An Overview of Physics Education Research on Problem Solving

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Abstract:

An overview of the research on solving tasks commonly used as "problems" in introductory physics courses is presented as an introduction to this domain of physics education research. This overview, which is not intended to be exhaustive, describes many aspects of the complex topic of investigating human problem solving to help identify issues of potential interest to researchers and instructors. The article identifies links between research in physics education and more general research on problem solving, and provides useful references. The article ends with a dozen open questions which the author believes deserve answers.

1. Introduction

Investigating humans engaged in problem solving is a very diverse and complex endeavor, and narrowing the focus to investigating students engaged in problem solving in introductory physics only reduces the scope a little. Early general problem-solving research established several aspects/facets of problem solving that apply to all domains. However, how these general aspects play out within a particular domain depends on the domain, and problem solving within a specific domain has features that are particular to that domain. Because there are many interactions between the problem-solving processes people use and their domain knowledge, presenting research on problem solving cannot be done in a nice, clean, sequential manner. Consequently, deciding how to structure this introductory overview presented the author with a rather challenging problem.

The result, after much thought and struggle, is the following organization: (A) General background on research on problem solving; (B) Aspects of how individuals solve physics problems, which consists of three parts—problem-solving steps, problem-solving strategies and problem-solving states of mind (this framework is borrowed from the ThinkFun® Game Club website: www.thinkfungameclub.com.); (C) Aspects of “teaching problem solving”; and (D) Miscellaneous aspects. Finally, we take stock of where we are in understanding problem solving in physics and identify some possible future research questions.

It is important to point out two things here at the beginning. First, even though the focus of this presentation is problem solving in physics, a number of the studies that are described do not involve physics. Nonetheless these studies, because problem solving is a concern for many fields and shares characteristics in those fields, do tell us useful things that can be applied to problem solving in physics.

The second thing to be aware of is that this presentation is not intended to be in any way exhaustive. If one is interested in pursuing research in this area it would be useful to consult the review by Maloney[1] and the resource letter by Hsu, Brewe, Foster, and Harper[2] to get a better perspective on what work has been done in physics. In addition, consulting problem-solving reviews derived from other perspectives[3-5] is critical for placing research on physics problem solving in the broader context of problem-solving research.

2. General Background

2.1 The Difficulty with Defining "Problem"

As stated above, problem solving involves a very diverse and complex set of processes, and one of the difficulties with interpreting the problem-solving literature is comparing and contrasting the activities associated with people working the spectrum of tasks that are called problems. As Adams points out, "The study of problem solving is almost impossible if you try to look at it as one thing that a person does. It has many facets and to study these it's useful to isolate and identify the individual facets."[6]

There are many definitions for problem in the literature and while they share a number of elements, they are not identical. In many studies the authors never provide a definition of what they are calling a problem, rather it is taken for granted that the reader understands and agrees with the researchers that the tasks presented to the subjects were problems for the subjects. This approach is "problematic."

One useful general definition of a problem is given by Hayes: whenever there is a gap between where you are now and where you want to be, and you don't know how to find a way to cross that gap, you have a problem.[7] It is useful to notice that this definition implies that tasks or situations are not in themselves problems. The problem arises when an individual interacts with the task or situation. Different people interacting with the same task/situation might not all find it to be a problem. The skills and knowledge an individual brings to a situation play a major role in whether that individual thinks of a situation as a problem.

While the Hayes description provides a general definition of a problem, it is actually of limited usefulness for comparing studies that have been done. The reason for this is that anything from an ill-defined, complex, multi-step task to a one-step knowledge recall true/false question can be considered a problem under this definition. However, while some of the processes employed by an individual responding to these two tasks might be the same, there will also be large differences in the reasoning used to deal with them, or with conceptual tasks versus computational tasks. This means that identifying clearly and explicitly what tasks/situations are being used to produce the interactions that qualify as problems for the subjects in a study is very important.

As an example of the variation in the nature of the reasoning that two different types of problems require, consider the following two physics tasks. In task A the mass of an object and the magnitudes and directions of two forces acting on the object are given, and the goal is to find the acceleration of the object. In task B the mass of an airplane, its landing speed and the magnitude of the net (braking) force it can have are given along with the length of the runway, and a question is posed: Can the plane land safely? These two tasks share a number of characteristics, but a specific difference is how the goal of the task is identified. For the first task the goal is to find a specific numerical value. The simplest way to achieve this goal is to use the equation which involves the given values and the unknown. No real analysis of the physical system and its behavior is needed. In contrast, in the second task the solver has to first determine how to use the given information to find something that will enable him/her to answer the question. In this case someone would need to think about what physical principles/relationships are relevant and which one(s) will lead to being able to answer the question. Other task formats (problem types), such as context rich problems,[8] Jeopardy problems,[9] or ranking tasks[10] require other and/or additional reasoning processes.

The wide range of tasks that qualify as problems for novices, who by definition have little domain specific knowledge for a field of study, is part of what makes investigating problem solving a complex domain of study. Adding to the complexity is the fact that problems can be classified in a variety of ways, such as conceptual or quantitative, or well-defined versus ill-defined. For the latter contrast, there is actually a spectrum along which problems can be placed.[11]

Using the definition of problem given above, there are three components of a problem: the initial state, the goal state, and procedures to eliminate the gap between them. A very well-defined problem would have all three aspects explicitly identified. For example, end-of-chapter numerical exercises fit this description of well-defined since the initial state (the given values), the final state (the quantity to be found) and the procedures to be used are all roughly specified. The procedures to be used are not exactly specified, but these tasks are often labeled with the section to which they relate and doing that specifies the procedures indirectly. (Actually one might argue that such tasks do not qualify as problems under the Hayes definition.) Problems that have one, or more, of these three features not explicitly identified then fall further along the spectrum toward ill-defined. The ill-defined end of the spectrum has situations where the problem solver may not even be sure there is a problem; such tasks require explicit identification and definition of the problem, i.e., the initial state, goal state, and nature of the gap, before an attempt can be made to solve them. The vast majority of academic problems are well-defined while real-world problems tend to be ill-defined to various extents.

In light of these aspects of the problem-solving domain, anyone studying the literature in this domain should take care to identify what tasks/situations the authors are calling problems and whether such identification is reasonable for the subjects involved in the study.

2.2 Types of Problems

Since the nature of the tasks/situations that can be problems for people is so broad, it is useful, at least at times, to have ways to classify and distinguish among those that share characteristics. Not surprisingly, considering the previously mentioned broad range of tasks that can be problems, there are various ways to classify problems. Perhaps the most obvious way is on the basis of the knowledge domain involved, i.e., physics or chemistry problems. However, there are other ways to classify problems that can be useful if we are interested in gaining insight into students' solving approaches and abilities.

One general dichotomous classification scheme has insight problems as one category and what might be called non-insight, or systematic, problems as the other category.[12] Systematic problems are those that are amenable to solution by persistent application of known procedures. An example would be the popular Sudoku puzzles for which even the most challenging versions can be solved by systematic application of the guess and test heuristic. (A heuristic is a general problem-solving procedure that one typically employs when he/she doesn't have specific actions available. Much more will be said about heuristics below.) In contrast, insight problems require, somewhere along the way, a breakthrough shift in thinking—the Aha moment—to solve.[13] The identification of these two types of problems immediately raises a number of questions such as: Does experience with systematic problems promote skill at solving insight problems in a domain? Does solving one insight problem facilitate solution of others? We will return to these questions later.

We have already mentioned above the classification of problems along a spectrum from well-defined to ill-defined. Another classification scheme which has some relation to the well-defined/ill-defined spectrum was developed by Johnstone.[14] This scheme for problem types is based on status for the solver of three components: data given, method to be used, and goal. These clearly correspond to the three parts, initial state, procedures for reducing the gap and goal state identified in the Hayes definition. The eight resulting problem types are:

Type	Data	Methods	Goals/outcomes
1	Given	Familiar	Given
2	Given	Unfamiliar	Given
3	Incomplete	Familiar	Given
4	Incomplete	Unfamiliar	Given
5	Given	Familiar	Open
6	Given	Unfamiliar	Open
7	Incomplete	Familiar	Open
8	Incomplete	Unfamiliar	Open

Looking at this scheme it is pretty clear that the vast majority of tasks that students encounter in academic situations are types 1 to 3, but real world problems are more commonly types 4 to 8. How do we help students develop the skills they need to tackle those types of problems?

Jonassen has developed a general typology of problems that includes 11 groups. The different types are: (1) logic problems, (2) algorithms, (3) story problems, e.g., the typical end-of-chapter word problems found in science, technology, engineering and mathematics (STEM) textbooks, (4) rule-using problems, (5) decision making problems (which usually require that problem solvers select maximal solutions from a set of alternative solutions based on a number of selection criteria), (6) troubleshooting, (7) diagnosis-solution problems, (8) strategic performance, (9) policy analysis, (10) design, and (11) dilemmas (problems which involve social and/or ethical conflicts).[4] He contends that his scheme "represents a developmental theory of problem solving" and that "How discrete each kind of problem is, and whether additional kinds of problems exist, is not certain."[11] Clearly not all of these categories are involved in scientific problem solving, but several—e.g., algorithms, story problems and rule-using problems—just as clearly are part of science.

Jonassen also argues that problems can be encountered as either discrete items or as aggregates. He contends that the latter are more common in real work contexts.[11] To what extent experience with one type of problem—troubleshooting—provides useful learning for dealing with a different type—story problems—is very much an open question.

2.3 The Difficulty with "Problem-solving Skills"

The difficulties described above about defining problems carries over into discussions about "problem-solving skills". Teaching "problem-solving skills" is fairly commonly cited as a major goal of physics, or mathematics or chemistry, instruction. However, determining what this means, and whether it can actually be done, is another matter. Even a rather quick exploration of the problem-solving literature brings to light the fact that there is a difficulty with even identifying what qualifies as a problem-solving skill. While many researchers, not to mention teachers, strongly believe that there are general problem-solving skills, those skills are always applied to, and with, knowledge from some specific domain. Consequently, identifying skills that operate in the same way in all domains is very difficult.

On one hand few people would argue with identifying such skills as being able to identify given information, being able to specify what the goal state is (stating the unknown), determining what concepts, principles, and relations are needed, and reviewing one's work as general problem-solving skills. However, these processes are so general that explicitly stating, and trying to teach, them is of limited value, unless we are talking about working with very novice subjects who lack both knowledge and general reasoning skills. In contrast, skills such as drawing appropriate representations (free-body diagrams, equipotential diagrams, or ray diagrams), translating qualitative situations into quantitative relations, or making assumptions in order to constrain a situation to one for which quantitative relations can actually be developed are more specific to context.

What is the value of identifying potential general problem-solving skills like planning, using heuristics and evaluating the solution, when the actual application of each varies widely depending on each situation? How useful is it for an individual to be able to plan, but not have the specific domain knowledge needed to accomplish the plan? On a related aspect, how do we effectively teach such general problem-solving skills, assuming it is worthwhile to do so? What has been called the dilemma of teaching problem solving nicely describes the difficulty of teaching problem-solving skills: "If a student is presented with problems which he/she can solve with his/her current problem-solving procedures they will see no need to change their approach. However, if they are presented with problems on which their current methods fail they are likely to simply give up and conclude that the problem cannot be solved."[8]

2.4 Student Epistemology and Problem Solving

Another aspect of investigating students' problem-solving approaches is the matter of student epistemology. As Hammer found, students tend to have one of two perspectives on learning physics: they either focus on getting the answer, or they work to have the physics make sense.[15] A student who is only interested in getting an answer will approach a typical end-of-chapter problem in general physics one way, i.e., find the equation to plug the given numbers into and solve. In contrast, a student who is trying to make sense of the physics will approach it a different way, e.g., by actually thinking about the physical system involved and how it is behaving.

In addition to the issue of how students frame what it means to "learn physics," there is also the issue of how they frame what "problem solving" means. We could think of the two perspectives on learning leading to different representations of what "solving a problem" means. For many students, in large part because of previous academic experience, solving problems means applying "plug and chug" to some given information to find a numerical answer. As we will see, plug and chug is a version of a common heuristic called means-ends analysis, but limiting one's perspective of what problem solving is to just this process constrains the individual's thinking entirely too much. Getting students who have such a perspective of problem solving to deal effectively with other tasks/situations that are also problems for them is another part of the dilemma of teaching problem solving.

How do we characterize the problem-solving skills of students who have these different representations of what solving a physics problem means? Is it reasonable to characterize their problem-solving skills on the basis of how they do physics tasks when they might actually construct the opposite representation of what solving the problem means if the task was an economic one?

There have also been investigations of how student epistemology affects the development of expertise, in contrast to just acquiring more experience. For example, Bereiter and Scardamalia have argued that sense-making and incorporating the understanding gained from doing so when solving a new problem leads to expertise. In contrast, focusing only on getting an answer leads to a restricted set of procedural skills.[16]

2.5 Methodology

In considering methods used in problem-solving research, the two basic categories of techniques—quantitative or qualitative—employed are the same as those used in other areas of educational research. Chief among the qualitative approaches is the think-aloud protocol, which has been a part of a number of studies. In think-aloud protocols the subject is asked to do a task while verbalizing everything they are thinking about as they do the task. Chi points out that there are two types of think-aloud protocols, "protocol analysis" where the researcher interferes minimally and "verbal analysis" where the researcher probes for self explanations.[17] Quantitative techniques often use paper and pencil tasks, such as open-ended problems, with relatively large samples. What tends to be different in the problem-solving work is the subject pool and the types of tasks used. The following sections describe four common approaches used to study problem solving in physics.

2.5.1 Expert-Novice Studies

This "technique" actually includes a variety of different investigations which share the fact that the performance of two groups of subjects is compared. One group, the novices, usually consists of students taking a general physics course. The other group, the experts, ranges from advanced undergraduate physics majors through graduate students, to experienced college physics professors.

Contrasting the think-aloud protocols of experts and novices working end-of-chapter tasks from general physics textbooks was one of the early approaches to investigating problem solving in physics.[18] The subjects were audio, or video, taped as they worked the task and described their thinking. The researcher's job in such investigations is to take notes and prompt, in a non-directive manner, the subjects if they are clearly thinking but not vocalizing. The researcher strives to be unobtrusive and non-directional. However, he/she can take note of situations where clarification or amplification is needed and develop questions to ask when the subject has completed the task.

An issue with those early expert-novice studies is the nature of the tasks the subjects were working on. For example, Larkin, et al. used end-of-the-chapter tasks from an introductory calculus-based physics text as their problems. These tasks could legitimately be called problems for the novices in their study. However, for the experts those tasks hardly qualified as problems because of the experts' experience and knowledge base. Consequently, what was actually being investigated was how large amounts of experience and an organized knowledge base alter the approach to what can be called basic tasks. And what was found was that the experts' experience and knowledge base—hierarchically organized knowledge of physics principles—enabled them to use a working-forward approach. This working-forward approach was guided by a qualitative analysis and specific physics representations. By contrast, the novices, since they lacked these resources, were left to make use of whatever heuristics they knew. What novices know in this situation is one version of a general heuristic called means-ends analysis, which is typically described as "plug and chug."[18]

In contrast to those early studies, Singh conducted an expert-novice study where the task could legitimately be called a problem for both groups. With one exception, none of the subjects in either group was able to solve the task in the time allowed. Consequently, this study tells us about the more generic, i.e., not specific physics, resources the two groups employed in their efforts to solve the problem. What was found was that in this regard the experts still had more resources available in the form of general heuristics, such as making simplifying assumptions, and specific heuristics, such as thinking in terms of conservation relations. [19]

2.5.2 Card-Sorting Studies

One technique that is unique to the problem-solving research was first used in the classic study by Chi, Feltovich, and Glaser.[20] The technique is sorting cards, each of which has a single "problem" on it, into categories on the basis of how the individual would solve the problem. These researchers chose 25 problems from the ends of the chapters of the mechanics section of a well-known general physics textbook and copied each one onto a separate index card. They then asked their subjects to classify the problems into as many groups as they thought reasonable based on similarity in how they would solve them and to name or describe each category. The researchers also compared how experts—graduate students—and novices—students enrolled in a general physics course—differed in doing the task. They found that the experts constructed categories based on the physical concepts and principles that would be used to solve the problems. In contrast the novices' categories tended to be based on "surface" characteristics of the problems such as objects on a ramp or pulleys.

2.5.3 Isomorphic Problem Studies

Another approach is to have subjects work problems that are isomorphic, i.e., share the same solution process, but which vary in “surface structure.” By systematically varying the features between problems one can investigate how those factors influence subjects’ techniques. For example, Singh used this approach to investigate how factors, such as quantitative versus conceptual task, context, and students’ alternative ideas about friction, affected students’ ability to transfer problem-solving skills from one task to another. [21] Subjects in the study solved more qualitative problems when they were coupled with an isomorphic quantitative problem, but the pairing did not promote more success with quantitative problems. Another major finding was that students’ alternate ideas about friction strongly interfered with their ability to benefit from isomorphic problems that do not involve friction.

2.5.4 Instructional Studies

Another approach that has been used, especially with computer tutors, is to attempt to teach subjects how to solve problems and carefully monitor what they do. One of the early investigations in this area was the study by Heller and Reif. [22] They taught a “prescriptive model” of problem solving, which focused on formulating a theoretical description, i.e., a physical representation, to a group of subjects. The subjects using the prescriptive model performed significantly better than a control group, and also better than a group using a modified version of the model. The prescription these researchers developed was very specific and restricted to problems involving the application of Newton’s second law.

Researchers at the University of Minnesota have developed an instructional program involving innovative problem tasks, students working in cooperative groups and a structured problem-solving strategy.[8,23] The innovative problem tasks are called “context rich problems” and while these tasks vary in several ways they do share some specific characteristics. (Context rich problems can legitimately be classified as ill-defined, although they are usually not radically so.) Among the shared features are: there is a “cover story” which explicitly places the problem solver into the story—this is done to provide motivation; there are no diagrams provided; the problems cannot be solved in a single step; and the goal is not explicitly identified, but rather has to be determined as part of the solution process.

The second part of the Minnesota program is a structured problem-solving strategy modeled on the Polya four step framework—understand, plan, carry out, and look back.[24] The elements of the Minnesota strategy are 1) focus the problem, 2) describe the physics, 3) plan the solution, 4) execute the solution, and 5) evaluate the answer. The strategy is explicitly used by instructors when they solve example problems for the students and the students are required to use it when they submit work. The third element in this program was the use of collaborative grouping, with assigned roles for the students, for working on the problems in discussion sessions.

Leonard, Dufresne, and Mestre reported on an instructional program which required students to write explicit strategies for how they would solve problems. The researchers established guidelines for what constituted a good strategy and gave students practice in writing these strategies. The results of the study demonstrated that students who wrote good strategies were better able to identify relevant principles and remember important ideas at a later testing.[25]

Section 2 Summary: Problem solving—even the restricted problem solving of students in introductory physics courses—is a very diverse and complex domain. What situations are considered to be problems depends on both characteristics of the task and characteristics of the human interacting with the task. Problems can be classified on the basis of a variety of features, but two important classifications are insight versus non insight and the spectrum of ill-defined to well-defined. While a general consensus exists that "problem-solving skills" exist, specifically identifying such skills is difficult since all problem solving involves specific domain knowledge. Student beliefs about the nature of learning physics and the role of problem solving in learning physics (i.e., their epistemology) affects how they approach problem-solving tasks. There are four general approaches—expert-novice, card sorting, isomorphic problems, and instructional studies—that have been used to investigate physics problem solving.

3. Aspects of Individuals Solving Physics Problems

This section will focus on what has been learned about how individuals function when solving problems. The author has chosen to use a framework from the ThinkFun® Game Club website. The ThinkFun® company makes a range of puzzles and games. Within the company's website there is a section devoted to exploring how games can teach problem solving where they state: "If a child can explain to you: the **Steps** s/he used to evaluate a problem, the **Strategy** s/he used to solve the problem, his/her **State of Mind** when working through the problem, then we believe s/he has learned the fundamentals of problem solving." The author believes this applies to our students also and provides a useful guide for research on individual differences in problem solving and for what we need to help students learn in order to be good problem solvers.

3.1 Problem-Solving Steps

In the ThinkFun® framework these are identified as: (1) understand, (2) choose, (3) do, and (4) inspect. Obviously these are quite similar to the four step Polya framework (see above). One can think of these aspects as the overall model of the task and the general strategy one adopts toward the problem. Several features of these steps, such as the fact that students tend to jump right into trying to solve a problem, have been investigated in physics.[18] The common failure of people to inspect or review their solutions is also known. [26] However, the focus will only be on one aspect—the development and use of representations to understand the problem—in this section. The reason for focusing on only this one aspect is to keep this review from getting any longer and to highlight the critical importance of representations.

With regard to the first step mentioned above, understand, a critical part of understanding a problem is the representation the solver constructs, i.e., the internal mental model of the problem situation that the solver forms. There are at least two types of representations associated with a problem and problem solving. One is the internal mental model of the situation which can lead to the type of stumbling block described in the next paragraph. However, there is also the representation of what the process of solving a problem involves, e.g., is the solver only trying to get an answer, or is there also some learning and understanding supposed to come from the process? More will be said about this later in the states of mind section.

The process of forming a representation, i.e., constructing a mental model, has been found to be a critical part of the problem-solving process. One of the common examples of a problem where this feature is critical is the Nine Dots problem. This task has nine dots laid out in a 3 x 3 grid and the goal is to connect all nine dots with four straight lines without lifting the pen or pencil from the paper. Most people find this task very challenging, because the representation they construct has the constraint that the four lines have to fit within the grid defined by the arrangement of the dots. Actually this constraint, which is not part of the task, makes the task impossible, so someone trying the task cannot make progress until they re-represent the task.

One of the first studies of student's approaches to problem solving in physics was that of Chi and her colleagues mentioned in the methodology section above. The subjects in their study were asked to classify a set of mechanics problems based on how they would go about solving them. Recall that the two groups produced different classification schemes. The experts classified the problems on the basis of the physical principle(s) needed to solve them. In contrast, the novices tended to classify the problems on the basis of the objects and their behavior, i.e., on the basis of the surface structure.[20]

Here we see one effect of the problem representation on novices' attempts to solve physics problems. Since the novices lack extensive domain knowledge, they are forced to construct representations that employ ideas and features, i.e., everyday or surface ones, with which they are familiar. In contrast, the physics experts' representations incorporate important physics concepts, principles and relations.

One of the reasons representations are so important in the problem-solving process is because it is actually the representation that the problem solver works on. If the solver constructs an effective representation he/she is on the way to solving the problem, but if the solver constructs an inappropriate representation he/she will not be able to make any real progress until they re-represent the problem accurately.[12] One implication of this feature of problem solving is that effectively representing and re-representing problems is an important problem-solving skill. And this skill includes being aware of the nature of representations and the possible need to re-represent a problem, especially if one is stuck and cannot identify the nature of the sticking point.[26]

Larkin also investigated the use of representations in physics problem solving by investigating how six graduate students approached a virtual work problem in classical mechanics. She found that several of the students started with one representation, e.g., conservation of energy or conservation of angular momentum, with which they analyzed the situation. These students found their initial representations unproductive so they changed one, or more, times until they got to the virtual work representation. Once they developed the virtual work representation they were able to write an equation and solve the problem.[27]

The term representation is actually used two ways when talking about problems. As mentioned above the issue of the internal model of the problem situation that a solver forms, i.e., the representation, is an important aspect of problem solving. However, the term "representation" can also be used to describe the format of the task, e.g., verbal, graphical, diagrammatic, etc. This aspect of physics problems has been the focus of some recent research. Meltzer investigated how students in an algebra-based physics class performed on quizzes which had essentially isomorphic problems in different formats (verbal, equation, graph, or diagram). He found that student performance varied with format and topic.[28]

Kohl and Finkelstein investigated how students handle different types of physics task formats such as verbal descriptions, equations, graphs, and diagrams. They found that experts and novices both used multiple external representations on the tasks involved in their study. However, subjects differed in how they used the external representations. Experts and successful novices spent time using the external representations to make sense of the physics in the task, while unsuccessful novices seemed to draw pictures and free-body diagrams out of a sense of requirement.[29] It is important to realize that, while there is a relation between these external representations such as graphs, free-body diagrams, ray diagrams, etc, and the mental model the problem solver forms, it is the latter that is usually meant when the term representation is used in discussions of problem solving.

3.2 Problem-Solving Strategies

One of the earliest works on problem solving was mathematician George Polya's *How to Solve It,* published in 1945.[24] In this volume Polya tried to teach a method for solving mathematics problems. His focus was general problem-solving approaches or procedures—heuristics—and he brought their importance in mathematical problem solving into focus. Polya actually presented two different levels of heuristics by identifying a broad four step—1) understand the problem, 2) make a plan, 3) carry out the plan, and 4) look back—approach, as well as more specific heuristics, such as working backwards or exploiting symmetry.

Polya presented a prescription for an effective problem-solving strategy. Few students actually employ such prescribed strategies. Rather they have their own natural strategies, which are commonly much less well organized and structured.

The problem-solving strategies identified in the ThinkFun® framework are more specific heuristics such as subdividing, working backwards and guess and check. The process of searching for a way to close the problem gap is where heuristics such as those just mentioned come into play most directly. In a study using the Chinese ring puzzle—a set of six interlocking rings on a oval loop—Kotovsky and Simon found that all of the subjects who successfully solved the puzzle went through a basically similar process of exploration. The subjects initially randomly made progress toward the solution and then backtracked, i.e., made moves that took them away from the solution, then forward again, and more backtracking. Subjects repeated this process until they figured out the structure of the puzzle, at which point they then proceeded directly to the solution. In essence they were exploring the task domain using various heuristics such as trial and error or means-ends analysis until they understood the task, which meant in this case determining the appropriate algorithm. Once that was identified, they applied the algorithm to solve the problem.[30]

In terms of general strategies, one classic study found a definite difference between experts and novices on the tasks usually assigned in general physics courses. Larkin and her colleagues conducted think-aloud protocols with experts and novices who were engaged in solving problems drawn from general physics textbooks. The researchers then developed computer programs that could basically reproduce the performance of the two groups. The two groups differed in the basic approaches they employed, with the novices using a working-backward strategy while the experts used a working-forward process.[18] What the researchers called the working-backward strategy—identifying the unknown, searching for an equations containing the unknown, seeing if all other quantities in the equation are known, etc. (By the way, this working-backward strategy should not be confused with the working-backwards heuristic.)—is familiar to anyone who has taught physics for even a short period as "plug and chug." "Plug and chug" is actually a specific example of a general heuristic process called means-ends-analysis. In contrast the experts (who were working on tasks better described as exercises for them) started with the given information, developed qualitative representations and worked toward the mathematical relation needed to solve the problem. The differences in the performance of the two groups are consistent with the knowledge, and knowledge organization, the two groups brought to the task.

Sweller and colleagues conducted a series of studies which led them to argue that student use of means-ends analysis on typical numerical goal problems was counterproductive in helping students develop a better conceptual understanding.[31-33] The main thesis of these studies was that the use of means-ends analysis—in the form of plug and chug—requires such a large fraction of an individual's cognitive resources that they have few resources left to devote to thinking about the concepts, principles, and relations involved in the task situation.

Heuristic use has not been studied much in investigations of problem solving in physics. An exception is one of the studies by Singh mentioned above. One of the differences she found when she gave experts and novices a task that was a problem for both groups was that the experts had better heuristic skills. The experts thought about behavior in extreme cases and had domain-specific general principles, such as conservation of energy to help them. The novices were often at a loss when their plug and chug approaches were inapplicable.[19]

3.3 Problem-Solving States of Mind

In physics education research this aspect of individual problem-solving efforts has received relatively little attention until recent work on student epistemology. In the ThinkFun® framework, believing that one is a good problem solver, being confident, persevering, being willing to take some risks and being confident are listed in this category. While there have been some studies of how well students persevere (e.g., Schoenfeld, who found that many students thought that if a problem could not be solved within ten minutes then it was essentially impossible), there is very little work on the other aspects mentioned.

However, Schoenfeld conducted several studies in which he explored the effect of having students engage in metacognition about their problem-solving efforts. From observation of many hours of video tapes of students solving problems in pairs or groups Schoenfeld identified the importance of control episodes, e.g., what planning solvers engaged in and whether they monitored and reviewed their work as they went. He found that students' problem-solving abilities could improve if they were explicitly taught about heuristics and how to use them.[34]

Schoenfeld also analyzed student problem-solving attempts using a system that identified four facets of the process: (1) resources, which is the body of knowledge an individual can bring to bear on the task, (2) heuristics, which are the rules of thumb for developing an understanding of the task, (3) control processes, which deal with issues of resource management during problem solving, and (4) beliefs, which are one's world view. The beliefs component of this framework obviously relates to students' epistemologies. As an example of how students' beliefs affect their problem-solving efforts, Schoenfeld found subjects took an empirical approach to a mathematical construction problem that is better addressed using deductive geometry, even though the students were subsequently found to be capable of making deductive geometric arguments.[34]

Tuminaro and Redish looked into student attempts to solve problems from the perspective of what they call epistemic games. They define an epistemic game as: "a coherent activity that uses particular kinds of knowledge and process associated with that knowledge to create knowledge or solve a problem". They found that students use a limited set of these games which "…appear to provide the students with guidance as to what knowledge and procedures to access and what to ignore."[35]

Section 3 Summary: When an individual works on a problem, there are three facets of their thinking that are critical: the steps they take, the strategy they use as a guide for the overall process and their state of mind about problem solving and their abilities to solve problems. The first, and arguably most important, step to solving a problem is forming a mental representation of the task. An inappropriate representation is an actual obstacle which must be corrected before effective work can occur. Novice strategies can not only be unproductive, they can actually be counter-productive. Students' beliefs about what constitutes a problem, how they are solved, and their abilities to solve problems strongly affect what they can accomplish.

4. Aspects of "Teaching Problem Solving" in Physics

4.1 Teaching "Problem Solving"

As Docktor and Heller point out: "Problem solving is one of the primary goals, teaching tools, and evaluation techniques of physics courses."[36] In this section we will look at some additional efforts to teach problem solving beside those described above. All such efforts involve some form of what can be called a global heuristic, having such steps as define the problem or draw a diagram.

Gaigher et al. report on a study of teaching a "structured problem-solving strategy" and its effect on improving students' problem-solving skills and conceptual understanding.[37] Their structured strategy, which they based on an "extended semantic model" of Greeno, has definite similarities to other frameworks, going all the way back to Polya. Their strategy has seven steps: 1) draw a simple diagram to represent the system, 2) indicate the data on the diagram, 3) identify the unknown variable, 4) analyze the problem in terms of physics principles, 5) write down the relevant equation(s), 6) substitute and solve, and 7) interpret the numerical answer. The authors argue that the results of the study "found that students who had been exposed to the structured problem-solving strategy demonstrated better conceptual understanding of physics and tended to adopt a conceptual approach to problem solving."

Cooper et al. describe a study where general chemistry students worked in small collaborative groups on qualitative analysis chemical problems. (Qualitative analysis here means the process of using physical and chemical characteristics to identify unknown compounds.) They report that student problem-solving ability improves as a result of the collaborative work. These researchers also looked at two characteristics of the students to see how they interacted with the problem-solving improvement and found that female students who were classified as "pre-formal in terms of logical thinking" and grouped with "concrete" students showed the largest improvements.[38]

Ogilvie has recently reported on the effect of instruction that employed context-rich problems on student heuristic usage. He found only a small reduction in the frequency of students' usage of means-ends analysis in the form of plug and chug. However, there was also a large increase in students using diagrams and thinking about concepts before searching out an equation.[39] The results of this study could provide useful insights into how to transform novices into experts.

4.2 Computer Tutors

There has been little research on the use of computer tutors to teach problem solving within PER until recently. Hsu and Heller have reported on preliminary work to develop computer tutors as "personal assistants for learning"[40] which is based on the idea of cognitive apprenticeship. The researchers are developing three types of tutors—(1) computer as coach for the student, (2) student as tutor for the computer, and (3) computer as assistant for the student who works more independently—and are currently in the process of evaluating the usefulness of the tutors.[41]

More work on this approach has been done within the computer science/AI community. VanLehn et al. report on using computer tutors in a “minimally intrusive” way to help students learn to solve physics problems.[42] The ANDES tutor developed by these researchers is a very different entity than the tutor described in the previous paragraph. ANDES incorporates an artificial intelligence system designed to try to ascertain the student’s mental and skill level. As such this system is not something an instructor can modify to use with his/her course and students.

Lee and colleagues report on a “Web-based homework tutor MASTERINGPHYSICS” which they argue uses a “Socratic tutoring style.”[43] While not exactly a computer tutor is the sense of ANDES, this system is designed to give assistance to students engaged in problem solving “when needed.” The authors found that students who used tutoring between an incorrect first submission and a second answer submission improved in performance. In contrast, students who did not avail themselves of tutoring actually showed a decline.

4.3 Learning from Worked Examples

Putting this topic here rather than in section B may seem strange, but this goes back to the intertwined nature of exploring problem solving. The reason for putting it here is that this aspect doesn’t deal directly with students solving specific problems, for the most part. There are two classic studies that reported important differences in how strong and weak students studied and used worked examples.[44-45] Both groups of researchers found that the stronger students would spend time when studying worked examples figuring out aspects of the example that they did not initially understand. In contrast, the weaker students tended to take essentially everything for granted and did very little deep processing, i.e., trying to make sense of what was presented in the example. Not surprisingly when the weaker students were engaged in solving problems for themselves they would go back and look for a worked example that they could use essentially as a “template” for the current problem. In contrast, the stronger students would only go back to worked examples when stuck on a specific step in their solution of a new problem, and they would go in search of a specific idea or process, not a complete map.

Chi subsequently developed a model of learning from worked examples that involves two processes: generating inferences and self-repairing. She contends that these processes come into play when the solution omits some information and the students are forced to explain to themselves what is missing. The more important of these is the self-repair where the student has a defect in his/her mental model that produces a conflict with the scientific model conveyed in the worked example. Students who do not recognize and address the conflict will obviously be limited in what they learn from the worked example.[46]

Following Chi's lead, Singh, Yerushalmi and colleagues have investigated how students do with self-diagnosis tasks.[47-49] Among their findings were that the more explicit the instructions the students get, the better their self-diagnosis; self-diagnosis performance did not correlate with performance on a transfer problem; and that students seldom engaged in self-repair during self-diagnosis.

Section 4 Summary: The results of the relatively few studies of teaching problem solving have been mixed, at best, so far. Work with computer tutors is really in its infancy, but likely to grow. Investigations of how students use worked examples indicate that such examples are useful for strong students who engage in deep processing while studying them. However, such examples are of much more limited value for weaker students.

5. Miscellaneous Aspects

5.1 Assessing students' problem-solving skills.

There have been several recent attempts to development an instrument to assess students' problem-solving skill or ability. These efforts have taken different directions.

Docktor and Heller have been developing a rubric to assess "written solutions to physics" problems. This rubric looks at five aspects of the solving process which the researchers argue are at least roughly independent of each other. The five aspects are: useful description, physics approach, application of physics, mathematical procedures, and logical progression, which the researchers also describe as "overall communication of an organized reasoning pattern". There are six numerical scores, 0 to 5, as well as two "not applicable" designations on the rubric. The rubric is still in the development phase but the developers contend: "The rubric provides more meaningful information than standard grading by indicating areas of student difficulty that can be used to focus coaching and improve problem writing."[36]

In contrast to the Docktor and Heller approach described above, Adams examined how subjects worked a complex problem in an everyday context. In other words, instead of developing a way of scoring students work on various physics problems, Adams had subjects work a specific task framed in an "every day" context and carefully examined their work. She identified a variety of skills and processes that the subjects used.[6]

5.2 Faculty Attitudes and Approaches to Assessing Problem Assignments

Henderson et al. have explored how faculty evaluate students' written problem solutions. One of the interesting things they found was that faculty often have difficulty following through on assigning low scores to students' solutions that do not contain explicit communication of component processes such as a diagram if the "answer" is correct. In addition, instructors have a tendency to assume, if the answer is present, that students have used appropriate reasoning or processes even if the students do not communicate the processes and reasoning they employed.[50]

Yerushalmi et al. investigated how instructors' beliefs and values influenced what problems they assigned in introductory physics courses. The researchers found that instructors' goals and beliefs were consistent with the findings of physics education research on how to develop competent problem solvers. However, many instructors failed to actually use such beneficial problem features because those features conflicted with other values held by the instructors, specifically clarity of presentation and reduction of student stress.[51]

5.3 Relation of Mathematics to Problem Solving in Physics

Problem solving in physics commonly involves the application of various mathematical procedures, so there are a number of issues related to the math-physics interaction worthy of investigation. One issue is how well students understand the mathematics on a conceptual basis and how that affects how they can use mathematics in working physics problems. Another issue is what Bing and Redish call the epistemic role of mathematics. As they state: "Mathematics thus fills many different epistemic roles for a physicist. It reflects physical relations, provides a calculation framework, forms a web of interconnected ideas, and provides a packaging system for encoding rules and previous results."[52]

Investigation into these issues is relatively new so there is much yet to learn.

Sherin has investigated how students understand mathematical equations when doing physics problems. He wanted to know what it means for a student to understand a physics equation. He argues that: "successful students learn to understand what equations say in a fundamental sense." He cites an example of two students who write a relation "$F_{up} = F_{down}$" to describe an object on the Earth falling with a terminal velocity and contends that the students have developed this relation from using a balance analogy. Sherin argues that people acquire knowledge elements he calls "symbolic forms" that connect conceptual understanding with equations.[53]

Bing and Redish investigated how students use mathematics when solving physics problems. They contend students use a process they call epistemological framing in which students decide the kind of knowledge, (e.g., do they need to do a calculation, or do they need to map the mathematics to the physical situation, or do they invoke authority, etc.), to employ at various points in the problem-solving process. They argue that students can get stuck when solving problems because they employ a particular epistemological framing rather than a more appropriate one which they also have as a resource. Bing and Redish contend that students have identifiable warrants (reasons) that guide the framing decisions for how to use mathematics when solving physics problems.[52]

5.4 Solving Ill-defined Problems

Researchers have started investigating how students approach ill-defined problems rather than just the typical well-defined textbook style problems. Shin, Jonassen & McGee studied how 9th grade students worked open-ended well-defined and ill-defined astronomy tasks. They found two useful predictors—domain knowledge and justification skill—for success on the well-defined problems. And while these same two predictors worked for the ill-defined problems, there were also two additional aspects—science attitudes and regulation of cognition—involved in making successful predictions.[54] For these authors regulation of cognition includes planning and monitoring skills.

Also recently, Fortus studied subjects, who all had at least a BA in physics, working on one ill-defined and three well-defined problems. The four problems all involved the same topic material. The author reports two main findings: (1) for all subjects the most difficult aspect of solving the ill-defined problem was the process of making constraining assumptions to convert the problem to a well-defined one, and (2) only subjects who had prior experience making such assumptions had much success.[55]

5.5 Problem Solving and Conceptual Understanding/Knowledge Organization

One of the common ideas among physics instructors (and mathematics and chemistry instructors) is that students "learn the physics" by solving problems. However, the research has shown that if the "problems" involved are the traditional end-of-chapter tasks this contention is not valid. Kim and Pak found that even students who had solved more than 1000 traditional tasks in preparation for college entrance exams still had most of the common alternate conceptions about how physical systems behaved.[56] Sherin explored the role of "intuitive knowledge" in physics problem solving. He found that subjects did not always follow formal physics rules, instead appealing to common sense as a guide for how to proceed.[57]

Similar work has been done in chemistry. Nurrenbern & Pickering found that students could solve numerical tasks, but struggle with essentially the same task when presented in a qualitative manner.[58] Sawrey and Cracolice, Deming & Ehlert followed up with further investigations of the relations and differences between effective problem solving and conceptual understanding.[59-60]

There have also been studies investigating the relation between knowledge organization and problem solving. Eylon and Reif explored how two different knowledge organizations—hierarchical versus single-level—affected recall and ability to use the knowledge in problem solving. Subjects working with the hierarchical knowledge organization were significantly better at using the knowledge on complex tasks.[61]

These studies, and others, make it clear that the relations between problem solving and conceptual understanding or knowledge organization are complex ones that we are only starting to understand.

Section 5 Summary: Several physics education researchers have turned their attention to investigating ways to assess student problem-solving skills, but only two approaches have been reported at this time. Studies of faculty attitudes to assessing problem assignments has revealed conflicts between what instructors believe is productive problem solving and the tasks they assign, as well as how they grade those assignments. Another recent topic of research is the relation between mathematics and physics problem solving. These early studies are revealing something about how complex this relationship is. Work has also started on how students approach ill-defined problems, but has already strongly indicated that additional skills, over and above those needed for typical well-defined physics problems, are needed. Another complex area that needs much more work is that of the relations between/among conceptual understanding, knowledge organization and problem solving.

6. Taking Stock

So where are we at this point in time? We have learned a good bit about what students do with ‘traditional’ introductory physics tasks and some things about other aspects of students’ efforts to solve the tasks presented to them. We are also starting to learn things about teaching problem solving and instructors’ approaches and values for this task. In presenting the rough summary below it is important to acknowledge that there are other aspects of problem solving that this review has not discussed, but which are worthy of research. Having said that, what can we say in summary at this time?

We know students use surface features to classify problems and choose solution approaches. We know that many students have an epistemology of “answer making” and consequently, represent the problem-solving process as a matter of finding the right equation to plug the given numbers into. We know good worked examples point out the connections between concepts, principles, relations, qualitative representations, and the problem-solving process. We also know that good students can use worked examples effectively by engaging in deep processing that produces learning about the connections among conceptual knowledge and mathematical representations, among other things. Along the same lines, we know that poor students tend not to benefit from studying worked examples, and so are left trying to “pattern match” assigned problems to worked examples when necessary. We know that problems that are isomorphic from a physics principle perspective will appear different to students based on their knowledge.

We know that students’ knowledge bases are a major source of the difficulty they have for several reasons. First, they lack domain knowledge, which means they are essentially forced to employ primarily "weak" methods such as general problem-solving heuristics. Second, what little domain knowledge they do have is not organized effectively to foster its use in either recognizing or working on the problems. Third, alternate conceptions (ideas such as "motion requires a force" or "acceleration means a change in speed") can prevent students from using appropriate domain knowledge because they believe in the conflicting idea more strongly than the proper idea.

In addition to having poor knowledge of physics, students also have little knowledge of problem-solving heuristics. Trial and error, working backwards, or plug and chug may be the only approaches many students know. Even when they are using these techniques, students are likely to be using them with little, if any, conscious planning.

Because of these student characteristics we know that standard end-of-chapter numerical exercises are indeed problems for them. Much of the work that has been done has employed this type of task as the problems for the subjects and much has been learned. It is also clear there is still more that can be learned about students' interactions with these tasks. However, it also seems that common use of these types of tasks in instruction is one of the reasons for many of the student difficulties found. Use of these 'problems' has led students to a representation of the problem-solving process as one of "plug and chug" and has done little to help students learn the relevant concepts, principles and relations.

We know that problem representations are critical to problem solving. It is also clear that formation of effective representations, especially for more challenging, or ill-defined, problems requires good domain knowledge. Representation, in the sense of problem format, has also been shown to affect students' work.

It is clear from the problem-solving work done to date, in all domains, that problem-solving expertise is domain-specific. That is, someone who is an expert at solving chess problems will not automatically also be expert at solving physics problems. Nonetheless there is evidence for general problem-solving skills, perhaps at several "levels." We can envision at least three levels, the most general level being reasoning processes at the level of metacognition, which is domain independent. The next more specific level is that of employing heuristics, such as guess and test or identifying and applying constraints, which can be applied in all domains, but which application requires some domain knowledge. And finally there are domain specific skills such as drawing free-body diagrams or ray diagrams and using a free-body diagram to write the Newton's second law relation for an object, or when analyzing a DC circuit distinguishing between global aspects—those that apply to the circuit as a whole, e.g., the battery potential—and local aspects—those that are particular to a specific circuit element, e.g., the potential difference across a single resistor in a series sequence.

Real progress has been made in understanding how students deal with various aspects of solving physics problems. However, as mentioned at the outset, the domain is so complex and diverse that we have actually just scratched the surface. The author would argue that there are a couple of things that should be made explicit in all investigations of problem solving. First, the specific nature of the tasks which are qualifying as problems in the investigation needs to be specified carefully. It should be clear whether the tasks are well-defined, or ill-defined, and if the latter, how. In a similar way, whether the tasks are primarily quantitative or qualitative needs to be explicitly identified. Second, there is a definite need when talking about teaching problem-solving skills to explicitly identify what those skills are rather than taking, as most physics instructors do, students' ability to solve typical numerical tasks as the operational definition of problem-solving skill.

7. Some Open Questions/Areas for Future Research

There are many open questions about problem solving in physics that someone could investigate at this point; given below is a quick dozen. Some would be straightforward to tackle, but a number would require a fair amount of ingenuity to research productively. The following list is just a small set of the possibilities.

- What, if anything, do students learn from doing typical end-of-chapter numerical exercises? If students learn little or nothing from doing these tasks, which are nonetheless still challenging for them, why take the time to have students work them?
- What are reasonable alternative tasks to traditional end-of-chapter numerical tasks that can promote conceptual understanding?

- Does having students solve different types of tasks/problems about the same concept help the students learn the concept? If yes, what types and number of problems are needed? If no, why not?

- What mathematical understanding is critical for solving what physics problems? For example, does a student have to have a conceptual understanding of divergence to be able to figure out whether a Gaussian surface encloses a charge?

- What is the value and role of worked examples? What types of worked examples are more effective for promoting conceptual understanding? What about promoting specific problem-solving skills?

- What is the value, if any, of explicitly teaching heuristics, and what heuristics are worth teaching?

- How do, if they do, “scientific reasoning abilities” relate to how well students can solve physics problems?

- What are the advantages/disadvantages of having students work problems in collaborative groups versus having them work the problems individually?

- To what extent, if any, does doing such well-defined problems as end-of-chapter numerical tasks actually build a foundation to improve ability with ill-defined problems?

- To what extent are heuristics important for dealing with ill-defined problems compared to well-defined problems? Which heuristics are important?

- Since problem solving serves three purposes in instructional contexts (a primary goal, a critical teaching tool, and an important evaluation technique) how well do instructors differentiate among these, and how well do students? In addition, to what extent, and how, is it necessary to do so?

- How do novices transition to expertise? What are the patterns in how peoples’ domain knowledge, problem-solving skills, epistemology, etc. morph from novice status to expert status?

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