Worksheet for Exploration 21.4: Entropy of Expanding Ideal Gas

In this animation, \( N = nR \) (i.e., \( k_B = 1 \)). This, then, gives the ideal gas law as \( PV = NT \). Restart

In thermodynamic processes, the entropy depends not on the path taken but on the end points. It is a "state function" (in contrast with heat and work which depend on the process). Since \( Q = \Delta U + W \) and \( \Delta U = (3/2)nR\Delta T \) (for a monatomic gas),

\[
\Delta S = \int \frac{dQ}{T} = \int \frac{(3/2)nRdT}{T} + \int \frac{PdV}{T} = nR[(3/2)\ln(T_f/T_i) + \ln(V_f/V_i)].
\]

Thus \( \Delta S = (3/2)n \ln(T_f/T_i) + n \ln(V_f/V_i) \) for an ideal monatomic gas (note that \( \ln \) represents the natural log, base e).

In the animations, note that the area under the PV diagram is equal to the work.

a. What is the work done in each case?

\[ W_{\text{isobaric}} = \_\_\_\_\_\_ \]
\[ W_{\text{isochoric}} = \_\_\_\_\_\_ \]
\[ W_{\text{isothermal}} = \_\_\_\_\_\_ \]

b. What is the heat absorbed or released in each case?

\[ Q_{\text{isobaric}} = \_\_\_\_\_\_ \]
\[ Q_{\text{isochoric}} = \_\_\_\_\_\_ \]
\[ Q_{\text{isothermal}} = \_\_\_\_\_\_ \]
c. What is the area under the associated TS diagram? (Note that the choice of the initial entropy is arbitrary.)

\[
\text{Area} \_{\text{isobaric}} = \_\_\_
\]

\[
\text{Area} \_{\text{isochoric}} = \_\_\_
\]

\[
\text{Area} \_{\text{isothermal}} = \_\_\_
\]

d. How does the change in entropy compare for the three processes?

e. Compare your measurements from the graphs to the calculated values found using the equation above for an ideal monatomic gas.

Another way to measure the change in entropy is to use \( Q = mc\Delta T \) or, for a gas, \( Q = CN\Delta T \). In this case,

\[ \Delta S = CN \ln \left(\frac{T_f}{T_i}\right). \]

f. Show that for the isobaric expansion where \( C = C_P = (5/2) \), you get this change in entropy.