Examining the Evolution of Student Ideas About Quantum Tunneling

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Abstract. We have studied whether repeated exposure to complicated physics concepts, such as quantum tunneling, fosters increased understanding. For three students, we have multiple interview, survey, and examination data over three years. We present data from a single student whose understanding of energy conservation in tunneling improved with repeated instruction, but whose ability to correctly sketch wave function solutions and discuss their meaning showed little progress.

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INTRODUCTION

A small but growing body of work has reported on student difficulties with quantum mechanics. Researchers and instructors have discussed student misconceptions regarding quantum mechanics [1], understanding of measurement in the context of quantum mechanics [2], general issues in tunneling [3], and interpretations of probability in understanding tunneling [4]. For three years, we have been studying student understanding of tunneling in quantum mechanics, a topic addressed in most introductory quantum courses.

Our research has included interviews [5,6], a survey [5], and examination questions all focused on the theoretical scenario of tunneling through a rectangular potential energy barrier. Surveys and interviews revealed that a majority of students believe energy is lost in the tunneling process, supporting Bao's findings [3]. Also, many have difficulty sketching and explaining the wave function corresponding to the real solution to the time-independent Schrödinger equation in the proximity of a rectangular potential barrier. Specifically, many students describe an "axis shift" [7] where students represent the supposed loss of energy of tunneling particles by lowering the axis of oscillation of the wave function describing transmitted particles (see Figure 1a and e, for example).

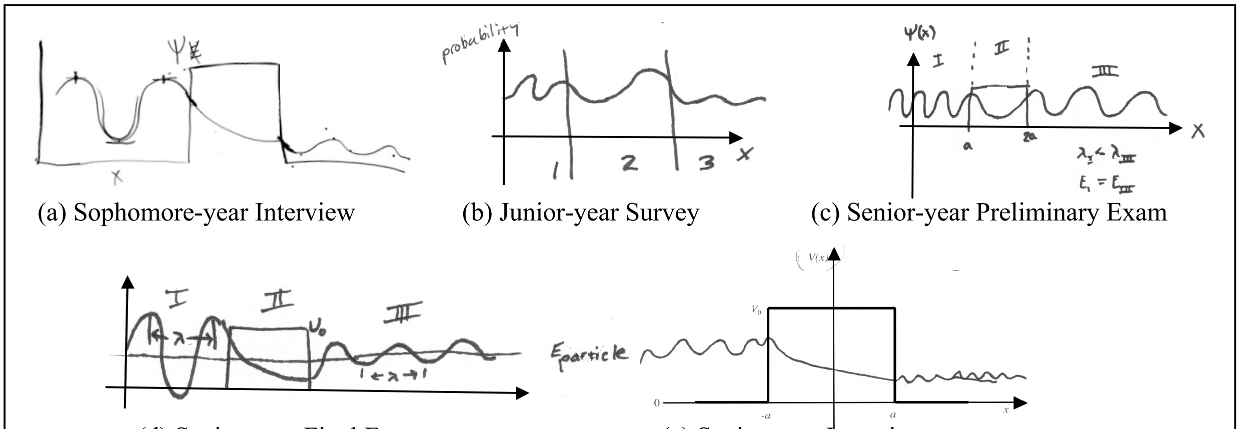
For three students, we have six "snapshots" of their thinking about tunneling over a multi-year period: two interviews, two surveys and responses on two exams. We examine the responses of one of these three students, Selena, because we have reported on her previously [5,6] and have gathered additional data that illustrate the difficulty of learning this topic.

CASE STUDY – "SELENA"

Selena graduated from the University of Maine with a Bachelor of Science in Engineering Physics. In the fall semester of her sophomore year, she completed the traditional, lecture-based introductory modern physics course, her first formal exposure to the phenomena of tunneling. She was first interviewed in March of her sophomore year and took the first version of the tunneling survey that April.

Based on the interview results from Selena and others, the survey was modified extensively to eliminate redundant questions and require students to sketch wave function solutions. In April of her junior year, she took the modified version of the tunneling survey.

In the fall of her senior year, she took quantum physics again. The instructor of the course was familiar with our previous findings, and modified his lecture-based instruction to emphasize the idea of energy conservation. Class discussion on rectangular-barrier tunneling included the idea of an ensemble of particles with different energies and the calculated proof that higher energy particles have a greater transmission probability, thus making the average



(a) Sophomore-year Interview (b) Junior-year Survey (c) Senior-year Preliminary Exam

(d) Senior-year Final Exam (e) Senior-year Interview

FIGURE 1. Selena's sketches of the wave function corresponding to tunneling through a rectangular potential barrier.

energy of the particles that have tunneled higher than the average energy of the incident particles. The instructor included questions from the survey on one of the preliminary exams and on the final exam. After completing her senior quantum course, Selena was interviewed again during the spring of her senior year.

Ideas About Energy

As a sophomore, Selena stated in her initial interview [6] that she believed energy was lost when particles tunneled through a rectangular potential barrier. The same answer was also given on the quantum tunneling survey responses in her sophomore and junior years. We are not surprised by these results, as she had no additional quantum physics courses in the interim that might have influenced her understanding of the scenario. Selena showed improvement on both senior-year exams. She indicated that the energy of tunneled particles remains the same as that of incident particles, since "energy is not lost in travel."

On the final exam, she also stated that the particles with the highest energy are more likely to be found on the far side of the barrier, though she did not say that this would make the average energy higher. When interviewed during her senior year, Selena stated that "the average energy of the particles from one side to another, here (pointing to the post-barrier region) will be higher because "only higher energy particles are able to tunnel." We note the incomplete interpretation of the previous semester's classroom discussion.

Sketching the Wave Function

Because the wave function solutions to the Schrödinger equation are emphasized in both quantum courses discussed in this paper, we included questions about this representation in our interview protocol and on the surveys. With the exception of the first version of the tunneling survey, all interviews, surveys, and exam questions asked the student to sketch the wave function solution to the time-independent Schrödinger equation. All of Selena's sketches are shown in Figure 1. We describe the relevant elements of each sketch in chronological order.

During her sophomore year interview, Selena first sketched a potential barrier (Figure 1a), superimposing her sketch of the wave function on top. Though the wave function exhibited generally correct characteristics in each region (sinusoidal-exponential-sinusoidal), problems existed. First, the superposition led her to first describe the vertical axis for the wave function as representing energy, later crossing out that axis label in favor of "ψ," which she referred to as "probability." Second, her sketch exhibited an "axis-shift," consistent with the sketches of many students who believe energy is lost in the tunneling process.

On the junior-year survey, Selena no longer superimposed her sketch on top of an energy barrier (Figure 1b). However, she drew the wave function as sinusoidal in all regions, with what appears to be an increased wavelength in the barrier region. Survey questions asked the respondent to show by sketching how the graph of the wave function is different when incident particle energy is increased. On both of these sketches (not shown), Selena drew a similar shape function, but shifted it, so as to make the function oscillate about a higher imaginary axis that is coincident with the given energy level of the particles.

On the first senior-year exam, Selena returned to superimposing her sketch of the wave function on a potential barrier shape, though she does label the axes "ψ *(x)*" and "*x*" (Figure 1c). It is unclear whether her function is sinusoidal or a sum of increasing and decreasing exponentials in the barrier region. It appears that she keeps the amplitude of the wave function the same in regions I and III. Though she stated that the energies in both of those regions are equal, she indicated incorrectly that the wavelength of the function is greater in region III. Her sketch, though sinusoidal, once again oscillates about some imaginary positive axis, perhaps coincident again with the energy level of the particles.

On the final exam, the wave function is more clearly a decaying exponential in the barrier region, though it decays far below the amplitude of the sinusoidal portion of the wave function in region III (Figure 1d). She indicated that the wavelength and energy are the same on both sides of the barrier, but she failed to label the given axes. She also sketched an axis of oscillation, labeling it "$E_{particle}$." She labeled sketches of the wave function on further questions involving increased incident particle energy similarly.

During the second interview, Selena sketched her representations of the wave function on the provided scenario sheets (which described the potential energy symbolically and graphically), once again placing the wave function on top of the potential steps and barriers (Figure 1e). Though the general shape of the wave function solution was correct for the rectangular barrier, her sketch once again includes the "axis shift" problem. When asked about the vertical axis label for the plot of a wave function, Selena replied that it is "potential energy." She admitted that this caused conflicts, however: "…which makes half of what I've said wrong, because I didn't keep the amplitude the same. But this is the part that I always get messed up with when I'm thinking about it, cause I remember pictures that look like this, but not necessarily where the axes were…"

Describing the Wave Function

In her first interview, Selena sketched the wave function corresponding to tunneling through a rectangular potential energy barrier. She described the "waves" on either side as being the same, and was questioned on exactly what she meant:

S: … uh, the wave function is dependent on the nature of the particle, not external conditions, so it's, it has to be the same wave on either side, but it's lost energy.

I: Would you describe these (pointing to the sinusoidal portions of the wave function on either side of the barrier, see Fig. 1a) as being the same wave?

S: Yeah, I draw badly, but sure, you know…

I: Is everything the same about them?

S: Well, there's less energy here, and depending on the size of this area the, um, wait a minute, that doesn't make any sense – this has less energy, the wavelength should be the same… I think.

I: Why do you think the wavelengths should be the same?

S: Because it's the same particle, and the wave function is describing the particle. And the particle, the only thing that's changed about the particle by going through the barrier is the amount of energy that it has, which is indicated by the height of the waves.

Two years later, discussion again addressed Selena's ideas about the wave function. She was questioned about the vertical axis label of a graph showing wave function as a function of position, and stated that it was "potential energy." She noted that this caused problems, since she had previously said energy is conserved. Though she stated, "if you just said these waves are psi of x, we have no problem," when pressed on the two models, she stuck to energy:

I: Would it be OK to write psi of x equals V of x? I mean, if it's energy, could we…

S: Yeah, it would, you could say… the psi of x for the potential is a constant starting at x equals zero.

Though she stated a qualitatively correct relationship between energy and wavelength earlier in the interview and also realized that several contradictions are solved by not equating the vertical axis of a wave function plot with energy, she remained determined to equate the two. In doing so, she changed many of her correct ideas discussed earlier in the interview, abandoning the careful logical connections in an attempt to match an explanation to her sketch.

Beliefs about Quantum

In both interviews, Selena made references to her beliefs about the nature of reality and quantum theory's ability to describe it. She consistently spoke of quantum mechanics as an imprecise science. In her sophomore year, she stated:

S: Uh, the math we have for describing these things is crappy, um, we don't actually know what's going on, we're assuming a whole lot of things, and, uh, according to the equations we have that work with observed stuff, we will not find it here.

Two years later, she had not abandoned that feeling, and stated:

S: I don't buy that Schrödinger's equation is a hundred percent right. Yeah, it works, but, you know, derivative twice with respect to position, derivative once with respect to time, it works, I think we, we're missing something.

DISCUSSION

Selena showed improvement on the energy-loss misconception, but little change on other concepts and beliefs about the validity of quantum physics. Selena held the idea that energy is lost for most of her undergraduate physics career but abandoned it, at least in verbal and written responses, in the presence of specific instruction on the rectangular-barrier tunneling scenario during a senior-level quantum mechanics course. It is not clear whether Selena's new answers arose from a solid conceptual understanding of the phenomena or merely memorization of phrases and ideas repeated by the instructor on multiple occasions. The phrase "only higher energy particles are able to tunnel," given during the second interview, suggests the latter. In reality particles of all energies possess a non-zero probability of tunneling; it is merely greater for higher-energy particles. Also, it may be that knowing "the answer" to the energy loss question introduced new confusions and created a willingness to abandon good ideas in an attempt justify her sketch.

As we previously discussed, Selena superimposes the wave function on the potential energy barrier in four of five sketches of the wave function, a representation not uncommon in quantum mechanics texts and computer simulations. While there is arguably some merit to this approach, we believe that representations of this form can cause unnecessary confusion and may have led to Selena's struggle with labeling the vertical axis of wave function sketches.

We note that Selena's sketches of the wave function are most correct during or shortly after instruction. While many features of her sophomore interview sketch are correct (function type in each region, axes label), these are not present in the sketch from the junior year survey. Her sketches on exams during the senior-level course show improvements in function shape, amplitude, and wavelength from the preliminary exam to the final. The final interview sketch returns, however, to the "axis shift" response given on the initial interview, and no apparent effort was made to match wavelengths for the portions of the wave function on either side of the barrier. This may suggest that without meaningful conceptual change persistent, incorrect ideas return in the absence of instruction. It may also indicate that questions tied to grades, such as the examination questions, yield more careful and reasoned answers than responses in volunteer interview sessions.

Selena uses "wave" and "wave function" interchangeably, suggesting that the two are not clearly distinguished in her mind. This may lead her to link energy with amplitude, an idea from classical waves. If she then remembers the general shape of the wave function for this scenario, which her sketches suggest she does, we can understand why she is adamant, during her first interview, that tunneling particles loser energy, and why she struggles, during her second interview, to reason about the wave function in when she knows that energy is conserved.

Selena seems to characterize the particle by its wave function and wavelength. Thus, if the particle has not changed, neither can its characteristic features. Her insistence during the first interview that the wavelengths of the two portions of the wave function are the same because "it's the same particle" may be an artifact of her quantum physics training, as many books and lecturers discuss an object's deBroglie wavelength as a means of deciding whether to analyze it with classical or quantum physics.

Finally, we see little evidence that Selena's belief in quantum physics as an accurate model of the world has changed over her undergraduate experience.

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