, students were also told that the final temperature is the same as the initial temperature for both processes, and that zero work is done in a free expansion, so that they could focus on the relevant questions without being distracted by unknown details of the processes involved. Students were first asked to determine the sign of the system entropy change for each process (the “sign task”). Additionally, students had to compare the system entropy changes for the two processes (the “comparison task”). Students were directed to explain their reasoning on both tasks.

These tasks were given to the students twice: as an ungraded pretest before instruction on the concept of entropy, and later after instruction in both entropy and the 2nd Law, slightly modified as part of a more comprehensive midterm examination question.

On both the pretest and post-test, student responses were couched almost exclusively in mathematical equations, especially on the exam where time was a constraint. Extensive material data, but no formulae, were provided. Additionally, student reasoning often referred to answers on previous sections of the exam question. The best explanations contained both verbal and mathematical reasoning.

The Sign Task

The intended purpose of the sign task was to probe student understanding of entropy and the 2nd Law. A system entropy change, ΔS , is defined as

$$\Delta S = \int dS \geq \int \bar{d}Q/T, \quad (1)$$

where $\bar{d}Q$ represents the heat transferred to the system, and T represents the temperature. The equality holds only for reversible processes, such as the isothermal expansion here. Heat must be transferred to the

system to maintain a constant temperature while the gas expands, doing work against the piston, so both $\bar{d}Q$ and T are positive; it follows that ΔS is *positive* as well. The free expansion is irreversible, yielding the inequality in Eq. (1). Since this process does no work and maintains a constant temperature, no heat is transferred to the system. Thus, ΔS must be positive for both processes.

In addition to the thermodynamic definition of entropy, the course instructor also introduced the statistical definition of entropy to help students develop a conceptual meaning for entropy. The entropy of a system is proportional to the logarithm of the system multiplicity, which is the number of available microstates for a given system macrostate. Multiplicity is a function of both the volume and temperature of the system: an increase in either of these properties *alone* yields a greater system multiplicity. For both gas processes in Fig. 1, the volume increases while the temperature remains constant. Thus, the multiplicity of the system, and hence the entropy, increases for both processes.

The Comparison Task

The comparison task was designed to probe student understanding of entropy as a state function. A thermodynamic variable is a state function if the value of the variable is a property of the equilibrium state of the system; thus, the change in a state variable depends only on the initial and final states and is independent of process (“path-independent”). Since entropy is a state function and both processes have the same initial and final states, the entropy changes of the gas in the two processes are *equal*. The state function property is quite prominent in thermodynamics, and the importance of student understanding of this concept cannot be overstated. Hence, student performance on a task dealing with the unfamiliar concept of entropy and changes in entropy over different paths is a

valuable indicator of a functional understanding of the state function property.

RESULTS AND DISCUSSION

Several features of student reasoning on these tasks are noteworthy, including applications of the 2nd Law, difficulties with the state function property, and the definition of entropy itself.

What is Entropy?

Analysis of the pretest reveals that most students were, predictably, unfamiliar with the correct thermodynamic concept of entropy before it was introduced in this class. Three students were unable to give any explanation for their answers on either task, stating they had no idea what entropy is. Two students linked the concept of entropy to the more familiar concepts of temperature and energy: for example, one student incorrectly ranked the isothermal entropy change as less than the free expansion entropy change because “the quasistatic nature of [the isothermal expansion] leads to less overall energy ‘consumption’ from the reservoir.” Nevertheless, and somewhat surprisingly, several students did have *some* idea of the role entropy plays in everyday phenomena, even though the specifics of this role were vague and unspecified. These students referred to a notion of ‘disorder’ in their reasoning about entropy. Two students correctly stated that the system entropy changes are both positive, reasoning that “in both cases, the disorder increases,” without any further explanation to indicate why disorder would increase during these processes, or what is meant by the term ‘disorder.’ (Other students also invoked ‘disorder’ in discussing entropy on questions not presented here. Similar results have been seen among general physics students in the introductory course at ISU [8].)

After instruction, all students correctly applied the thermodynamic definition of entropy in answering the sign task for the isothermal expansion. Two students stated that the entropy change for the free expansion was zero since $dQ = 0$, failing to recognize that the equality in Eq. (1) does not hold for irreversible processes. Some students additionally used the statistical definition of entropy, as discussed below.

The Statistical Definition of Entropy

Before instruction, only the one student who had previously completed *Statistical Mechanics* correctly used a statistical interpretation of entropy to determine the sign of the system entropy change, stating “multiplicity is a function of temperature and volume,”

and that “more possible locations for molecules is a larger multiplicity and thus a larger entropy.” However, three students out of seven (including the one above) used ideas such as multiplicity and microstates to determine the sign of the system entropy change on the exam. This increase is noteworthy since the majority of textbook and classroom determinations of entropy changes used the thermodynamic rather than the statistical definition.

Nevertheless, only one of these three students (not the veteran of *Stat. Mech.*) also correctly compared (equated) the two system entropy changes on the exam. The other two students made incorrect comparisons, mentioning only the volumetric aspect of multiplicity in their solution. These students did not explicitly state that the final volumes were equal, so it is not clear whether this fact was acknowledged. Had they also appropriately acknowledged the effects of temperature, they might have correctly equated the entropy changes.

This finding suggests that although students may be more amenable to using the statistical, rather than the thermodynamic, definition of entropy to answer conceptual questions, their understanding of the statistical definition may still be incomplete. Furthermore, these results and the pretest results related to ‘disorder’ above, taken together, support the idea that students may perceive the volumetric aspect of multiplicity as a refinement of their notion of entropy as ‘disorder.’

The Free Expansion Process

On both the pretest and the post-test, the free expansion process presented a number of difficulties to students. Although all relevant information about the two processes was included in the question given as a pretest, some students disregarded the information provided for the free expansion. For example, two students stated that the temperature decreases in a free expansion, even though it was specifically stated that temperature remains constant.

Although all students correctly answered all question parts related to the isothermal process on the post-test, the free expansion garnered few correct responses, and incorrect responses contained multiple errors. Prominent in student reasoning was the incorrect idea that the free expansion does work, causing the temperature of the gas to decrease. One student stated that “the rapid expansion cools the gas,” thereby lowering the temperature. The inclusion of insulation on the diagram for the free expansion also may have been problematic, as a number of students relied on that to determine that $dQ = 0$, even though the process would be identical without insulation.

Entropy, *System*, and the Second Law

The sign task also allowed us to investigate student ideas about the Second Law of Thermodynamics. This law states that the entropy of the *universe* (system *plus* surroundings) must always increase in any irreversible process, and can only remain constant if a process is reversible. The 2nd Law is mute on the *individual* entropy changes of the system and the surroundings.

Among student pretest responses there was an incorrect tendency to apply the 2nd Law to the system rather than the universe. Indicative responses were that the change in entropy for the isothermal process was zero (incorrect) since the process was reversible, and that of the free expansion was positive (correct) due to the irreversibility of that process. This line of reasoning implies an understanding of the 2nd Law in form but not in context. It is unclear whether this difficulty is specific to the interpretation of the 2nd Law or is a more general difficulty with the concepts of *system* and *entropy*. Similar issues with the system concept are seen in our *Statistical Mechanics* course, and have been reported in other contexts [9]. This confusion was virtually nonexistent on the post-tests, however. We plan to investigate this topic further both in thermal physics and in other areas of science.

Entropy as a State Function

The concept of *state function* is essential throughout the formalism of thermodynamics. Consequently, we were interested to see how students applied the state function property to entropy on the comparison task. On the pretest, only one student correctly completed the comparison task. After instruction, only one additional student gave a correct response. Notably, these two students were the only ones to explicitly use the idea that entropy is a state function in their solution reasoning. Invoking the path independence of state functions provides the only means of reaching the correct comparison in this task.

There was no significant improvement in student performance on the comparison task due to instruction. It is somewhat surprising that more students did not apply the strategy of the path independence of state functions to this task, since it was repeatedly encountered during instruction, class discussion, and homework. While this result contrasts with previous work in the context of the 1st Law, in which the state function concept is often applied indiscriminately to both the path-independent change in internal energy and the path-dependent variables work and heat [3,4], it corroborates more recent findings dealing specifically with the concept of entropy [10].

CONCLUSION

Our preliminary investigations have revealed ways in which students think about entropy in thermodynamics. Prior to instruction, some students appear to link entropy to other more familiar concepts such as temperature and energy, while others link it to ‘disorder.’ Post-instruction results show that students are largely capable of learning and applying the thermodynamic definition of entropy, and show an affinity for the statistical definition of entropy (in a course overwhelmingly focused on the thermodynamic definition), preferring it in verbal explanations. Students also initially misapply ideas dealing with the 2nd Law, confusing the entropy of the universe with the entropy of a system; however, many of these difficulties are successfully addressed with instruction.

The concept of entropy as a state function, however, remains nearly as elusive post-instruction as it was pre-instruction. The state function concept is extremely important for students to understand in the larger context of classical and statistical thermodynamics.

Future research will investigate the prevalence of student difficulties with entropy specifically and state functions in general, and will explore teaching methods and curriculum design that may improve student facility with both. Case studies of individuals will also be conducted to document the evolution of state function concepts as they are developed throughout the course.

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