

Resource Plasticity: Detailing a Common Chain of Reasoning with Damped Harmonic Motion

Eleanor C Sayre¹, Michael C Wittmann^{1,3}, and John E Donovan II^{2,3}

¹*Department of Physics and Astronomy, University of Maine, 5709 Bennett Hall, Orono, ME*

²*Department of Mathematics and Statistics, University of Maine*

³*College of Education and Human Development, University of Maine*

Abstract. As part of ongoing research into cognitive processes and student thought, we have investigated the interplay between mathematics and physics resources in intermediate mechanics students. We present evidence from a reformed sophomore-level mechanics class which contains both tutorial and lecture components. In the context of writing Newton's Second Law for damped harmonic motion, students discuss the signs of the spring and damping forces. Using a grounded theory approach, we identify a common chain of reasoning in which a request for reasoning is followed by elaborative sense-making and checks for consistency, finishing with an optional appeal for group consensus. Our analysis provides evidence for a description of student thinking in terms of Plasticity, an extension of Resource Theory.

Keywords: cognitive modeling, damped harmonic motion, advanced students

PACS: 01.30.Ib, 01.40.Fk, 01.40.Ha

INTRODUCTION

Intermediate Mechanics is a particularly rich course in which to study the interaction between students' ideas about mathematics and their ideas about physics: students often enter with a good intuitive grasp of the physics, but have not yet applied sophisticated mathematics. At the University of Maine, intermediate mechanics is a one-semester course with two lectures and one tutorial¹ each week. About half of the students are concurrently enrolled in Differential Equations; the other half have already taken it.

We follow a three-pronged approach to studying students' ideas about mathematics and physics: data collection of students using mathematics and physics, which drives cognitive model building, which informs curriculum development.

Our primary data comes from three sources. Video data of short interviews track small groups of students through the semester. Video data and field notes of homework help sessions (HHS) examine groups of students as they work through their homework with the help of a TA. It is important to note that the role of the TA is to facilitate students' understanding of the topic, and not merely explore their existing ideas, in contrast to the role of the researcher in the interviews.

We supplement our video and field note data with exams and homeworks.

We use a grounded theory approach to model student ideas about specific topics (such as coordinate systems, air resistance, and Lagrangians). These models drive more general theoretical work expanding Resource Theory²⁻⁵ and connecting it to other theories in Mathematics Education Research (such as the Theory of Reification⁶ and the RBC model for abstraction⁷). This theoretical work informs curriculum development.

Following the general outline for the Project, this paper will motivate and explain a general theoretical development with a specific example taken from damped harmonic motion.

THEORY DEVELOPMENT

Resource Theory is a constructivist schema theory which bridges neuro-cognitive models of the brain and results from education research to describe the phenomenology of problem solving.⁴ Resources are small, reusable pieces³ of thought that make up concepts and arguments.² Resources have internal linked structure⁸ made up of other resources. To be considered a resource, an idea must have sufficient duration and stability to be reused. Resources can be epistemological, metacognitive, or content-oriented.⁹

The physical context and cognitive state of the user determine which resources are available to be *activated*. The activation of resources occurs when their invocation, express or implicit, is used to support or form an argument. Once activated, they form a network with graph-like structure.⁸ If their internal structure is explorable (but currently not explored) by the user, they may be called concepts. If their internal structure is no longer explorable, they may be called primitives. A large body of literature has identified both concepts and some kinds of primitives.⁴

“Plasticity” is a continuum which extends Resource Theory to describe the development of resources and connect their use to observables. The two directions in the continuum are *more solid* and *more plastic*. A solid resource can be considered a durable concept: its connections to other resources are plentiful, and its internal structure is unlikely to change under typical use. Existing literature details solid resources and their properties well.^{2,4,5,8,9} Plastic resources, in contrast, are less durable in time or less stable in structure. While a solid resource may remain unchanged for years, a plastic one may only last "until the exam." Consider the path that you take to drive home from work. To you, this path is quite solid, and you find yourself getting home without much thought. In contrast, your colleague who visits you for the first time must explicitly look for landmarks, check the route against your directions, and figure out how to get to your house. For your colleague, getting to your house is plastic.

The more plastic a resource is, the less likely the user is able to apply it to new situations. More explanation is needed to justify and explain its use. The more solid a resource is, the more likely the user is to refer to the resource in diverse contexts without explaining its internal structure. The plasticity of a resource is independent of its veracity.

We use the RBC model⁷ to inform and improve the plasticity continuum, noting that it was originally intended to describe the reification and abstraction of concepts. The RBC model proposes three epistemic actions through which abstraction occurs and which may be inferred from behavior. These three actions - *recognizing*, *building-with*, and *constructing* - are dynamically nested. Recognizing, the simplest action of the three, occurs when a student realizes that a "familiar mathematical notion, process, or idea ... is inherent in a given mathematical situation."¹⁰ These recognized cognitive objects are the resources in Resource Theory. Recognition is thus synonymous with activation. Content resources such as these need not be restricted to mathematics; physics is another appropriate subject area. The specifics of which resources are recognized gives insight into students'

thought structure. Ease of recognition is therefore a marker of solidity.

Once a familiar idea has been recognized, a student may build-with that idea to solve a local goal, such as solving a problem or justifying a statement. Several resources may need to be recognized and built-with at once. Under Resource Theory, activated resources form a web or graph⁸ that may be built on the fly. Thus, recognizing is akin to activation, and building-with is akin to building graphs on the fly. Because building-with and recognizing are two separate actions, the RBC model can describe behavior when students mention an idea, but don't appear to know what to do with it.

In contrast to building-with, constructing has purpose and duration beyond solving a local goal. Constructing creates a less-local, more abstract entity. As a construction becomes more durable, it becomes more consolidated and is no longer necessarily built on the fly. It becomes a resource in its own right, and therefore can be recognized or built-with in later local goals. The new-formed resource may be quite plastic, but as further constructions are added to it and as it compiles further, it can become more solid. Thus constructing is a mechanism for increasing the solidity of specific resources. Extremely solid resources - *rigid* resources -- have been so tightly compiled that their internal structure is not readily accessible to the user.

The process of abstracting and consolidating resources can be of long duration and difficult to show in a brief paper. Instead, we focus on an example of recognizing and building-with which shows different levels of plasticity in one encounter.

EXAMPLE

In this example, taken from a Homework Help Session, a student ("Bill") works through the sign of a velocity-dependent drag force in damped harmonic motion (Figure 1). Immediately prior, Bill asks to go over why the drag and spring forces are negative in the statement of Newton's Second Law:

$$\sum \vec{F} = -k\vec{x} - c\vec{v} \quad (1)$$

On his first attempt, he is unable to explain why the forces are negative, and he confuses himself while trying to figure it out. He starts over, explaining that

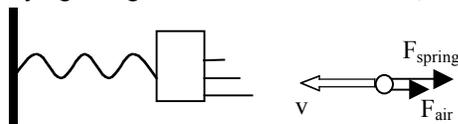


FIGURE 1. An object undergoing damped harmonic motion. The object is to the left of equilibrium, and moves leftwards.

the drag force opposes velocity and the spring force opposes the displacement, but he is unable to connect his description to the algebraic signs of the forces. While correct, his explanation confuses him again. To refocus discussion, the TA asks about $-cv$ first, holding $-kx$ for later. On his third attempt, Bill's explanation is longer, and he considers leftwards movement and rightwards movement separately. He references an implied coordinate system in which the positive direction is to the right. Bill's explanation takes 37 seconds:

- 1 TA: So if we think about the cv term, why is
- 2 it minus cv ?
- 3 Bill: $cv...$ (writes $-cv$) $cv...$ um... before...
- 4 (draws equilibrium line and base) this velocity
- 5 is going to be this way (draws arrow pointing
- 6 right) and the air resistance is going to be that
- 7 way (draws arrow pointing left). Alright, so
- 8 the velocity is positive and the force is
- 9 negative and when the velocity becomes
- 10 negative (draws arrow pointing left) the force
- 11 is positive (draws arrow pointing right) so it
- 12 changes the sign around. Of this (points at
- 13 $-cv$). In here. Alright? Is that right? If this
- 14 is the....

Bill's third attempt is correct, complete, and does not confuse him. After figuring out the sign of the air resistance force, discussion moves to reversing the implied coordinate system and figuring out the sign of the spring force. When finding the sign of the spring force, Bill employs a similar argument, but he uses it much more readily and facilely. For a minute analysis of the argument and its implications about the plasticity of two resources, *force sign* and *coordinate systems*, we examine Bill's successful reasoning about the air resistance force.

Common Chain

In this chain of reasoning, a request for reasoning is followed by elaborative sense-making and checks for consistency. It finishes with an optional appeal for group consensus.

Bill uses this chain five times in a productive six-minute episode, explicitly asking for agreement four times, and explicitly receiving it three times. Other students in other clips also use the chain to formulate and explain their reasoning.

In this example of the chain, we learn about the plasticity of two resources, *force sign* and *coordinate systems*.

Request for reasoning

The chain starts when one participant (here, the TA) asks another (here, Bill) about a previous statement, focusing the discussion (ll 1-2). The chain does not necessarily start with a refocusing question; it can also start with a request for a definition or a statement that an existing definition is incomplete.

Sense-making: elaboration

Bill responds by elaborating on his previous statement, making sense of the physics as he goes (ll 3-13). His hesitancy at the start (line 3) (which is more apparent in the video than the transcript), together with his earlier difficulties with these questions, signify that he is making sense of the situation as he goes, rather than repeating a past explanation. Bill is performing a build-with action. His sense-making indicates that his ideas about the signs of these forces are not solid – they are neither predetermined nor readily available. Furthermore, the detail of his description indicates that his *force sign* resource is plastic.

Sense-making: consistency check

Bill nests a consistency check within his elaboration (ll 7-12). By so doing, he explicitly tests if these newer ideas about the sign of the air resistance force are consistent with differently articulated previous work involving coordinate systems and directionality. Because Bill is facile enough with these ideas to test their consistency, they are sufficiently compiled to be resources, not mere fluid ideas. However, because he needs to test explicitly, *force sign* is plastic to him.

Justification through activity

Bill further justifies his response through reference to an activity: choosing and using a coordinate system (ll 8-12). His tacit use of a coordinate system as justification implies that Bill's *coordinate systems* resource is more solid than his *force sign* one. The implied nature of his coordinate system is further evidence of its solidity.

Social norm: agreement

Bill finishes his chain with an explicit social call for agreement (ll 13-14), signaling that he sees his reasoning as sufficient. Because his call is explicit, he does not see his reasoning as inherently self-obvious, further implying that his *force sign* resource is plastic.

As previously noted, the chain does not always conclude with an explicit verbal call for agreement. Sometimes, the other participants voice their assent without the call. At other times, the chain simply ends without verbal agreement. From the video data we have, which show a top-down view of the work surface and the participants hands, but not their faces, we cannot tell if non-verbal consensus is reached. Had we frontal views, we could capture more gestures and perhaps evidence of non-verbal consensus.

Discussion

The scope of this mechanics problem – the sign of the drag force in damped harmonic motion – is extremely small. However, it is often difficult for sophomore level physics majors and therefore strongly emphasized in the curriculum and homework. This interaction starts because Bill recognizes both his own confusion and the problem's importance to the course. Perhaps in other settings, different contexts would lead to different resources' activation, and this chain may not be employed. For example, had Bill activated his *knowledge-from-authority* epistemological resources instead of his *knowledge-as-invented-stuff* resources,⁹ he might have responded to the TA's request for reasoning very differently. He might have responded that the damping term is negative because the professor said so. Such a response precludes our observation of the extensive sense-making used in the chain.

The scope of this interaction – 37 seconds within a six-minute episode – is also quite small. In that short time frame, we see evidence of some resources' plasticity, but we cannot expect to see their level of plasticity change. In the longer episode, Bill reuses the chain to reason about the sign of the spring force, $-kx$, and to reverse his tacit coordinate system, making leftwards the positive direction. Each time, his reasoning is shorter and less verbally detailed, indicating solidification. Semester-long tracking of some resources may uncover more evidence of plasticity changing.

We have chosen to use plasticity and the RBC model to describe this interaction. Another description can be made using epistemic games, in which the chain of reasoning is identified as a specific game. However, the RBC model combined with plasticity provides a richness of description about the state of the resources involved as well as convenient language for their development during an interaction, a class period, or a semester-long course.

CONCLUSIONS

In nearly-novel situations¹¹ such as this one, students can reason successfully with the tools available to them given enough opportunity. These resources are used with varying levels of assuredness and detail, indicating different levels of plasticity.

As teachers, we are interested in helping students solidify appropriate resources. As researchers, we are interested in detailing the interactions between resources with differing plasticity to give us better insight into the working of student minds. In both roles, connecting student behaviors to differing levels of plasticity allows us to hone our observational skills and build better models of our students and their learning, which may lead to better curricular design.

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