

## Experiment III

### TRANSMISSION LINES

#### 1. Introduction

Consider the problem of transporting power or information signals over long distances. In principle only a pair of wires is needed to complete the circuit and allow transmission of current. Unfortunately, unless some care is devoted to the arrangement of the wires, the performance may be unsatisfactory. The transmission of power may be very inefficient or, in the case of information, the signals may be severely distorted and attenuated.

Below we discuss the general form of the problem and show that by judicious choice of parameters very desirable characteristics can be achieved. These considerations are relevant whenever AC currents are transmitted over distances where there is appreciable attenuation or change of phase. For a 60 Hz power line this means many miles, but for a fast pulse a few nanoseconds wide it means a fraction of a foot.

#### II. The General Problem

In general, a pair of wires has some distributed series resistance and inductance as well as a distributed parallel conductance and capacitance.

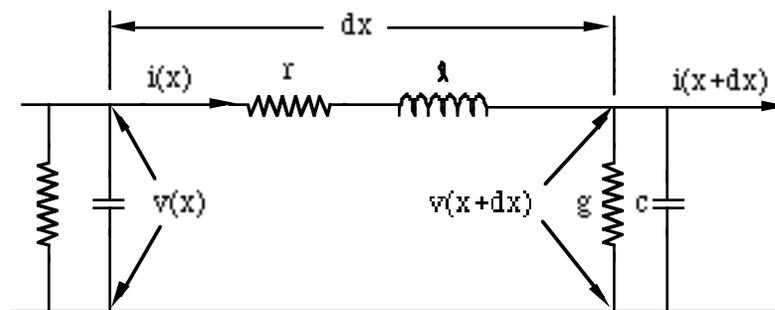


Fig. 1 Representation of a length of coaxial cable with:

- $r$  = series resistance per unit length
- $l$  = series inductance per unit length
- $g$  = shunt conductance per unit length
- $c$  = shunt capacitance per unit length

Then, applying Kirchoff's voltage law,

$$v(x + dx) - v(x) = - \left[ r dx i(x) + \ell dx \frac{\partial i(x)}{\partial t} \right]$$

and from the current law

$$i(x + dx) - i(x) = - \left[ g dx v(x) + c dx \frac{\partial v(x)}{\partial t} \right]$$

$$\text{i.e.,} \quad \frac{\partial v}{\partial x} = -ri - \ell \frac{di}{dt} \quad (1)$$

$$\text{and} \quad \frac{\partial i}{\partial x} = -gv - c \frac{\partial v}{\partial t} \quad (2)$$

Since Fourier analysis allows us to express any time dependence as a superposition of sine waves of different amplitudes and frequencies we consider one sinusoidal frequency component

$$v(x) = v_0(x)e^{j\omega t}$$

$$i(x) = i_0(x)e^{j\omega t}$$

where  $v_0$  and  $i_0$  are complex functions, independent of time and  $j = \sqrt{-1}$ .

Inserting these forms into equations (1) and (2),

$$\begin{aligned} \frac{\partial v_0}{\partial x} &= -ri_0 - j\omega \ell i_0 & \frac{\partial i_0}{\partial x} &= -gv_0 - j\omega cv_0 \\ &= -i_0(r + j\omega \ell) & &= -v_0(g + j\omega c) \\ &= -i_0 z \quad (3) & &= v_0 y \quad (4) \end{aligned}$$

where  $z \equiv r + j\omega \ell$  and  $y \equiv g + j\omega c$

$$\begin{aligned} \frac{d^2 v_0}{dx^2} &= v_0 z y & \frac{d^2 i_0}{dx^2} &= i_0 z y \\ \frac{d^2 v_0}{dx^2} - v_0 z y &= 0 & \frac{d^2 i_0}{dx^2} - i_0 z y &= 0 \end{aligned}$$

Let  $\gamma = \sqrt{zy} = \alpha + j\beta$

$$v_0(x) = V_1 e^{\gamma x} + V_2 e^{-\gamma x} \quad (5), \quad i_0(x) = I_1 e^{\gamma x} + I_2 e^{-\gamma x} \quad (6).$$

Thus,

$$v(x) = V_1 e^{\alpha x} e^{j(\omega t + \beta x)} + V_2 e^{-\alpha x} e^{j(\omega t - \beta x)} \quad (7)$$

$$i(x) = I_1 e^{\alpha x} e^{j(\omega t + \beta x)} + I_2 e^{-\alpha x} e^{j(\omega t - \beta x)} \quad (8)$$

The solution is a linear combination of damped sine-waves traveling in the positive or negative x-direction. The attenuation constant  $\alpha$  and the wave velocity  $\omega/\beta$  depend on the constants  $r, g, \ell, c$  which describe the transmission line.

There is an important relationship between the voltage and current on the transmission line. Consider waves propagating in a given direction, e.g., the +x direction:

$$v(x) = V_2 e^{-\gamma x} e^{j\omega t}$$

$$i(x) = I_2 e^{-\gamma x} e^{j\omega t}$$

From equation (3) the current is related to the voltage.

$$i_0 = -\frac{1}{z} \frac{\partial v_0}{\partial x}$$

$$= +\frac{\gamma}{z} V_2 e^{-\gamma x}$$

thus,

$$I_2 e^{-\gamma x} = \frac{\gamma}{z} V_2 e^{-\gamma x}$$

$$\frac{V_2}{I_2} = \frac{z}{\gamma}$$

$$\frac{V_2}{I_2} = \sqrt{\frac{z}{y}}$$

A similar result could have been obtained for waves traveling in the -x direction. The quantity

$$Z_0 = \sqrt{\frac{z}{y}}$$

is called the characteristic impedance of the line since it is the proportionality constant between voltage and current for a given point on the line.

### III. Special Cases

#### (a) A Lossless Transmission Line

The propagation parameter  $\gamma = \sqrt{(r + j\omega\ell)(g + j\omega c)}$  so if  $r$  and  $g$  are small compared with  $\omega\ell$  and  $\omega c$  respectively, we have

$$\begin{aligned}\gamma &= \alpha + j\beta \\ &= 0 + j\omega\sqrt{\ell c}\end{aligned}$$

Thus,

$$i(x) = I_1 e^{j\omega(t + \sqrt{\ell c} x)} + I_2 e^{j\omega(t - \sqrt{\ell c} x)}$$

which corresponds to waves with velocities  $\pm 1/\sqrt{\ell c}$  and no attenuation. Note too that this velocity is independent of frequency so there is no change in the shape of signals (no dispersion). In this experiment we will study the extent to which this condition is realized in practical transmission lines.

The characteristic impedance for such a line is

$$\begin{aligned}Z_0 &= \sqrt{\frac{r + j\omega\ell}{g + j\omega c}} \\ Z_0 &= \sqrt{\frac{\ell}{c}} \quad \text{for the lossless case.}\end{aligned}$$

#### (b) A Semi-Infinite Transmission Line

Consider a line driven by a sinusoidal voltage source at one end. Because of attenuation in the infinitely long line, we must have

$$|v(x)| \rightarrow 0 \quad \text{as } x \rightarrow \infty$$

Thus the solution from equation (7) must be

$$v(x) = V_2 e^{-\gamma x} e^{j\omega t}$$

and  $v(x)/i(x) = Z_0$  at any point on the line. There is no wave traveling in the  $-x$  direction because there is no mechanism to induce such a component.

**(c) A Finite Length Line Terminated in its Characteristic Impedance**

Suppose a source is connected to one end of the line and a lumped impedance, whose magnitude is  $Z_0$ , is connected across the other end. A wave propagates out from the source, having  $v(x)/i(x) = Z_0$ . In the terminating impedance this same relation holds and thus all the energy of the wave is absorbed in the load.

**(d) A Finite Length Line Terminated in an Arbitrary Impedance**

Suppose a "load" impedance  $Z_L$  is connected to the end of the line. As the initial wavefront travels down the line toward the load  $v(x)/i(x) = Z_0$ . At the load, however, Ohm's law requires  $v(L)/i(L) = Z_L$ . The only way to satisfy this requirement is to add a reflected wave from the load. Let  $v_i$  and  $i_i$  describe the incident wave and  $v_R, i_R$  describe the reflected wave.

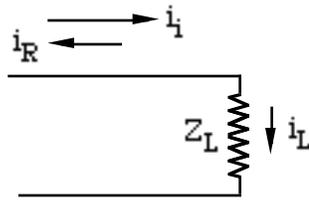


Fig. 2 The end of a transmission line: showing the incident and reflected currents and the current through the load impedance.

Kirchoff's current law applied at the end of the line is:

$$i_i = i_R + i_L \tag{9}$$

Ohm's law at the load is:

$$v_i + v_R = i_L Z_L \tag{10}$$

$$\frac{v_i}{i_i} = \frac{v_R}{i_R} = Z_0 \quad (11)$$

Note the disappearance of the minus sign because  $I_R$  is taken in the opposite sense to  $i_i$ .

Thus,

$$v_i + v_R = (i_i - i_R)Z_L$$

$$v_i + v_R = \left( \frac{v_i}{Z_0} - \frac{v_R}{Z_0} \right) Z_L$$

$$v_i \left( 1 - \frac{Z_L}{Z_0} \right) = -v_R \left( 1 + \frac{Z_L}{Z_0} \right)$$

$$\frac{v_R}{v_i} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Also, from equation (11)

$$\frac{v_R}{v_i} = \frac{i_R}{i_i}$$

so that

$$\frac{i_R}{i_i} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

This ratio of reflected voltage (current) to incident voltage (current) is called the reflection coefficient  $\rho$ .

$$\rho = \frac{Z_L - Z_0}{Z_L + Z_0}$$

Note that if the transmission line is left open (unterminated)  $Z_L \rightarrow \infty$  and  $\rho = +1$ . Thus the reflected voltage wave has the same magnitude and sign as the incident wave. If the line is short circuited  $Z_L = 0$ , giving  $\rho = -1$ , so the reflected wave has the same magnitude but opposite sign to the incident wave. The reflected wave travels back toward the source and may be again partially reflected depending on the impedance it encounters at the source end of the line. The final potential at any point on the line is the superposition of the incident wave and all orders of reflected waves.

Sometimes a so-called "zig-zag diagram" is a useful way to keep track of the wave amplitude at various times and positions on the line. Let  $\rho_s$  and  $\rho_R$  be the reflection coefficients at the sending and receiving ends of the line, respectively.

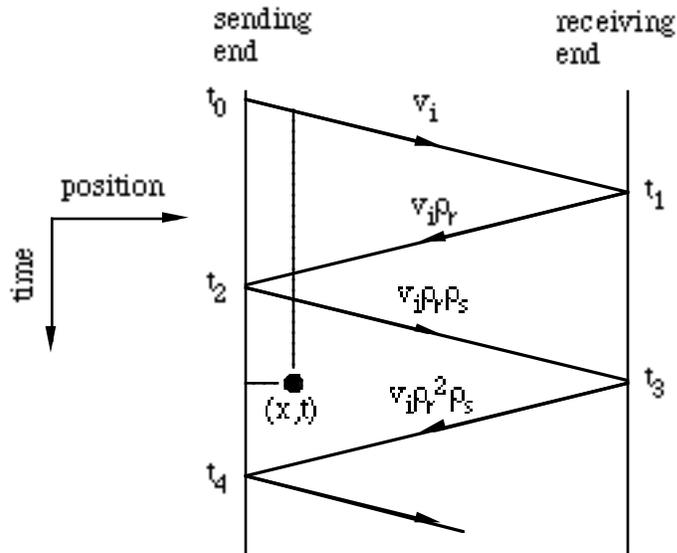


Fig. 3 Zig-zag diagram showing wave amplitudes at various times and positions along a transmission line .

If a constant voltage  $v$  is applied to the line at  $t = t_0$ , at a time and position  $x$  as shown on the diagram:

$$v(x, t) = v_i + v_i \rho_r + v_i \rho_r \rho_s$$

## EXPERIMENTAL PROCEDURE

### A. RG58 transmission line

You are provided with four lengths of RG58 transmission line. This line is a coaxial cable with a solid central wire surrounded by a braided shield.

1. Use the impedance bridge to measure  $r$ ,  $\ell$  and  $c$  of the RG58 transmission line. When measuring the capacitance  $c$ , leave the end of the line open (unterminated). When measuring the inductance  $\ell$ , and the resistance  $r$ , place a shorting cap on the end of the cable. To obtain  $g$  (the

reciprocal of the resistance between the central wire and the outer shield) remove the shorting cap and use a digital multimeter as an ohmmeter.

2. Compute the characteristic impedance and propagation speed for the cable. You should be aware that these parameters can be a function of frequency. You will see in the studies below that the performance of the line deteriorates at high frequencies. The performance is still described by the formalism given above except that the values of  $r$ ,  $g$ ,  $\ell$ , and  $c$  must be modified.
3. One sample cable is in the room. Measure its length and assume that all cables similarly marked are the same length as the sample cable.
4. Be sure that the variable time base and voltage controls on the scope are in the calibrated position throughout the experiment.
5. Place BNC Tee connectors on both vertical input channels and on the external trigger input of the scope. Place an attenuator on the output of the pulse generator. Connect the variable output of the pulse generator to the external trigger of the scope and on to the channel A input. Connect the long RG58 line between the channel A and B inputs. Place a  $50\Omega$  terminator on the open side of the Tee on Channel B. Trigger the scope externally and display the two channels alternately. Set the pulse generator to deliver 50 nsec pulses every 100  $\mu$ sec. Be sure to understand the display on the scope.
6. Measure the transit time through the line. How does this propagation speed compare with your predictions from the measured line parameters? Sketch the pulses from channels A and B in your notebook, recording pulse heights (voltages) and relevant times.
7. Remove the termination from the receiving end of the line. Sketch your scope display. How does the voltage at the receiving end compare with the initial pulse height at the sending end? Explain your findings. Give qualitative arguments for the value of the current at the receiving end. From the pattern of reflections observed on the scope deduce the output impedance of the attenuator.
8. Place a shorting cap on the receiving end. Sketch the scope display. Measure and explain the voltages and polarities you observe.

9. Remove the shorting cap from the receiving end of the line. Connect a  $50\Omega$  terminator in parallel with the sending end by using an extra Tee. What is the effective output impedance now? This is often called the impedance "driving the line." Sketch the pulses and explain their sizes and polarities. Are your observations consistent with calculated reflection coefficients? Estimate the attenuation in the line. What happens to the pulse shape after many reflections? Why?
10. With the setup of part 9, increase the pulse width to at least  $10\ \mu\text{sec}$ . Record and explain your observations.
11. Study the high frequency attenuation of the RG58 line by replacing the pulse generator with a radio-frequency sine-wave generator having an output impedance of  $50\Omega$ . Terminate the receiving end in  $50\Omega$ . How does the ratio  $V_{\text{out}}/V_{\text{in}}$  vary with frequency? What can you say about the high frequency attenuation of this cable?

**B. HH1600 transmission line**

1. This line has been mounted in a box with  $1.7\ \text{k}\Omega$  resistors added as shown on the box to provide termination at both ends. Measure the parameters  $r$ ,  $\ell$ ,  $g$  and  $c$  for this line and calculate the impedance and propagation speed.
2. Measure the propagation speed in the cable. Measure the attenuation as a function of frequency.