Contrasts in Student Understanding of Simple E&M Questions in Two Countries

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Abstract. We administered a survey on electricity and magnetism to two populations of undergraduate students: one from Ohio State University, the other from Bucharest University (Romania). The survey had two multiple part questions. One question invited use of Gauss's Law in several different region. A bare majority of students could solve the simplest problem, that of the electric field inside a conductor. The other question asked about the force on and trajectory of charged particles in regions of magnetic field. These latter questions rely on understanding the Lorentz force and on transfer of general knowledge from classical mechanics studied earlier. Our results show that mechanics knowledge learned earlier does not transfer to electricity and magnetism. Transfer of learning about electricity and magnetism in both countries as measured by our instrument is less successful than we, as teachers, would have wished.

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I. INTRODUCTION

Much work has been done in the past decade in considering the way students approach electricity and magnetism (E&M) problems. Much of this research took place in other countries. For example, Törnkvist et al. [1] found that Swedish students have great difficulties with the concept of field. Electric field lines in the neighborhood of charges and materials constitute a representation that is not generally understood by more advanced university students. Viennot and coworkers found that students have difficulty understanding the principle of superposition [2] and applying it to electric fields. Eylon and Bagno [3,4], working with Israeli teachers and students, found three critical areas of deficiencies in students concerning E&M: (1) qualitative understanding the importance of central ideas; (2) conceptual understanding of the relationships between the electric field and its sources; and (3) ability to apply central relationships in problem solving.

Raduta [5] identified seven general classes of E&M misconception in an extensive literature survey. In addition, Raduta further identified four areas of student "misconceptions" that had been missed or understudied in current research. These are:

A. mathematics-related misconceptions—e.g., misuse of mathematical tools,
B. tempting analogies between electric and magnetic fields—e.g., electric field : charge :: magnetic field : "charge",
C. lack of ability to see the connection between Maxwell's equations and the laws—e.g., Gauss's Law for electric fields; Ampère's law for magnetic fields, the Biot-Savart law for magnetic fields, Faraday's law connecting changing electric and magnetic fields, and Coulomb's law for electric forces, and
D. geometry of the Lorentz force law—e.g., belief that **F** is always perpendicular to **v**.

Despite the body of work, much further work remains to be done on this subject.

We are interested in the similarities and differences between American students and those in other countries. Our null hypothesis is that, overall, students would exhibit no differences between countries. We here take advantage of Raduta's commuting between the U.S. and Romania to begin to perform some tests of possible similarities and differences between students studying physics in these two countries while investigating student understanding of E&M.

We decided to determine whether university physics students who had studied E&M really were able to apply and understand the Gauss and Lorentz force laws, identified by Raduta as underresearched areas. We asked one relatively straightforward (multiple part) question about each topic on a survey administered to over 50 students from each country.

American students attend Ohio State University and had all completed the first quarter of the engineering sequence, which focuses on classical mechanics, and had just finished E&M in the second quarter. The Romanian students were second-year physics students at the University of Bucharest. Generally, the material learned in the last years of Romanian high school physics resemble that learned in the first two years of American university physics courses, so these students should be considered somewhat more advanced than the U.S. students.

In Sec. II, we consider the Gauss's Law question, presented to 74 American and 52 Romanian students (some data are still being evaluated; we report on 8 of these responses here). In Sec. III, we discuss the Lorentz force question using results from all 74 Americans and 52 Romanians. Three students from each country were interviewed in detail about their ideas. In Sec. IV, we summarize and discuss our results.

II. THE GAUSS'S LAW PROBLEM

We first discuss how students approached a Gauss's Law problem (given below). Gauss's Law relates the flux of a field (the surface integral of the field lines penetrating a surface) to the sources of the field enclosed within the surface and is found in student textbooks in both countries. The problem was presented as follows:

A small solid insulating sphere of radius a whose surface is uniformly charged with positive charge +q is surrounded by a larger hollow sphere whose inner surface (radius b) is uniformly charged with charge -q.

a. Determine the electric field at distance r from the center of the sphere when $r < a$.

b. Determine the electric field at distance r from the center of the sphere when $a < r < b$.

c. Determine the electric field at distance r from the center of the sphere when $b < r < c$.

d. Determine the electric field at distance r from the center of the sphere when $r > c$.

e. If you know that the electric potential at $r = c$ is $V(c)$, what is the electric potential for $r > c$?

It is easy to show that **E** is zero for (a) and (c). For (b) and (d), charge is contained within the Gaussian surface, so $E = +kq/r^2$ at any point. For case (e), we integrate to find

$$V(r) = \int_c^r -E_r \, dr = \int_c^r \frac{-kq}{r^2} \, dr = \frac{+kq}{r} - \frac{+kq}{c} + V(c).$$

We have not yet completed the analysis of our results from this question, so we present the preliminary analysis. An overview of our results from the five parts is given in Table 1, which gives details of student answers, and Fig. 1, which shows the numbers of students correctly answering the parts of the Gauss's Law question. It is not apparent from Table 1, but no student correctly answered all parts. Table 2 provides more detail on student answers (all students together).

TABLE 1 Preliminary results from the Gauss's Law question, Americans (A): 74, Romanians (R): 8

Characterization of answers	**A**	**R**
Mostly correct, explanations given	12	2
Some right, some explanation provided, misuse of formulas	3	1
At least one answer correct (usually a or c), explanations and formulas provided	14	0
At least one answer correct (usually a or c), no explanations but formulas provided	19	0
Formulas used, all of them incorrect	8	0
Formulas and explanations, incorrect	5	0
Some explanations, but incorrect	2	0
Blank, or wrong answers; no formulas or explanation provided	11	5

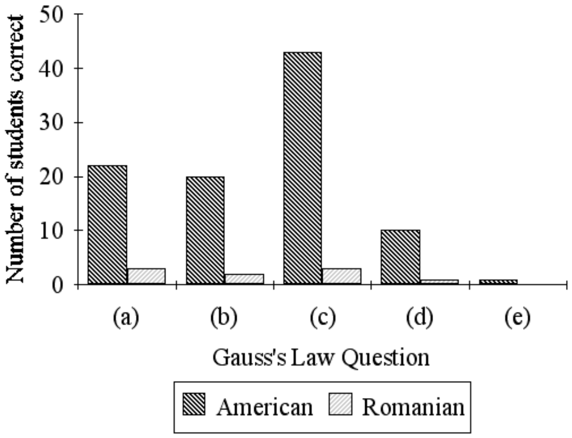

FIGURE 1. Differences in student answers for the Gauss's Law question; Americans (N = 74), Romanians (N = 8).

The "correct" designations were assigned generously: for example, the statement "the electric field is zero because it is within the object" was counted correct in part (c). It should also be noted that students could have simply memorized the maxim "there is no electric field inside a conductor" and produced the correct answer by applying it. Given the lack of detail in student answers (discussed further below), we were not able to identify which students might have found the answer by memorization. Clearly instruction at both universities had left students ill-prepared to find the potential from the field (e, Table 2).

TABLE 2. Numbers of students answering each part correctly.

Task	Americans Correct	(%)	Romanians Correct	(%)
a	22	29%	3	38%
b	20	27%	2	25%
c	43	58%	3	38%
d	10	14%	1	13%
e	1	1%	0	0%

III. THE TRAJECTORY PROBLEM

We turn to the second question:

There is a charged particle inside a region containing a constant uniform magnetic field.

a. What is the magnetic force (magnitude and direction. acting on the charged particle if the initial velocity is zero? What is the trajectory of this particle?

b. What is the magnetic force acting on the charged particle if the initial speed of the charge is v (known, but unspecified here) and the direction is parallel to **B**? What is the trajectory of this particle?

c. What is the magnetic force acting on the charged particle if the initial speed of the charge is v (known, but unspecified here) and the direction is perpendicular to **B**? What is the trajectory of this particle?

d. What is the magnetic force acting on the charged particle if the initial speed of the charge is v (known, but unspecified here) and the angle between **v** and **B** is α? What is the trajectory of this particle?

Expected solutions are $\mathbf{F} = 0$ in a and b, so the velocity remains constant. For case c, $F = qvB$, and the trajectory is a circle about the axis along which the magnetic field lies. For case d, $\mathbf{v}_{||}$ remains unchanged (as in b), while **v** makes the particle move in a circle (as in c), so the particle's trajectory is a spiral.

Students do not distinguish in many cases between a scalar and a vector—students often have a scalar on one side of the equal sign and a vector (or a vector product) on the other side. Even if a student made this mistake, we decided to include them as having a correct answer (but only if everything else was correct).

The results on this set of questions are summarized in Fig. 2. Romanian students are less knowledgeable about magnetic forces than American students, but slightly more likely to be able to identify the trajectories caused by the force when the velocity is perpendicular to the force. Romanian students who answered correctly were more likely to use a greater number of words to express their answers than American students. American students preferred to "let equations do the talking" (see Table 1). As a further example of the differences, we present contrasting excerpts from a Romanian "good" answer to the trajectory problem,

(a) The magnetic force for a charge in an uniform field is: $\mathbf{f} = q\mathbf{v}\times\mathbf{B}$. If $v = 0$, than $f = 0$, and it will not be accelerated in the field, hence we can't speak of direction of the force, but we can say that the magnitude is always zero.

(b) $\alpha(\mathbf{v},\mathbf{B})$, $v \neq 0$, $\mathbf{f} = q\mathbf{v}\times\mathbf{B} = qvB \sin \alpha$; $\alpha(\mathbf{v},\mathbf{B}) = 0$ ... $\mathbf{f} = 0$; Hence the trajectory is a straight line parallel the lines of magnetic field. The equation of the motion will be: $x = x(0) + vt$, where $v = ct$.

and an American "good" answer,

(a) $\mathbf{F}(L) = q\mathbf{v}\times\mathbf{B}$; if $v = 0$, then $F(L) = 0$;   to magnetic field.

(b) $\mathbf{F}(L) = q\mathbf{v}\times\mathbf{B} = qvB \sin \alpha = qvB \sin 0 = 0$; across magnetic field.

Overall, Fig. 2 shows how little students from both groups understand charged particle trajectories, even after instruction.

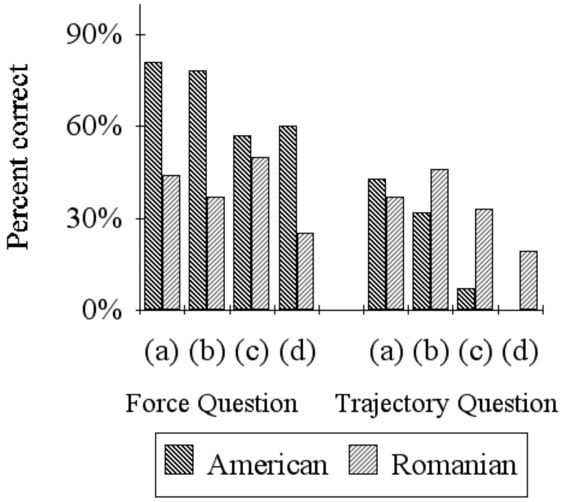

FIGURE 2. Number of American (N = 74) and Romanian (N = 52) students with correct answers to the force and trajectory parts of the question.

To probe this question further, three American and three Romanian students were interviewed and videotaped while answering the Lorentz force questions. They were asked to explain out loud their solutions for each part of the trajectory problem as they thought about them, and we attempted to observe differences between these two small (we hope representative) samples of the larger student populations. From the transcripts, Americans were more likely to speak "telegraphically," while Romanians were more likely to expand their answers (that is, to use longer phrases, which contained a greater number of words) and to refer to matters off the topic. Our results for the analysis of part (c) of this problem [for which $\mathbf{v}$ $\mathbf{B}$] are presented in Table 3 and do not contradict the results obtained in the written survey.

TABLE 3. Characteristics of students' interview answers to the trajectory problem. Names were changed; A: American; R: Romanian.

Student		**Phrases used**	**Words used, part c answer**	**Times referred to other subjects**
Mike	A	short	36	0
Barney	A	short	40	2
Timothy	A	long	68	1
Artenie	R	long	89	4
Ilie	R	long	58	1
Cristi	R	long	67	3

IV. SUMMARY AND DISCUSSION

McDermott, Shaffer and coworkers have attempted to address some student difficulties with field representations and Gauss's Law through Part II of their *Tutorials in Introductory Physics* [6]. Much of their work has been presented at AAPT meetings.

These questions we asked appear simple to physicists. Our results are sobering. Few students were able to answer satisfactorily, even though we interpreted their answers in the best possible light. Few of the answers were complete and none were completely correct. The best results for Gauss's Law occurred for answers that might have been "memorized." We are especially concerned about the lack of student proficiency in solving Gauss's Law problems. Many physics teachers avoid this by skipping Gauss's Law entirely, but that makes it very difficult to justify Maxwell's equations. Most students are unlikely to be exposed to this wonderful example of unification elsewhere in their careers. Given its practical and theoretical importance, it would be near tragic for them to lose that opportunity. The results drawn from the second problem show students' inability to find the trajectory corresponding to a given magnetic force.

These questions—two easy standard problems—were given to the American students two weeks before the end of the quarter, when one would expect students to be comfortable with the main concepts of E&M and students had just studied Gauss's Law. While Romanian students (as second-year students) were more advanced in terms of their coursework than the American students, they exhibited similar limitations. One would have expected them to have been better able to connect their mechanics and E&M knowledge than we found to be the case. The null hypothesis cannot be ruled out except for the better ability of American students to identify the force in the Lorentz force question, where the majority of American students were correctly able to identify the forces, while the majority of Romanian students were not. Overall, the results showed that both groups lacked E&M knowledge.

We are left with many questions about these topics.

•What can we do to teach more effectively? How might we continue to teach Gauss's Law but increase our students' ability to understand the deep connection to Maxwell's equations? Are those who would drop Gauss's Law from the elementary course altogether correct that the gains of doing so are outweighed by the difficulties?

•Why aren't students able to retain simple ideas about kinematics for even one quarter? This compartmentalization has often been observed in other, broader, contexts (for example, between mathematics and student application of their mathematics knowledge in physics, or between knowledge from chemistry courses and application of that same knowledge in physics courses). How can we better communicate the unity of the physical approach?

•Are the (conceivably major) stylistic differences we believe we have observed between these two samples of students doing just two problems representative of the outcomes in other countries? What is the connection between culture, if any, and student approach?

•Are these results characteristic of all international groups? Clearly, there are a lot of points of similarity; both groups of students can be said to belong to the Western tradition of thought, and perhaps groups differing more in culture would perform differently.

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