The Swinging Atwood's Machine: A Computational Approach

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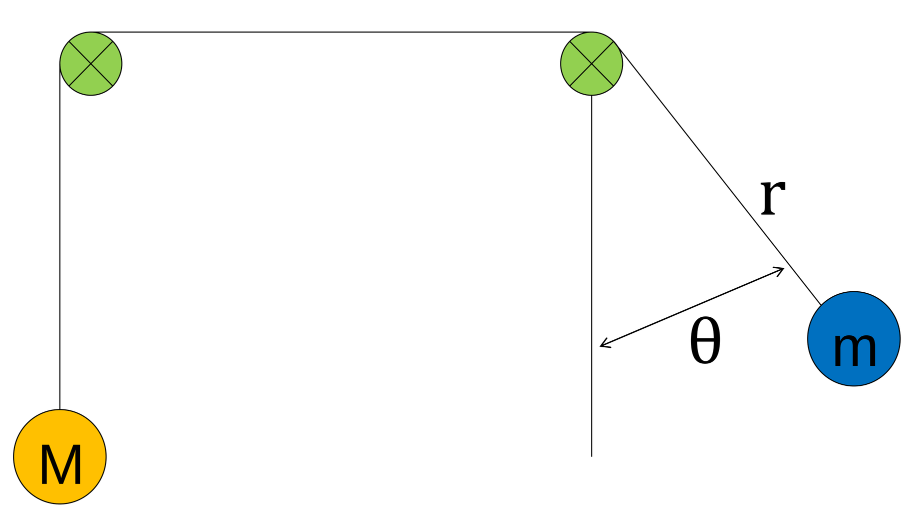
Introduction

The swinging Atwood's machine is an extension of the introductory physics problem in which two masses are connected by a string over an ideal pulley. In this extension of that problem, one mass is displaced by an angle theta. By displacing the mass by theta, the tension of the connecting string changes (unlike the standard problem) as the one mass swings which causes the other mass to move up and down. By changing the ratio of the two masses and the initial angle of displacement, the trajectory of the swinging mass changes. This Java-based simulation allows one to explore the effect that changing the initial conditions has on the path of the swinging mass. The effects of changing the initial conditions will be studied through the use of phase space and Poincaré plots. This simulation will also be compared to experimental results by the same author.

Equations of Motion



The Lagrangian, which is equal to the kinetic energy minus the potential energy (L = T-V), is:

$$\mathcal{L} = \frac{1}{2} M \ddot{r}^2 + \frac{1}{2} m \left(\dot{r}^2 + r^2 \dot{\theta}^2\right) + gr\,(m \,\mathrm{Cos}\,\theta - M)$$

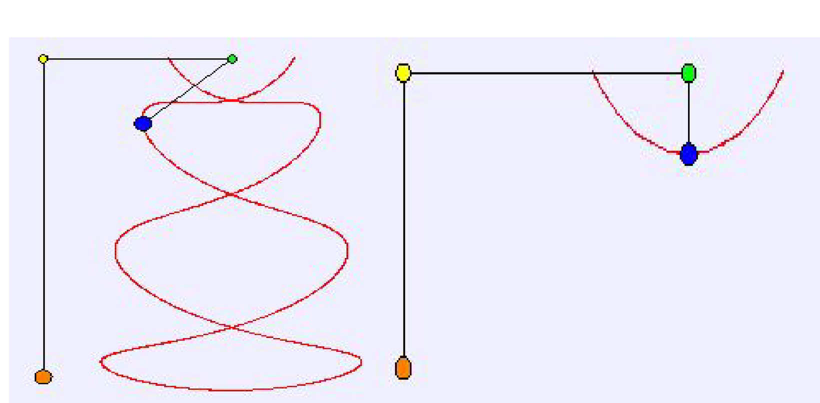
The equations of motions can then be found using the Euler-Lagrange equation. The equations of motion for the two degrees of freedom are therefore:

$$\ddot{r} = \frac{r\dot{\theta}^2 + g\,(\mathrm{Cos}\,\theta - \mu)}{(1 + mu)} \qquad \ddot{\theta} = \frac{2\,\dot{r}\,\dot{\theta} + g\,\mathrm{Sin}\,\theta}{r}$$

Where $\mu = M/m$.

Types of Motion

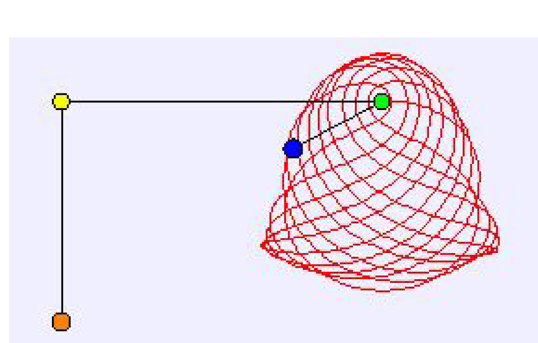
Periodic trajectory: when the path of mass m repeats after a certain time. $r(t+\tau)=r(t)$ and $\theta(t+\tau)= \theta(t)$ for t>0.



$\mu=1.1185$, $\theta=\pi/2$, rDot=0.0

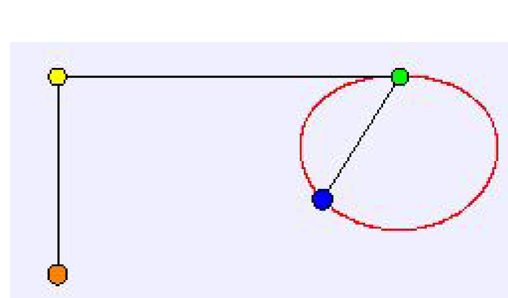
$\mu=1.665$, $\theta=\pi/2$, rDot=0.0

Singular trajectory: when r goes through zero. Mass m collides with the pulley and mass M must reverse direction. r(0)=0.



$\mu=2.3$, $\theta=\pi/2$, rDot=4.0

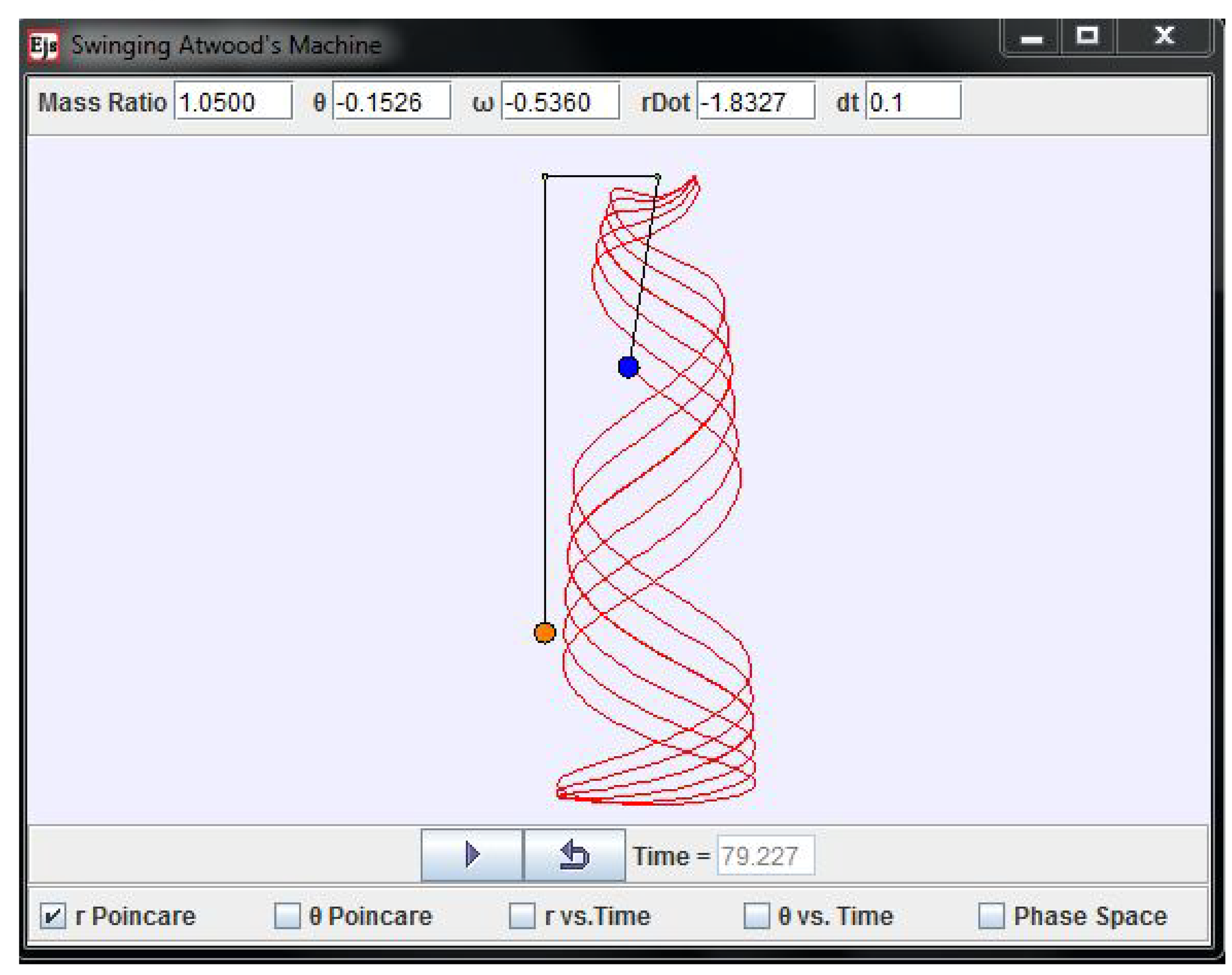
Terminating trajectory: when r returns to zero. $r(\tau)=r(0)=0$.



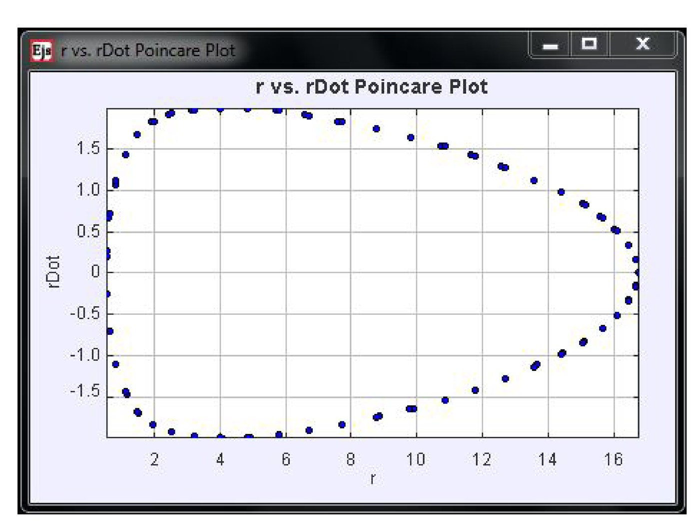
$\mu=3.0$, $\theta=\pi/2$, rDot=4.0

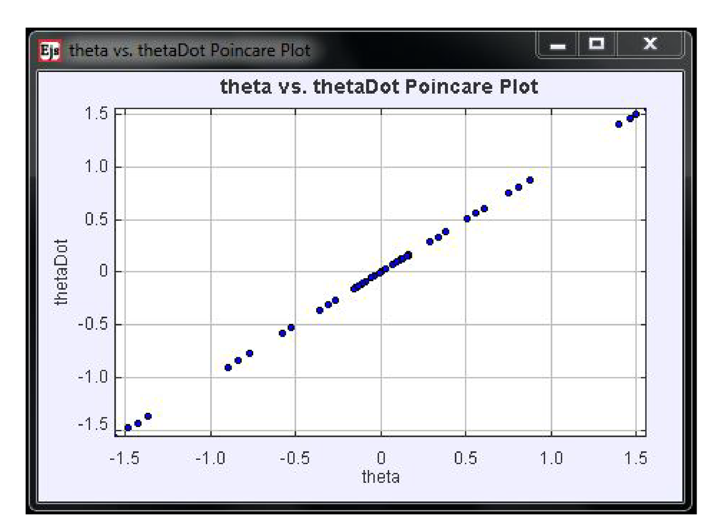
Simulation

The simulation allows a large amount of control and manipulation by the user. The user can change the mass ratio, the initial angle, angular velocity, string length and radial velocity of the mass.

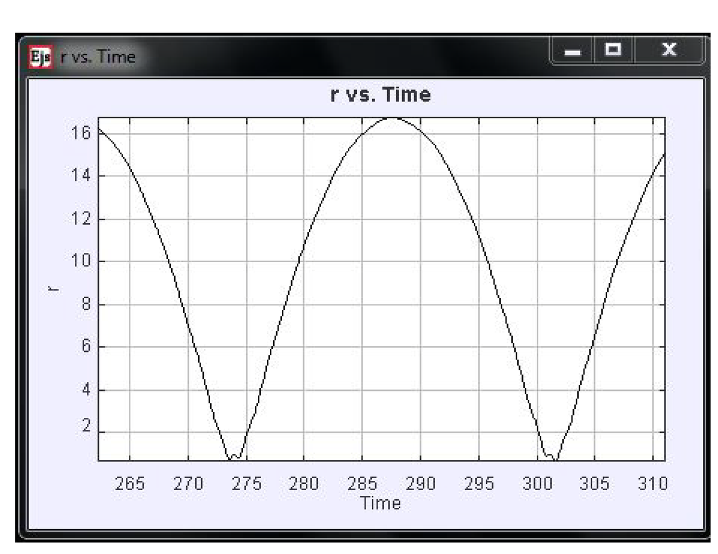


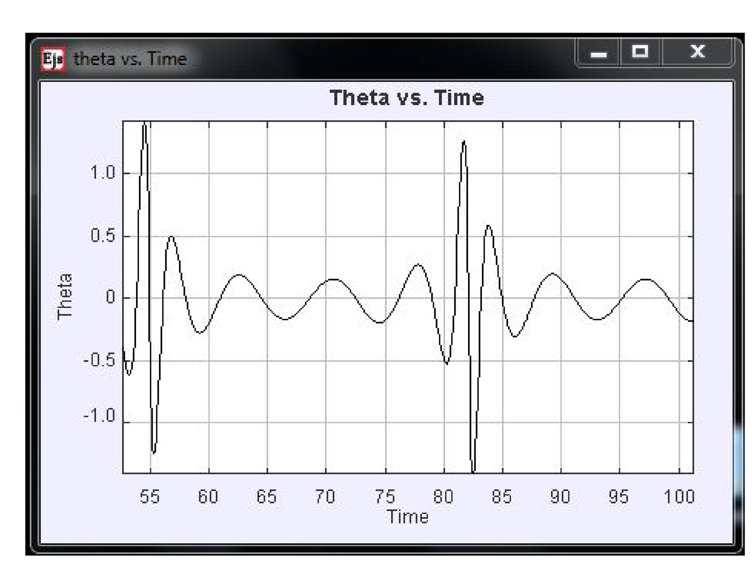
Since the swinging Atwood's machine is a non-linear system, a Poincaré plot is useful in analyzing the motion. A Poincaré map plots the first derivative of a degree of freedom versus that degree of freedom at a certain event in the motion. Those events are θ=0 and rDot=0, respectively.





Observing the r versus time and the theta versus time plots are also valuable in examining the motion of the system. These plots show how one particular degree of freedom changes over time.





References

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"Teardrop and heart orbits of a swinging Atwood's machine". Nicholas B. Tufillaro. *American Journal of Physics.* 26 Aug 1993.

Wolfgang Christian's EJS adaption of ``An Introduction to Computer Simulation Methods: Applications to Physical Systems," by Harvey Gould, Jan Tobochnik, and Wolfgang Christian

OSP Collection on the ComPADRE Digital Library: http://www.compadre.org/osp/

Easy Java Simulations: http://www.um.es/fem/EjsWiki/