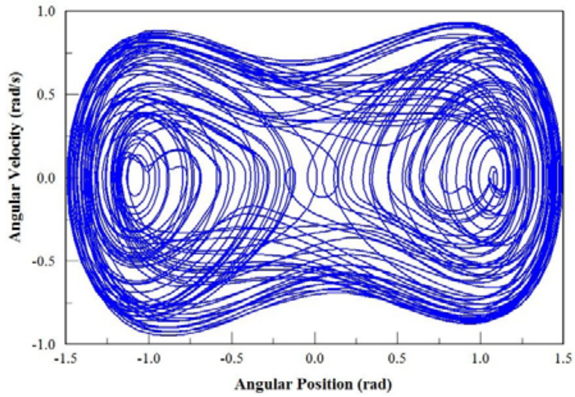

FIGURE 5. Simulated phase-space for the Duffing oscillator plainly shows the existence of the double-well potential.

Clearly, the simulated data are much cleaner than the experimental data shown in Fig. 4. In either case the two minima are definitely distinguishable.

Another important approach to analyzing the data in this experiment was generating Poincare sections for the oscillator. Similar to the phase space diagram, the Poincare section is also a plot of the angular velocity of the pendulum versus its angular position, but only one point is plotted for every period of the driving force. In this way, certain features of the long-term behavior of the oscillator become clear.

Period-doubling effects can be observed by the clustering of points in the Poincare section; two distinct groupings would correspond to period-two motion, four distinct groupings correspond to period-four motion, and so on. Fig. 6 shows an example of period-six motion. This occurred at a driving frequency of 1.4 Hz.

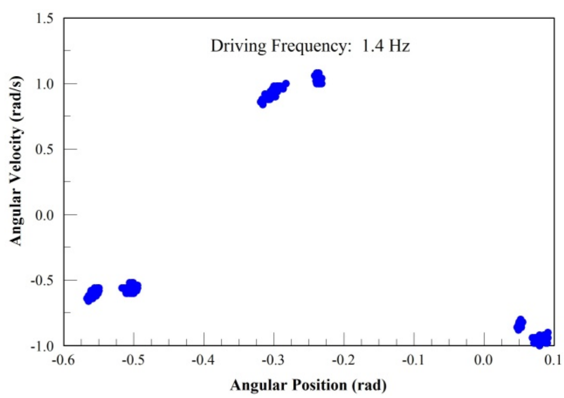

FIGURE 6. Experimental Poincare section of the Duffing oscillator showing period-six motion at a driving frequency of 1.4 Hz.

Chaotic motion can be observed by a lack of distinct clustering, and is accompanied by the emergence of a shape called a strange attractor, over which the points are spread relatively uniformly. An example of a strange attractor that corresponds to the chaotic motion of the pendulum for a frequency of 1.2 Hz is shown in Fig. 7.

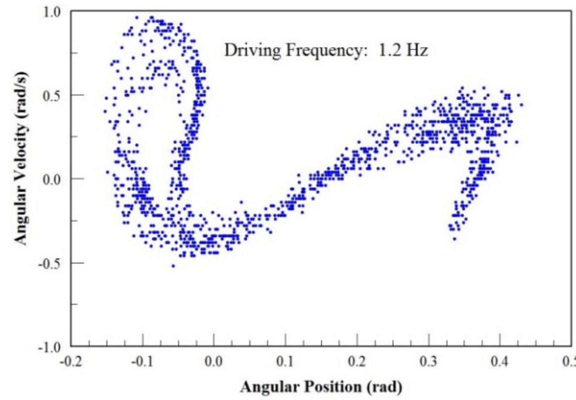

FIGURE 7. Experimental Poincare section for the Duffing oscillator driven at 1.2 Hz shows formation of a strange attractor.

DISCUSSION

This novel version of a driven, inverted, flexible pendulum provides a simple mechanism by which chaotic behavior can easily be produced and measured. The phase-space diagrams and Poincare sections show that the motion measured was truly chaotic at 1.2 Hz. The apparatus is cheap and easy to build from parts found in most undergraduate physics laboratories.

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