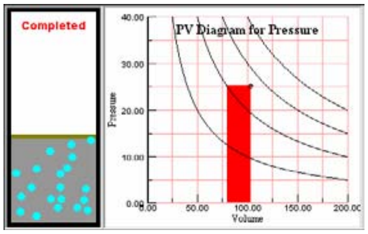
Worksheet for Exploration 20.6: Specific Heat at Constant Pressure and Constant Volume



In this animation, N = nR (i.e., $k_B = 1$). This, then, gives the ideal gas law as PV = NT.

For an ideal monatomic gas, the change in internal energy depends only on temperature, $\Delta U = (3/2)nR\Delta T = (3/2)N\Delta T$.

You may place results for a,b,c in the table below. You should be able to calculate each column before going on to the next column.

a. Calculate the change in internal energy for the three cases.

b. What is the work done in each case? As a reminder, $W = \int P\, dV$, and pressure can (and does in many instances) depend on volume. Calculate the work done in each case using the following two methods, then compare your answers.

- Graphically: To find the work done is to determine the area under the curve ("area" of the red region on the graph). After estimating the area by counting the grid blocks, click the checkbox above to show a calculation of the area (by numerical integration) on the simulation. Explain any significant differences between your estimation and the numerical integration.
- Analytic solution (a little bit of calculus required): When heat is added at constant pressure (isobaric process), then P, pressure, in the above equation for work is simply a constant of integration. When heat is added at a constant volume (isochoric process), the work done is zero. Why? When heat is added at a constant temperature (isothermal), use the ideal gas law (PV = NT) and write the pressure as a function of volume: NT/V (where N and T are constant) and then you can integrate (the answer involves a natural logarithm).

c. Using the first law of thermodynamics, $Q = W + \Delta U$, calculate the heat input and show that it is the same for all three cases.

	$W_{graphical}$	$W_{physlet}$	$W_{analytical}$	ΔU	Q
Const P					
Const V					
Const T					

The specific heat capacity of a material is a measure of the quantity of heat needed to raise a gram (or given quantity) of a material $1^{o}C$. For a gas, it requires a different amount of heat to raise the same amount of gas to the same temperature depending on the circumstances under which the heat is added. If the same amount of heat is added, the final temperatures of the constant pressure and constant volume expansions are quite different (and, for a constant temperature, heat is added but the temperature does not change!).

d. In which case does the heat input raise the temperature the most? Why?

So, if the specific heat capacity of an ideal gas is to have any meaning at all, it must be defined in terms of the process: specific heat at a constant volume or specific heat at a constant pressure.

e. Go back to your calculation of heat in (c). Calculate the constant of proportionality between heat input and the change in temperature for the constant volume and constant pressure cases: $Q = (\text{Constant})N\Delta T$.

Proportionality Constant$_p$=_______

Proportionality Constant$_V$=_______

f. What is the constant in each case? Why is the constant for an expansion at constant pressure greater? (Hint: Think about whether the heat is used only to change temperature or to change temperature and do work.)
   i. Here you should use your expressions for internal energy and for work to develop a theoretical prediction for the constant. This should agree with the results in the paragraph below (and with your measured proportionality constants above).

Generally, we write the heat capacity as a molar heat capacity (where n is the number of moles) and find that for constant pressure $Q = C_P n\Delta T$ and $C_P = (5/2)R$, and for constant volume expansion $Q = C_V n\Delta T$ and $C_V = (3/2)R$.

We began this discussion by noting that for an ideal monatomic gas, the average internal energy is (3/2)T. This comes from kinetic theory and the equipartition of energy where the 3 comes from 3 degrees of freedom. For a diatomic gas, the average internal energy is (5/2)T because there are two more degrees of freedom (rotation).

g. How are the heat capacity at a constant pressure and heat capacity at a constant volume different for a diatomic gas?