Using Johnson-Laird's Cognitive Framework of Sense-making to Characterize Engineering Students' Mental Representations in Kinematics

Bashirah Ibrahim and N. Sanjay Rebello

Department of Physics, Kansas State University, 116 Cardwell Hall, Manhattan, KS 66506-2601

Abstract. The study investigates the kinds of mental representations constructed by engineering students at Kansas State University when solving problems in the context of kinematics. A cohort of 19 students completed six non-directed tasks posed in different representational forms (mathematical, linguistic and graphical) requiring the generation of linguistic or mathematical models. Individual interviews were conducted immediately after completing the tasks. Based on the students' actions when solving the problems together with their interview responses, two main profiles emerged from the data. The profiles were then related to Johnson-Laird [1] cognitive framework for inferring about the categories of cognitive structures. The framework proposes three types of internal constructs: propositional representations, mental models and mental images. It is argued that comprehension occurs upon the construction of mental models. However, this study revealed that a majority (11 in 19) of the participants use propositional representation while the remaining students construct a mental image.

Keywords: mental representations, kinematics, problem solving
PACS: 01.40.Fk

INTRODUCTION

In physics education research, studies concerned with visualization and its role in teaching and learning have focused mainly on the use of multiple representations and exploring their effect on problem solving performance [2] and conceptual understanding [3]. Research studies have also investigated students' handling of multiple representations [4] as well as particular depictions [5, 6]. Further, studies have explored problem solving performance with a change in representation [7]. The importance of external representations lies in the fact that they "can contribute to students' understanding of physics concepts by attaching mental images to these concepts" [8] (p. 467).

Ainsworth [9] argues that the use of external representations encourages mental model construction which is a crucial element in the learning process. Additionally, according to Gobert [10], for learning to take place when using an external representation, its cognitive version needs to be generated. Consequently, understanding takes place only when these two kinds of representations agree with each other. However, the link between an external representation and its mental version is not straightforward and various factors have been identified affecting this process [11, 12, 13].

This study aims at contributing to the body of knowledge concerned with the relationship between internal and external representations. It uses students' external manifestations when solving different paper and pencil tasks to infer their categories of mental representations. The Johnson-Laird [1] cognitive framework of sense-making underpins this work. It consists of three kinds of mental representations. Propositional representations made up of syntactic structures, for example equations and definitions, which are abstract and meaningless when not integrated within a context. Mental models are analogical representations of real world situations or events which are constructed by perception or imagination. They are personal and tentative in nature. Mental images, based on observations and experience, are internal views of mental models with more visual-spatial information. Comprehension occurs with the construction of mental models which provide a context for the application of propositional representations and mental images hence allowing connections to be made between them.

Greca and Moreira [14] used this framework to classify the cognitive constructs of their sample when solving problems in electricity and magnetism. The present work investigates the following research question: what are the categories of mental representations that engineering students taking a calculus-based physics course construct when attempting kinematics tasks posed in different representational formats?

METHODOLOGY

Data collection instrument

A total of six non-directed kinematics tasks were structured in different representational forms namely mathematical, linguistic and graphical, requesting the generation of either a mathematical solution or a written response. No guidelines were presented to the participants. They were asked to devise their own strategies for attempting the problems. Two of the questions dealt with solving for a value using information provided in a statement. Two of the problems required the formulation of a written response from an acceleration-time graph as well as a kinematic equation for position. The remaining questions required a quantitative solution from motion graphs (position and velocity) and a kinematic equation for velocity respectively.

Background of sample

A cohort of 19 students (one female and 18 male) from a first semester calculus-based physics course, Engineering Physics 1, participated in the study. Except for three of the students, all of the participants had completed a physics course in high school. A total of 14 of them were concurrently enrolled in either Calculus 2 or Calculus 3, while the remaining students were concurrently completing other mathematics courses. Apart from mathematics and biology, the majors included computer, mechanical, industrial and chemical engineering.

Administration of instruments and individual interviews

Participants completed six tasks over two sessions. They were administered while the students were covering the topic of kinematics in class. Each session consisted of three questions which were completed individually and in strict sequence. Upon completing a problem, the participants were not allowed to go back to the previous tasks. Each session lasted for one hour. The participants were allocated 20 to 30 minutes to attempt the problems and individual interviews which lasted 15 to 20 minutes were conducted immediately after completion of the tasks.

Individual interviews were conducted as the interest lies in gaining insight into individual student's problem solving strategies which will inform the kinds of mental representations constructed. The interview questions were designed particularly to gather and have a clear determination of the students' actions (strategies) when attempting the different tasks together with the underlying reasons for adopting these actions. The questions were also geared towards clarifying ambiguous responses or words used by the students.

Analysis

We completed a qualitative analysis of the data. For tasks requesting the generation of a written response, the analysis focused on whether a description or an explanation highlighting the concepts was formulated. From problem statements requiring a quantitative solution, consideration was made for the absence or presence of a diagram and its purpose in the problem solution. When a value was worked out from a mathematical formulation or a motion graph, the analysis takes into account whether the problem was solved using a qualitative or quantitative approach. "Profiles" [15] were then constructed based on the students' actions when attempting the questions. For a particular student, the strategies employed to complete each of the six tasks were considered together and summarized thus described leading to a profile. This process was repeated for all 19 students whereby the descriptors were refined. The profile allocation was repeated independently by another researcher. An inter-reliability rate of 85% was obtained. Two main profiles emerged from the data. They were then related to Johnson-Laird's [1] cognitive framework of sense-making allowing inferences to be made about the cohort's categories of cognitive structures. The interview responses act as additional supporting evidence to facilitate the classification of the students' mental representations according to the cognitive framework.

RESULTS

Table 1 provides an overview for the categories of cognitive constructs that engineering students in a calculus-based physics course operate with when solving kinematics tasks.

The analysis revealed that more than half (11 in 19, 58%) of the sample used propositional representation while around 42% (8 in 19) of the students constructed a mental image when handling different formats of kinematics problems.

Students classified as using propositional representations are characterized by their prioritization for manipulating symbolic representations and pattern matching of information. When interpreting the kinematics equation for position or the acceleration-time graph, a written response in the form of an explanation for the physics concepts was formulated. However, when dealing with the position equation, derivatives were additionally applied in a routine manner to determine an expression for velocity and a value for acceleration. For tasks posed in mathematical and graphical forms requiring a quantitative solution, calculus or kinematics equations were employed. When probed into alternative problem solving strategies, the focus was still on the use of symbolic representations. There was no recognition of qualitative ways for attempting the tasks although there may be awareness of the concepts involved in terms of the slope and area under a velocity-time graph. When solving for a value from the problem statements, preference (seven in 11 students) was given to the direct manipulation of equations. A list is made of all the given and required information together with the formulae which can fit maximum information. The reasons provided for not including a diagram range from the simplicity of the concept, the extent of information presented to the students' routine pattern for problem solving. However, four of the 11 participants included a diagram in their solution with the sole purpose of better visualizing and understanding the situation described in the problem. The visual and mathematical representations were handled independently of each other.

TABLE 1. Profiles for the categories of mental representations in the context of kinematics**.**

Kinds of mental representations	**Interpretation of function and graph**	**Strategy when solving for a value from a function or a graph**	**Strategy when solving for a value from a problem statement**	**Total (%)**
Propositional	An explanation highlighting the physics concepts is formulated. Additionally, calculus is applied in a routine manner when presented with a function.	Equations or calculus are used to solve the problems. Although there may be awareness of the concepts involved in a velocity-time graph, it is not recognized that the problems can be solved qualitatively.	Equations are directly used with pattern matching of given and required information. No diagram is included out of habit, because of the extent of information provided and the simplicity of the concept involved.	7 (37%)
			Diagrams are inclusive in the problem solution with the purpose to better understand and visualize the situation. They are dissociated from the equations used.	4 (21%)
Mental image	An explanation highlighting the physics concepts is formulated. Additionally, calculus is applied in a routine manner when presented with a function.	Equations or calculus are used to solve problems. Although an expression for acceleration is obtained qualitatively from the given velocity-time graph, the problem is solved using kinematic equations.	Either diagrams are inclusive in the problem solution but not related to the mathematical representation or no diagram is explicitly included since the situation was simple and easy to visualize mentally.	8 (42%)
			Total	**19 (100)**

Students who constructed a mental image are described by their use of qualitative reasoning and emphasis on the generation of a diagram but still prefer the manipulation of mathematical formulations. When interpreting the kinematics equation or graph, an explanation was provided together with the mechanical manipulation of calculus when dealing with the equation. Although the students were able to articulate that the area and slope under a velocity-time graph yields the displacement and acceleration respectively, they failed to recognize and use the former concept when handling the problem presented with position-time and velocity-time graphs. An expression for acceleration was obtained qualitatively but then substituted in kinematics equations for solving the problem. When working out a value from the task posed in the form of a velocity equation the use of calculus or other symbolic representations predominates with no spontaneous recognition of qualitative ways to solve the problem unless hinted. Finally, when handling tasks structured in linguistic form requiring a quantitative solution, either the situation was visualized mentally or a diagram was included but not related to the equations used.

CONCLUSIONS

An alternative method for exploring and analyzing students' comprehension of particular concepts is highlighted. Consideration was made to both the cognitive aspect and the external manifestations of students when solving various tasks posed in different representational modes.

The predominance of propositional representation among engineering students was also identified in the work of Greca and Moreira [14] where a different context was used. Based on the cognitive framework, it can be argued that the cohort in this study do not have an understanding of the various concepts presented by the different kinematics problems. Only mental images and propositional mental representations were identified indicating that the visual and symbolic representations are handled independently. None of the participants was found to construct a mental model.

IMPLICATIONS & LIMITATIONS

The study indicates that students characterized as using propositional representation and mental image apply equations as the principal problem solving strategy regardless of the representational format of the task. Usually, students with mental model are described by their prioritization for qualitative reasoning and application of qualitative approach for problem solving. Diagrammatic representations play a key role in their problem solution and are related to the mathematical expression used. To encourage the use of these problem solving approaches, students should be explicitly taught and provided with opportunities to apply the various ways in which problems posed with particular representations can be handled. Emphasis should be placed on the use of qualitative approaches. The central role of visual representations for qualitative reasoning as well as for generating quantitative solutions should be highlighted. Consequently, the possibility of students constructing a mental model during problem solving may be promoted. Moreover, comprehension of concepts can be ensured as according to the cognitive framework, understanding occurs upon the generation of a mental model.

The non-identification of students who constructed a mental model may be due to the small sample size. It is assumed that a larger cohort will provide the possibility of capturing students with all three kinds of cognitive constructs. Also, only two of the six questions administered were concerned with a qualitative solution (written response). An even distribution of questions requiring qualitative and quantitative solutions will be an advantage.

Future work in this area of research is now focusing on exploring students' handling of diverse representations, in particular whether there are any differences in how students with different kinds of mental representations deal with multiple external representations.

ACKNOWLEDGMENT

This material is based in part upon work supported by the National Science Foundation under Grant Number 0816207. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect views of the National Science Foundation.

REFERENCES

1. P. Johnson-Laird, *Mental Models*, Harvard University Press, Cambridge, 1983.
2. P. B. Kohl and N.D. Finkelstein, *Physical Review Special Topics, Physics Education Research*, 1, 010104, 1-11 (2005).
3. A. Van Heuvelen and X. Zou, *American Journal of Physics* **69**, 184 - 194 (2001).
4. D. Rosengrant, A. Van Heuvelen and E. Etkina in 2006 *Physics Education Research Conference*, edited by P. Heron, J. Marx, and L. McCullough, AIP Conference Proceedings 818, Melville, New York, 2006, pp. 49-52.
5. R. J. Beichner, *American Journal of Physics* **62**, 750-762 (1994).
6. L.C. McDermott and P. Shaffer, *American Journal of Physics* **60**, 994 - 1003 (1992).
7. D.E. Meltzer, *American Journal of Physics* **73**, 463 – 478 (2005).
8. L. T. Escalada and D. A. Zollman, *Journal of Research in Science Teaching* **34**, 467- 489 (1997).
9. S. Ainsworth, *Learning and Instruction* **16**, 183-198 (2006).
10. J. D. Gobert, "Leveraging technology and cognitive theory on visualization to promote students' science learning and literacy," in *Visualization in Science Education*, edited by J. K. Gilbert, Dordrecht: Springer, 2007, pp. 73-90.
11. I. M. Greca and M.A. Moreira, *International Journal of Science Education* **22**, 1 - 11 (2000).
12. D. N. Rapp, "Mental models: Theoretical issues for visualizations in science education," in *Visualization in Science Education,* edited by J. K. Gilbert, Dordrecht: Springer, 2007, pp. 43-60.
13. W. Schnotz and M. Bannert, *Learning and Instruction* **13**, 141 - 156 (2003).
14. I. M. Greca and M.A. Moreira*, International Journal of Science Education* **19**, 711 - 724 (1997).
15. B. Ibrahim, A. Buffler and F. Lubben, *Journal of Research in Science Teaching* **46**, 248 - 264 (2009).