Perspectives in Quantum Physics: Epistemological, Ontological and Pedagogical

An investigation into student and expert perspectives on the physical interpretation of quantum mechanics, with implications for modern physics instruction.

Charles Raymond Baily

BA (1995), MS (2002) University of Colorado at Boulder

A thesis submitted to the
Faculty of the Graduate School of the
University of Colorado in partial fulfillment
of the requirements for the degree of
Doctor of Philosophy
Department of Physics
2011

This thesis entitled

Perspectives in Quantum Physics:
Epistemological, Ontological and Pedagogical

written by Charles Raymond Baily

has been approved by the Department of Physics

(Noah Finkelstein)

(Michael Dubson)

(5/2/2011)

The final copy of this thesis has been examined by the signatories, and we find that both the content and form meet acceptable presentation standards of scholarly work in the above mentioned discipline.

HRC Protocol # 0205.21

ABSTRACT

Baily, Charles Raymond (Ph.D, Physics)
Title: Perspectives in Quantum Physics: Epistemological, Ontological and Pedagogical
Thesis directed by Associate Professor Noah D. Finkelstein

A common learning goal for modern physics instructors is for students to recognize a difference between the experimental uncertainty of classical physics and the fundamental uncertainty of quantum mechanics. Our studies suggest this notoriously difficult task may be frustrated by the intuitively *realist* perspectives of introductory students, and a lack of *ontological flexibility* in their conceptions of light and matter. We have developed a framework for understanding and characterizing student perspectives on the physical interpretation of quantum mechanics, and demonstrate the differential impact on student thinking of the myriad ways instructors approach interpretive themes in their introductory courses. Like expert physicists, students interpret quantum phenomena differently, and these interpretations are significantly influenced by their overall stances on questions central to the so-called *measurement problem*: Is the wave function physically real, or simply a mathematical tool? Is the *collapse of the wave function* an *ad hoc* rule, or a physical transition not described by any equation? Does an electron, being a form of matter, exist as a localized particle at all times? These questions, which are of personal and academic interest to our students, are largely only superficially addressed in our introductory courses, often for fear of opening a *Pandora's Box* of student questions, none of which have easy answers. We show how a transformed modern physics curriculum (recently implemented at the University of Colorado) may positively impact student perspectives on indeterminacy and wave-particle duality, by making questions of classical and quantum reality a central theme of our course, but also by making the beliefs of our students, and not just those of scientists, an explicit topic of discussion.

ACKNOWLEDGEMENTS

This work has been supported in part by NSF CAREER Grant No. 0448176 and the University of Colorado. I wish to express my most sincere and unending gratitude for the cooperation of all the modern physics instructors and students who made these studies possible, but particularly our own students from the Fall 2010 semester, who were involuntary but enthusiastic participants in this dissertation project. I am also indebted to all the members of the Physics Education Research group at CU, for their continued insight and interest over the span of four years. Most importantly, I am grateful for the enduring support of my advisor and mentor, Noah Finkelstein.

TABLE OF CONTENTS

CHAPTER 1

Perspectives in Quantum Physics

"Why do some textbooks not mention *complementarity*? Because it will not help in quantum mechanical calculations or in setting up experiments. Bohr's considerations are extremely relevant, however, to the scientist who occasionally likes to reflect on the meaning of what she or he is doing." – Abraham Pais [1]

I. Introduction

I.A. Notions of Classical and Quantum Reality

Albert Einstein considered the aim of physics to be "the complete description of any (individual) real situation (as it supposedly exists irrespective of any act of observation or substantiation)." [2] His statement on the *purpose* of science speaks also of his predisposition toward thinking of the universe itself in terms of *realist* expectations: there *is* an objective reality that exists independent of any human observation. In other words, a *complete* description (or theory) of that objective reality is minimally comprised of elements in one-to-one correspondence with physical quantities (such as position or momentum) that are assumed to have definite, objectively real values at all times. [3]

Such assumptions about the nature of reality are built into the equations of classical mechanics – in describing the position of a free electron with a given momentum at some later time, it is already assumed the electron was initially located at some definite, single point in space (x_0), and that its specific momentum (p) *predetermines* its definite location (x) at all later times (t):

(1.1) $$x(t) = x_0 + \frac{p}{m_e} \cdot t$$

Just as with the assumption of *universal time* for all observers in Galilean relativity, these classical assumptions are based on intuitive notions grounded in everyday experience, and it may not occur to classical thinking that these are even assumptions to begin with, or anything other than axiomatic.

Contrast this with an expression from quantum mechanics for the wave function (Ψ) describing a free electron with definite momentum (and therefore definite energy, E) as a function of position and time:

(1.2) $$\Psi(x,t) = A\exp\left[\frac{i}{\hbar}(p \cdot x - E \cdot t)\right].$$

Although the electron's momentum is well defined (Einstein would say it has *reality*), its location may only be described in terms of the probability for where it might be found when observed, which (according to Born's probabilistic interpretation of the wave function) is given by the modulus squared of this complex exponential:

$$(1.3)\quad \rho[x]=|\Psi(x)|^2=\left(A^*\exp\left[-\frac{i}{\hbar}p\cdot x\right]\right)\cdot\left(A\exp\left[\frac{i}{\hbar}p\cdot x\right]\right)=|A|^2=\text{constant}$$

(where the energy/time term has been suppressed). When its momentum is certain, the probability density for its location is constant, and the electron has an equal likelihood of being found anywhere in space – in the mathematics of quantum physics, the location of this free electron is not well defined, and the outcome of a position measurement cannot be predicted with any certainty. If, as Einstein assumed, the electron always exists as a localized particle and is indeed located at a specific point in space at all times (its position also has reality), then a probabilistic (statistical) description of the true state of that electron must be considered *incomplete*. [3] A physical quantity that has some definite value, but is not described by a theory, is known as a *hidden variable*.

According to quantum mechanics, the observables **p** and **x** are *incompatible* (their mathematical operator representations do not commute):

$$(1.4)\quad [\hat{p},\hat{x}]=\hat{p}\cdot\hat{x}-\hat{x}\cdot\hat{p}=\frac{\hbar}{i}\qquad\leftrightarrow\qquad\Delta x\cdot\Delta p\geq\frac{\hbar}{2}$$

and so the position and momentum of a particle cannot be simultaneously described with arbitrary precision. If, in this scenario, the electron does not actually exist at some single, definite location until observed, then the theory of quantum mechanics is *not necessarily* an incomplete description of that reality.

In 1935, Einstein (along with Boris Podolsky and Nathan Rosen, collectively known as EPR) posited a second assumption about the nature of reality (which they considered to be "reasonable"): "If, at the time of measurement, two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done in the first system." [3] This intuitive *assumption of locality* says that the outcome of a measurement performed on some System A can have no influence (or dependence) on any measurement performed on some other System B that is sufficiently isolated from the first. With their *condition of completeness* and the *assumption of locality* in hand, EPR argued that the position and momentum of a particle can be logically demonstrated to have simultaneous reality, and that the quantum mechanical description is therefore incomplete.

Originally formulated in terms of position and momentum measurements, the EPR argument has been reframed [4] in terms of spin measurements performed on systems of *entangled* particles. We imagine a pair of spin-1/2 fermions (Particles A & B) somehow formed in a state of zero total spin angular momentum and traveling in opposite directions.[1] Individual measurements of each particle's spin projection along any given axis will always yield one of two values (*up* or *down*, +1 or −1, however we choose to designate them). Moreover, spin measurements

[1] The argument does not depend on how this is done, but one method would involve preparing a positronium atom in a singlet state, and then dissociating the electron-positron bound state in such a way that the total linear and angular momentum of the system are conserved. [5]

performed on these entangled fermion pairs will always yield opposite values, so long as the measurements are performed along the same axis. In this way, a spin measurement performed along the z-axis for just one of the particles is sufficient for predicting with 100% certainty the outcome of a spin measurement performed on the second particle along that same axis; the actual measurement on the second particle need not be performed, but can be done so as to confirm the predicted outcome. The same is true for spin measurements performed along the x-axis, or any other axis we choose, so long as the axis of orientation is the same for both analyzers. Quantum mechanics says the operators for the x-component and the z-component of spin angular momentum are non-commuting, and therefore obey a similar uncertainty relation as with position and linear momentum (the components of spin angular momentum for a particle cannot be simultaneously specified along two different axes with arbitrary precision):

(1.5) $\left[\hat{S}_x, \hat{S}_z\right] \neq 0 \quad \leftrightarrow \quad \Delta S_x \cdot \Delta S_z \neq 0$

Now suppose the spin of Particle A is measured along the z-axis: an outcome of +1 for Particle A means that a similar measurement performed on Particle B will *always* yield the result of −1, *before* any such measurement on Particle B is actually made. The *assumption of locality* says that any measurement performed on Particle A can have no causal influence on the outcome of any measurement performed on Particle B.[2] EPR would then argue that the z-component of spin for Particle B must have had a definite (real) value at the time of its separation from Particle A, and that this value can be found without disturbing Particle B in any way. If the measurement on Particle B is instead performed along the x-axis, EPR would conclude that the spin projection for Particle B is now simultaneously specified along two different axes, both x (by the second measurement on Particle B) and z (by the first measurement on Particle A); they therefore have simultaneous reality, which is precluded in the quantum mechanical description. [Eq. 1.5] It follows that quantum mechanics offers an incomplete description of the objectively real state of Particle B.

In defense of the completeness of quantum physics, Niels Bohr took issue mainly with EPR's claim of *counterfactual definiteness* – there can be no definite statements (according to Bohr) regarding the outcomes of quantum measurements that haven't been performed. [6] He further insisted that no definitive line could be drawn between the measurement apparatus and the system being measured: "An independent reality in the ordinary [classical] physical sense can […] neither be ascribed to the phenomena nor to the agencies of observation." [7] Bohr ultimately went so far as to redefine the purpose of science: "It is wrong to think that the task of physics is to find out how nature is. Physics concerns what we can say about nature." [8]

---

[2] Assuming the two measurements are performed at space-like separations (the second measurement lies outside the light cone of the first), then special relativity precludes any cause-and-effect relationship between the two events.

I.B. Philosophy or Science?

It is generally agreed in the physics community that Bohr emerged triumphant in this debate, [8] though many physicists of today might feel hard-pressed to say exactly why. If anything, it has been argued that the positivistic[3] aspects of the *Copenhagen Interpretation* [9] (often referred to as the *orthodox* interpretation of quantum mechanics [10]) have contributed to its popularity over the years by allowing physicists to set aside questions of completeness and locality, and instead just use the wave function to "shut up and calculate!" [11] All the same, it was anyways widely believed that J. von Neumann had successfully ruled out the possibility for hidden quantum variables in 1932. [12]

Such beliefs went largely unchallenged [4] until the appearance in 1964 of a groundbreaking paper by J. S. Bell, who had come to realize that Einstein's assumptions were not just a matter of philosophical taste, and could be put to experimental test. [13] In his own discussion of the EPR argument, Bell maintained the assumption of locality in his demonstration that a more complete description of an entangled system of particles could never be specified in terms of hidden variables (a set of one or more unknown parameters, λ). If the result for Particle A is a function of the orientation of its Stern-Gerlach analyzer (unit vector ***a***) and the hidden parameters (λ); and if the outcome for Particle B is similarly a function of both the orientation of its Stern-Gerlach analyzer (***b***) and of λ, we may write this as

(1.6) $A(a,\lambda)=\pm 1 \quad \& \quad B(b,\lambda)=\pm 1$,

where A and B represent the measurement outcomes for Particles A & B, respectively. The assumption of locality may expressed as

(1.7) $A \neq A(a,b,\lambda) \quad \& \quad B \neq B(a,b,\lambda)$

which says merely that A cannot depend on how the other analyzer is oriented, and similarly for B. The anticorrelated nature of measurement outcomes along similar axes may be written as

(1.8) $A(a,\lambda)=-B(a,\lambda)$.

We may then find the expectation value in this local hidden variable (HV) theory for the *product* of the two measurements, by summing over all possible values for the hidden variables, weighted by some probability distribution for the hidden parameters (ρ):

(1.9) $E_{HV}(a,b) \equiv \left\langle \left(\vec{S}_1 \cdot a\right)\left(\vec{S}_2 \cdot b\right)\right\rangle_{HV} = \int d\lambda\ \rho(\lambda) A(a,\lambda)\, B(b,\lambda)$

We now show that the product of the hidden variable expectation values (where locality has been assumed) must obey an inequality that is violated by the predictions of quantum mechanics. We start by writing down the expression:

(1.10) $E_{HV}(a,b)-E_{HV}(a,c)=\int d\lambda\ \rho(\lambda)\left[A(a,\lambda)B(b,\lambda)-A(a,\lambda)B(c,\lambda)\right]$

where ***c*** is some other unit vector along which the spin projection might be measured. Using (1.8) this may be rewritten as

---

[3] In this context, we are referring to a refusal to speculate on that which can't be observed (measured).

(1.11) $E_{HV}(a,b)-E_{HV}(a,c)=-\int d\lambda\ \rho(\lambda)\left[A(a,\lambda)A(b,\lambda)-A(a,\lambda)A(c,\lambda)\right]$,

and then factored by recognizing that the square of any measurement outcome must be equal to +1, so that

(1.12) $E_{HV}(a,b)-E_{HV}(a,c)=-\int d\lambda\ \rho(\lambda)A(a,\lambda)A(b,\lambda)\left[1-A(b,\lambda)A(c,\lambda)\right]$.

We must also have that:

(1.13) $\left|A(a,\lambda)A(b,\lambda)\right|\le+1$

so that taking absolute values in Eq. 1.12, and using the fact that $\rho(\lambda)$ is normalized, gives what is now known as *Bell's inequality*:

(1.14) $\left|E_{HV}(a,b)-E_{HV}(a,c)\right|\le 1+E_{HV}(b,c)$.

The quantum mechanical expectation value for the product of spin measurements is

(1.15) $E_{QM}(a,b)\equiv\left\langle\left(\vec{S}_1\cdot a\right)\left(\vec{S}_2\cdot b\right)\right\rangle_{QM}=-\ a\cdot b=-\cos(\phi)$,

where ϕ is the angle between the unit vectors a and b. The equivalent expression for (1.14) in terms of the quantum mechanical (QM) expectation values is

(1.16) $\left|E_{QM}(a,b)-E_{QM}(a,c)\right|\le 1+E_{QM}(b,c)$.

There are a variety of angles for which this quantum mechanical inequality holds, but for the simple case where the three vectors are situated at 60^0 to each other, so that $\hat{a}\cdot\hat{b}=\cos\left(60^0\right)$, $\hat{b}\cdot\hat{c}=\cos\left(60^0\right)$ & $\hat{a}\cdot\hat{c}=\cos\left(120^0\right)$ we find:

(1.17) $1\ =\ \left|\frac{1}{2}-\left(-\frac{1}{2}\right)\right|\ \le\ 1-\frac{1}{2}\ =\ \frac{1}{2}$

which clearly violates Bell's inequality. Because quantum mechanics correctly predicts the observed expectation values (see below), it follows that at least one of EPR's assumptions (realism and/or locality) is not valid when describing quantum phenomena. If locality is instead *not* assumed in the above argument:

(1.18) $A=A(a,b,\lambda)\quad\&\quad B=B(a,b,\lambda)$

(the outcome for each measurement depends on the orientation of *both* analyzers), there are many functions (A & B) for which the quantum mechanical expectation value (Eq. 1.15) is reproduced, [14] and so it is the assumption of locality that must be set aside, leaving open the possibility for *non-local* hidden variable theories [4]

In 1969, Clauser, et al. generalized Bell's theorem to realizable experiments by allowing for detector inefficiencies, and for the possibility that the measurement correlations are imperfect (less than 100%). [15, 16] From all of this we may conclude that: (A) No local hidden variable theory can reproduce all of the predictions of quantum mechanics; and (B) An experiment may now be devised to differentiate between the two. Various refinements have been made, and a number of loopholes closed over the years, [17-21] but the first definitive test of the assumptions of *Local Realism* was made in 1981 by Alain Aspect and colleagues, [22] when they measured the polarization correlation rate for entangled photon pairs emitted in a radiative atomic cascade. [Fig. 1.1]

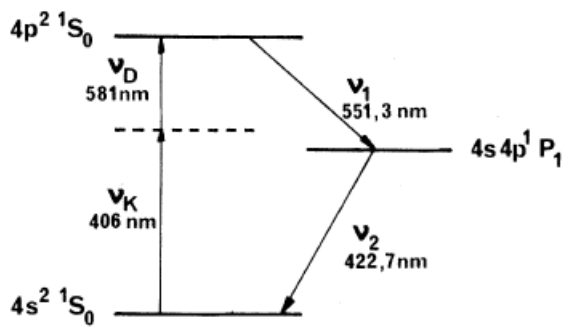


FIG. 1.1. Relevant energy levels of calcium. The atoms are excited by a two-photon absorption process (ν_K and ν_D), and then decay by the emission of two visible photons (ν_1 and ν_2) that are correlated in polarization. [22]

In this experiment, entangled photon pairs were created using a calcium 40 cascade that yields two visible photons (υ_1 and υ_2). The calcium atoms were pumped to the upper level of the cascade from the ground state by two-photon absorption; the average decay lifetime of the intermediate decay state is τ = 4.7 ns. An atomic beam of calcium (with $\rho = 3\times10^{10}$ atoms/cm^3) was irradiated at 90^0 by two laser beams with parallel polarizations, the first a krypton ion laser (λ_K = 406.7 nm), then with a Rhodamine 6G dye laser tuned to resonance for the two-photon process (λ_D = 581 nm). With each laser operating at 40mW, the typical cascade rate was $\sim 4\times10^7$ per second. [22]

In its ground state, calcium 40 has two valence electrons outside a closed shell; with their spins oppositely aligned, the total angular momentum (spin plus orbital) of this state is J = 0. The upper level of the cascade is also a J = 0 state, and the intermediate state is J = 1, so that the excited atom has two possible decay paths (m = +1 or m = −1) on its way to the ground state. By conservation of angular momentum, any photon pair (υ_1 & υ_2) that *happen to be emitted back-to-back* in this process must therefore have the same circular polarization: either both right-handed (R) or both left-handed (L). The entangled state of the two photons may then be written as:

(1.19) $|\Psi_{12}\rangle = |R_1\rangle|R_2\rangle + |L_1\rangle|L_2\rangle$.

Einstein would argue that each atom always decays by either one path or the other, so that each photon pair is produced in just one of the two polarization states with equal probability (determined by some hidden parameter), but that we cannot know which one until the photon pair is observed. He would say that the superposition state describing each photon pair is a reflection of *classical ignorance* (a lack of knowledge regarding the true state of the photon pair). Bohr would argue that the superposition state is a reflection of a more *fundamental uncertainty*, and that each photon pair exists in an indeterminate superposition state until measured. Observing only one of the two photons instantly *collapses* the superposition at random into just one of the two definite states with equal probability. The collapse must be instantaneous if the two measurements occur at space-like separation, since there would be no time for a signal to travel between the two photons regarding how they should behave when they encounter a polarizer.

Aspect measured the rate of coincidental detection of back-to-back photon pairs with the same type of polarization along a variety of relative angles, and found that these measurements violated the generalized Bell's inequality by more than 13 standard deviations, providing strong evidence against *any* local hidden-variable theory. [Fig. 1.2]

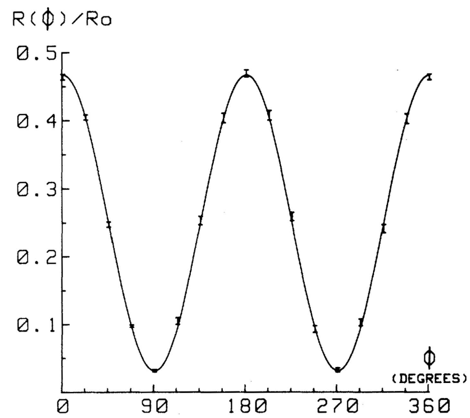


FIG. 1.2. Results of the first Aspect experiment testing Bell's inequality – normalized coincidence rate as a function of relative polarizer orientation. Error bars represent one standard deviation, and the curve drawn through the data points is not a best-fit curve, but rather what is predicted by quantum mechanics. [22]

I.C. Wave-Particle Duality and Ontological Flexibility

In arguing for the incompleteness of quantum mechanics, Einstein was essentially questioning whether quantum mechanics could be used to describe the real state of individual particles, or merely a statistical distribution of measurement outcomes for an ensemble of similarly prepared systems (e.g., a coherent beam of single photons or electrons), where the final distribution of results is determined by some set of unknown, hidden parameters (initial position and/or momentum, for example). Does the *instantaneous collapse of the wave function* represent a change in knowledge of the observer regarding the true state of an individual system, or does it represent a physical transition for that system from an indeterminate state to one that is definite? Erwin Schrödinger famously questioned exactly when this so-called *collapse* is supposed to take place, when he ironically proposed a thought-experiment in which a macroscopic object (in this case, a cat in a box) is imagined to be in a superposition of two states (dead or alive) right up until the moment it is observed (when we open the box). [23] By 1950, Einstein had few allies remaining in the assault on realism in physics, as he expressed in a letter to Schrödinger from that time:

> "You are the only contemporary physicist, besides Laue, who sees that one cannot get around the assumption of reality – if only one is honest. Most of them simply do not see what sort of risky game they are playing with reality – reality as something independent of what is experimentally established. They somehow believe that the quantum theory provides a description of reality, and even a complete description; this interpretation is, however, refuted most elegantly by your system of radioactive atom [plus] cat in a box, in which the Ψ-function of the system contains the cat both alive and [dead]. Is the state of the cat to be created only when a physicist investigates the situation at some definite time? Nobody really doubts that the presence or absence of a [dead] cat is something independent of observation. But then the description by means of the Ψ-function is certainly incomplete, and there must be a more complete description." [24]

The practical significance of EPR's argument (and its refutation via Bell's Theorem) was not truly realized until the mid-to-late 1970's – as reflected in how their paper had a total of only 36 citations in *Physical Review* before 1980, but added 456 more citations in the period from 1980 to June 2003. [25] A similar trend can be seen [Fig. 1.3] in the belated, sudden increase in citations of Bell's paper, "On the Einstein-Podolsky-Rosen Paradox." [26]

It was the development during the 1970's and onward of experimental techniques for isolating and observing single quantum objects like photons, electrons, and atoms that caused physicists to take ideas about "quantum weirdness" seriously. According to Aspect: "I think it is not an exaggeration to say that the realization of the importance of entanglement and the clarification of the quantum description of single objects have been at the root of a *second quantum revolution*, and that John Bell was its prophet." [27]

Long before any such experiments were possible, physicists were already arguing for their preferred interpretations of quantum mechanics in terms of the individual behavior of quanta. In his own book on quantum mechanics, Dirac considers a thought experiment wherein individual photons are directed toward a beam splitter, and have equal probability of being transmitted or reflected. The quantum mechanical wave describing the probability for detecting the photon coherently splits at the beam splitter (it is both reflected and transmitted), but the result of any detection "must be either the whole photon or nothing at all. Thus the photon must suddenly change from being partly in one beam and partly in the other to being entirely in one of the beams." [28] Dirac argued this as a point of principle, despite there being no specific experimental evidence at the time for this assertion.

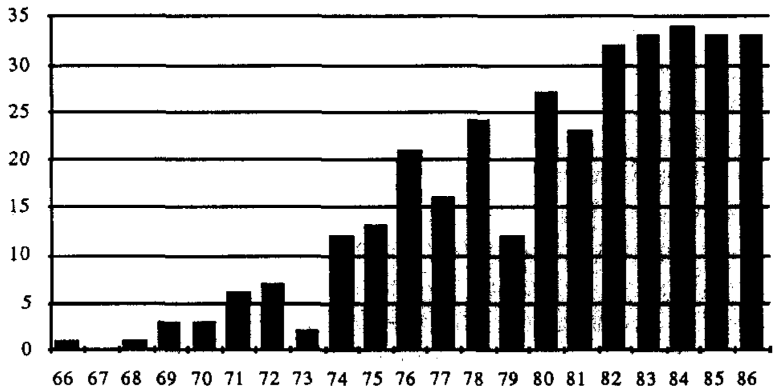


FIG. 1.3. Number of annual citations [1966-1986] of "On the Einstein-Podolsky-Rosen Paradox," by J. S. Bell, *Physics* **1**, 195 (1964). [26]

Definitive evidence for such behavior was most elegantly demonstrated by Grangier, Roger and Aspect in 1986 [29] using the same calcium 40 cascade photon source used in Aspect's first experiments testing the assumptions of Local Realism. [Fig. 1.4] Their first experiment was designed to demonstrate the particle-like behavior of photons; the second was meant to demonstrate the wave-like behavior of photons in a nearly identical situation. The experimental setup was along the lines proposed by Dirac in the thought experiment described above.

In each of these two experiments, the first photon (ν_1) emitted in the calcium cascade serves as a trigger when detected in PM1, and the electronics opens a gate for a time equal to twice the lifetime of the intermediate state ($2\tau \sim 10$ ns), telling counters N_A & N_B to expect a second photon (ν_2); a coincidence counter (N_C) is triggered if both photomultipliers fire during the short time the gate is open. The path to the beam splitter (BS1) from the source is collimated such that the second photon must have been one that was emitted back-to-back with the first, which greatly reduces the luminosity of this "single-photon" source. A set of mirrors (M_A & M_B) direct the second photon toward either PMA (it was reflected at BS1) or PMB (it was transmitted at BS1).

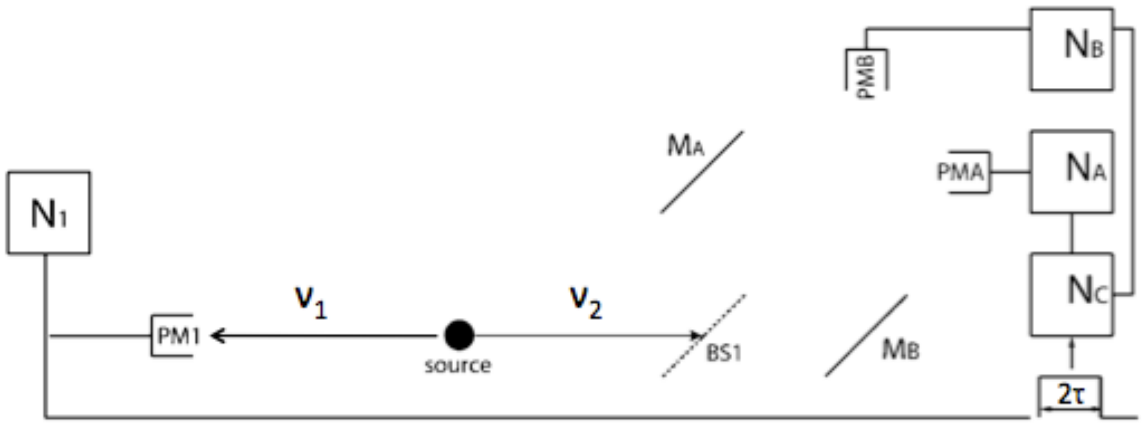


FIG. 1.4. Schematic diagram for the first anticoincidence experiment by Grangier, et al. PM1, PMA & PMB are photomultipliers; N_1, N_A, N_B &N_C are counters; BS1 is a beam splitter; M_A and M_B are mirrors. [29]

If the photon energy were coherently split at the beam splitter (wave-like behavior) it would be expected that energy would be deposited into the photomultipliers coincidentally, and that they would therefore fire together more often than separately. If the photon were instead either transmitted or reflected at the beam splitter (but not both; particle-like behavior) we expect the photomultipliers to always be triggered separately, so long as only one photon is in the apparatus at a time. We can quantify how often this is happening by defining an *anticorrelation parameter* (α):

$$(1.20)\quad \alpha \equiv \frac{P_C}{P_A \cdot P_B}$$

where P_A is the probability for PMA to fire, P_B is the same for PMB, and P_C the probability for both to fire during the time the gate is open.

- If individual photons are always detected in only one photomultiplier or the other (particle-like behavior), then $\alpha = 0$ since P_C must be zero (there is zero probability that the two detectors click together during the time the gate is open).
- If the detectors are firing randomly and independently, then $\alpha = 1$, since P_C is just the product of P_A and P_B. This would be consistent with either many photons being present in the apparatus at once, or with waves depositing energy over time and randomly triggering the detectors.
- If there is a clustering of counts (higher than random probability that both detectors click together; consistent with wave-like behavior), then $\alpha > 1$ (i.e. P_C is greater than just the product of P_A and P_B).

The results for this first experiment show that, more often than not, photons are being detected in either one photomultiplier or the other during the time the gate is open, which is consistent with the predictions for particle-like behavior ($\alpha \geq 0$), while being inconsistent with the predictions for wave-like behavior ($\alpha \geq 1$). [Fig. 1.5 – the solid curve represents the predictions of quantum mechanics; error bars represent one standard deviation. It is necessary to extrapolate the measurements to "single-photon" intensity ($\alpha = 0$) since the apparatus has a *dark rate* of ~300 counts/second.] We *interpret* these results as meaning that each photon must always take one path or the other on its way to detection – it is either reflected at the beam splitter or transmitted (but not both).

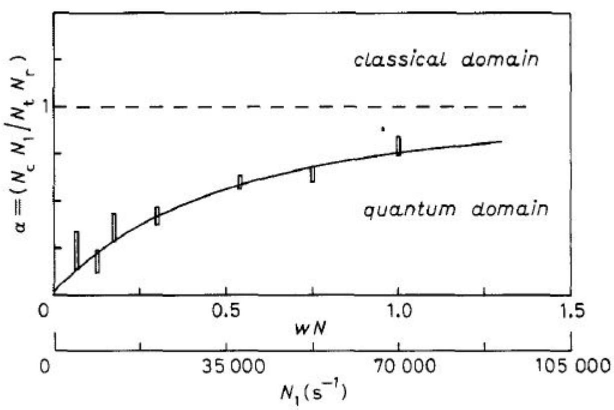


FIG. 1.5. Results from the first photon anticoincidence experiment performed by Grangier, et al. The anticorrelation parameter plotted as a function of the counting rate in PM1 (equivalently, the luminosity of the "single-photon" source). [29]

The experiment can be run a second time after a slight modification is made: inserting a second beam splitter into the paths taken by the photons (BS2). [Fig. 1.6] With BS2 in place, a photon might reach PMA by transmission at BS2 (Path A – it was reflected at BS1) or by reflection at BS2 (Path B – it was transmitted at BS1). Either way, a detection in PMA or PMB yields no information about the path taken by a photon to get there. According to quantum mechanics, the probabilities for photon detection in either PMA or PMB are oppositely modulated, as a function of the pathlength difference between Paths A & B. This means that, for certain pathlength differences (δ), *all* of the photons are detected in PMA and *none* are detected in PMB; and there are intermediate phases where detection in either photomultiplier is equally likely. [Fig. 1.7]

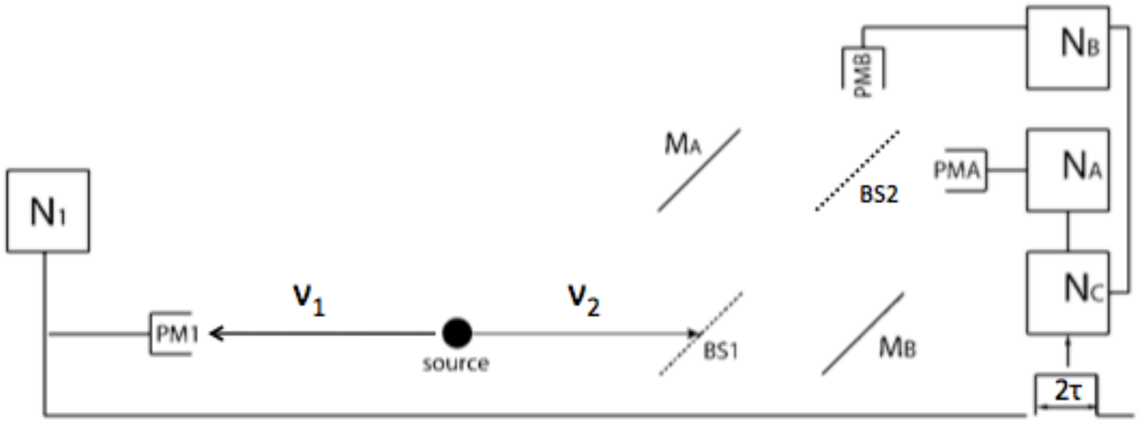


FIG. 1.6. Schematic diagram for the second anticoincidence experiment by Gragier, et al. PMA, PMB & PM1 are photomultipliers; N_1, N_A, N_B &N_C are counters; BS1 and BS2 are beam splitters; M_A and M_B are mirrors. [29]

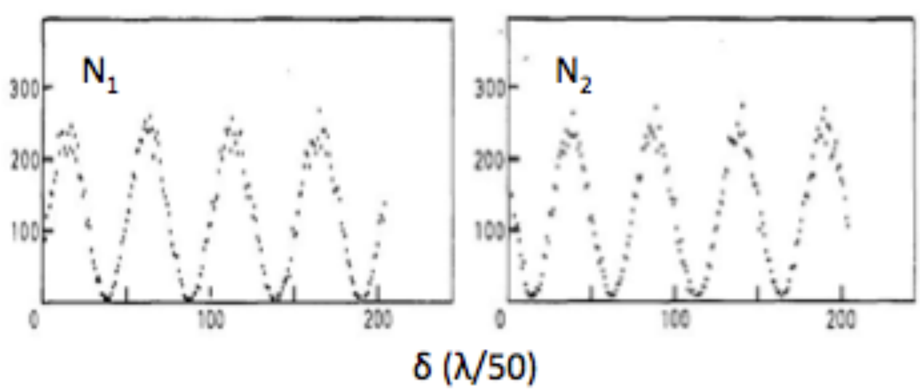


FIG. 1.7. Results from the second photon anticoincidence experiment performed by Grangier, et al. Counting rates at 15-second intervals for each of the two counters N_1 (left) and N_2 (right) as a function of path length difference (δ - in units of $\lambda/50$). For this experiment, $\alpha = 0.18$. [29]

We *interpret* these results as meaning that each photon is coherently split at each beam splitter – it is both reflected *and* transmitted at BS1 (wave-like behavior, in contradiction with our conclusions from the first experiment) for, as the argument goes, how else could changing something about Path B affect the behavior of the photons that were supposed to have only taken Path A? For this second experiment, the anticorrelation parameter was small (α = 0.18), and so we must conclude that each photon is interfering with itself along the two paths (as opposed to many photons interfering with each other).

How are we to make sense of these two experiments, when the results seem to indicate contradictory behavior for the photons at BS1? How does each photon know whether BS2 is in place or not (whether we are conducting the first experiment or the second) when it first encounters BS1? Dirac would argue that every photon is coherently split as a delocalized wave at each beam splitter in *both experiments*, and that in each case the wave instantly collapsed down to a point when interacting with a detector.[4] Bohr would argue (more philosophically) that each photon is, from the very beginning, interacting with the entire apparatus as a whole, and that it behaves as it does at the first beam splitter (particle-like or wave-like) according to which type of behavior is allowed for which type of experiment.

In the end, these are all questions of *ontological category attribution* – it is clear that photons sometimes exhibit particle-like behavior, and sometimes exhibit wave-like behavior, depending on the experiment. Is it possible for photons to simultaneously behave as both particle and wave, for them to simultaneously *straddle* two (classically) distinct ontological categories?[5] A famous thought-experiment was proposed by Wheeler in 1978 [30] (and realized by Hellmuth, et al. in 1987 [31]) to test for this possibility. Imagine a photon entering the apparatus when only one path is available (the photon must take a single, definite path) from source to detector, but then a second path is opened up at the last moment (suddenly, two paths are available). If the photon had already "chosen" to take a single path at the first beam splitter, there should be no opportunity for the photon to interfere with itself, and no interference should be visible in the detectors.

In the actual experiment, [Fig. 1.8] a short-pulsed laser (less than a billionth of a second, with an average of one photon per pulse) was directed at a beam splitter, and the light then passed through 10-meter long optical fibers (in order to increase the transit time by ~30 ns). A Pockels cell (PC-A) in conjunction with a Glans prism was used to effectively insert and remove a path. When a voltage is applied to the Pockels cell, it rotates the plane of polarization of the light within five nanoseconds; the Glans prism then deflects away photons whose polarization has been altered, while transmitting unrotated photons. Therefore, when a voltage is

---

[4] Dirac did not take this "mental model" as a literal description of what was happening, but instead considered it to be a picture that helps to make sense of the situation: "One may extend the meaning of the word '*picture*' to include any way of looking at the fundamental laws which makes their self-consistency obvious." [28]

[5] Ontology concerns itself with the categorization of concepts, physical entities and processes according to their fundamental properties. Entities with similar characteristics belong to similar categories or sub-categories.

applied to the Pockels cell, there is only one path by which a photon could reach the second beam splitter; with no voltage applied, both paths are possible. By randomly applying and removing voltages to the Pockels cell at the required frequency, it was possible to change the nature of the experimental setup after each photon had encountered the first beam splitter.

They found that when the experiment was run with initially only one path open (voltage applied), but then switched to both paths open after the photon had already encountered the first beam splitter (voltage removed), the photon still behaved as though two paths had been available all along, and interference was observed in the detectors. [Fig. 1.9] Wheeler argued that the photon's "choice" as to how to behave at BS1 (like a particle or a wave) must have been made *after the fact* (hence the term *delayed-choice experiment*, which may also refer to the delayed choice made by the observer of which experiment to conduct). [30]

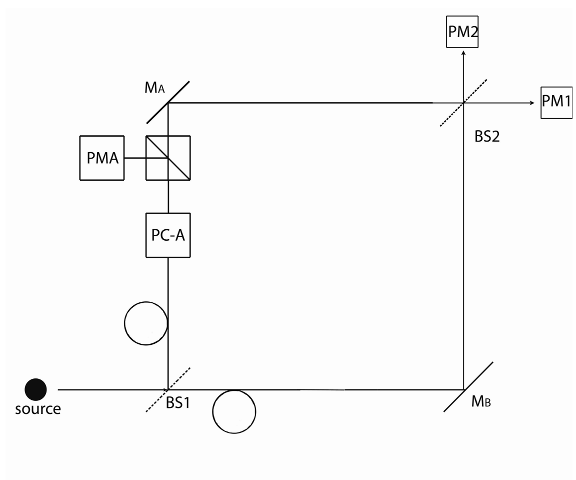


FIG. 1.8. Schematic diagram of the delayed-choice experiment conducted by Hellmuth, et al. PC-A is a Pockels cell used to rotate the plane of photon polarization when a voltage is applied; a Glans prism is used to pass unrotated photons, and to reflect away rotated photons. Only one path to BS2 is available with the voltage applied. [31]

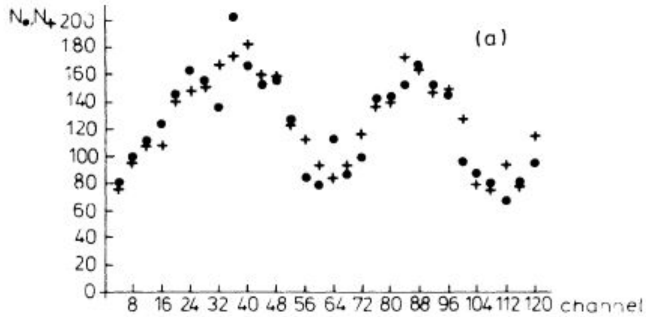


FIG. 1.9. Counting rates in "normal" mode [dots; no voltage applied throughout] and in "delayed-choice" mode [crosses; second path is unblocked after photon encounters BS1] as a function of path length difference. A clear interference pattern is observed in both data sets. [31]

It seems that no matter how an experiment is devised, we observe the behavior of quanta to be particle-like in some circumstances, and consistent with our expectations for classical waves in others, but we cannot demonstrate both types of behaviors simultaneously. Dirac has preemptively offered his interpretation of these experiments: each photon coherently divides at each beam splitter as a delocalized wave, interferes with itself when more than one path is available, and then instantly collapses to a point when interacting with a detector. Niels Bohr would characterize this *dual wave-particle* behavior as *complementary* (but exclusive) features of our ultimately classical understanding of an abstract quantum world. No single classical ontological category (particle or wave) can account for all the results of quantum experiments, but the union of these two complementary concepts allows for a generalized description of the whole. Like the *Yin* and *Yang* of Chinese philosophy, Bohr saw *Complementarity* as an epistemological[6] tool with broader implications; for example, he considered *truth* and *brevity* to be complementary concepts (the more you have of one, the less you have of the other). [1]

This complementary wave-particle duality is not limited to massless photons, but can be seen in the behavior of all kinds of matter. [32-34] A double-slit experiment performed with single electrons [35] is isomorphic to the experiments described above involving single photons. In this experiment, single electrons are passed through two slits and detected one at a time at seemingly random places, yet an interference pattern still builds up over time. [Fig. 1.10] A *matter-wave* interpretation of this result would insist that each electron propagates as a delocalized wave and is coherently split at both slits, interferes with itself, then becomes instantly localized in its interaction with the detecting screen. The *Copenhagen Interpretation* would say each electron's behavior at the two slits must

[6] Epistemology concerns itself with the nature of knowledge, and how it is acquired. In simplest terms, it addresses the question: How do we know what we know?

be understood in terms of classical waves, and the nature of the detecting apparatus reveals a *complementary* electron behavior that can only be understood in terms of classical particles. Changing the nature of the experimental setup (e.g., blocking one of the slits, removing one of the paths) changes how the behavior of each electron is to be described over the course of the experiment.

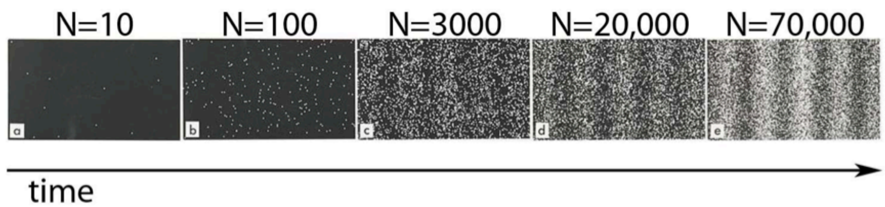


FIG. 1.10. Buildup of a single electron interference pattern. Single electrons are initially detected at seemingly random places, yet an interference pattern is still observed to build up after detecting many electrons. [35]

As epistemological tools, both interpretations are of similar use, in that we may employ either to decide which type of behavior will be observed in a given situation, without actually conducting the experiment: there should be no interference effects when only one path from source to detector is available; when two (or more) paths are allowed, interference will be observed. The two interpretations differ, however, in the physical meaning behind any switch between ontological descriptions of the behavior of quantum entities. In a *matter-wave* interpretation, each electron is viewed as a quantized excitation of a matter-field that (randomly) deposits its energy at a single point in its interaction with a detector. This *instantaneous collapse of the wave function* is viewed as a physical process by which these quantized excitations transition from a delocalized state (wave category) to one that is localized in space (particle category). *Complementarity* views this collapse as a moment when new information is available to the observer regarding the state of the quantum system in its interaction with the measurement apparatus (the line between which is arbitrarily drawn). The experiment reveals two sides of a more abstract quantum whole, each in analogy to classical behavior (particle- or wave-like), but any switch between ontological categories occurs only in the mind of the observer describing the system. Dividing the behavior of quantum entities into (classically) separate ontological categories is seen as a method for making sense of the decidedly nonclassical behavior of quantum entities, in terms of classical concepts intuitively associated with particles and waves.

However you choose to look at it, it should be clear that a proper understanding of quantum physics requires some degree of flexibility in the assignment of ontological categories when describing the behavior of quantum systems, and epistemological tools must be developed for understanding when each type of assignment is (or is not) appropriate for a given situation. A variety of formal interpretations of quantum theory may then be regarded in terms of coherent *epistemological and ontological framings* that guide the process of category assignment according to context. In this way, many difficulties in the conceptual understanding of quantum mechanics may be understood as stemming from varying degrees of commitment to *epistemological and ontological resources* that are in themselves neither right nor wrong, but which lead to incorrect or paradoxical conclusions when inappropriately applied to the description of quantum phenomena. We will see how this view has implications for the teaching and learning of quantum mechanics among introductory modern physics students.

II. Epistemology and Ontology in Physics Instruction

Research into student learning has shown that, in contrast to the straight-forward acquisition of facts or skills, there are particular topics in science that are notoriously difficult for students, and where traditional modes of instruction have been demonstrated to be ineffective. Such difficulties in student learning are most generally thought of as stemming from any number of *prior ideas* held by students, which mediate the learning process, and which in some way or other must *change* before a proper (scientifically normative) understanding can be achieved. Precisely what it is that must change during this process of learning, whether it be concepts, beliefs, epistemological framings, or ontologies, is where education researchers primarily diverge. [36]

One line of research posits that many of the conceptual barriers faced by students in learning classical physics can be traced to unproductive or inappropriate degrees of commitment to *ontological category assignments*, and issues of *category inheritance*. It has been noted, for example, that *emergent processes* (such as electric current, resulting from the net motion of individual charged particles) are often alternatively conceptualized by students as *material substances* (electric current as a fluid that can be stored and consumed). [37] The general idea is that, whenever learners encounter some unfamiliar concept, they engage in a (conscious or unconscious) process of ontological categorization, whereby they sort the concept according to whatever information is available at the time. This information may include (but is not limited to) the context in which the concept is introduced, its similarity or co-occurrence with other concepts, or language patterns that give indications to its ontological nature. Once an ontological category (or sub-category) for that concept has been decided upon, it is believed that learners will then automatically associate with that concept the attributes of other concepts that fall within that same category – the new concept *inherits* the characteristics of other concepts that are ontologically similar in the mind of the learner. Many student difficulties in understanding emergent processes in classical physics can then be viewed as arising from the misattribution of properties intuitively associated with material substances. According to Chi, when the category assignment held by the learner is sufficiently distinct from the targeted (scientifically accepted) category, the process of reassignment cannot come about in gradual steps, and the learner must set aside their initial conceptualization in favor of a new conceptualization with other attributes. This *incompatibility hypothesis* motivates Chi's description of *radical conceptual change* in novice learners. [38]

A key question surrounding Chi's hypothesis is: What happens with the original ontology that is to be replaced? In their empirical work, Slotta and Chi make no real assertions regarding the ultimate fate of the original ontologies that are to be ignored by novice students, [39] though they have mentioned that

> "…physics experts do maintain substance-based conceptualizations in parallel with their more normative *process-like* views. In their everyday reasoning, physics experts often use substance-like models of heat, light, and electricity, although they are well aware of the limitations of such models, including when the models should be abandoned. Thus, if the early *substance-like* conceptions are not actually removed or replaced, we can interpret conceptual change as a matter of developing new conceptualizations alongside existing ones and understanding how and when to differentiate between alternatives." [40]

Slotta and Chi are therefore not only allowing for the possibility of *parallel ontologies* in student and expert thinking, they are insisting that productive use can be made of them by experts with a certain amount of sophistication in the flexible use of multiple ontological attributions for a single concept. [39]

Gupta, et al. have recently taken issue with the views of Slotta and Chi on ontologies in student and expert thinking, [41] most specifically with their delineation of ontologies into distinct, normative categories that remain *static*. [42] Gupta, et al. assert that not only do experts and novices often bridge between parallel ontologies, but that in many situations, clear distinctions between ontological categories don't even apply. Their view on *dynamic ontologies* claims that delineations between ontological categories and their associated attributes are not necessarily *rigid* in the minds of both experts and novices, and that they often *blend* material and process conceptualizations in their reasoning. They further take issue with the assumption that any one "scientific concept *correctly* belongs to a single ontological category." [42]

The differences in these two models of learning and cognition can be seen as analogous to the differences between material substances and emergent processes as ontological categories. A view of ontologies as distinct and stable *structures* (which is one way of accounting for the observed robustness of common student misconceptions) is contrasted with a dynamic view of flexible and adaptive ontologies that emerge in real time through the *coordinated activation* of cognitive resources (that are in themselves neither right nor wrong). In this way, the stability of misconceptions observed by Reiner, et al. [37] may be understood as resulting from contextually stable and coherent patterns of resource activation. [43] It is therefore the pattern of resource activation within a given context that must change in the minds of learners, and Gupta, et al. argue this may come about in gradual steps, so that matter-based reasoning can slowly lead to process-based reasoning. [41, 42]

It is possible these two perspectives are not entirely incompatible in the context of classical physics instruction; they may disagree on questions of meta-ontology (ontological attributions as stable cognitive structures versus emergent cognitive processes), but both agree that the learning of new concepts is mediated (and sometimes hindered) by prior knowledge (students do not enter the learning environment as blank slates), and that conceptual difficulties in learning physics often arise from the misattribution of ontological characteristics to unfamiliar concepts. And both agree that a degree of flexibility in switching between ontological attributions is not only possible, but also a *desirable* aspect of expert-like thinking. In the context of quantum physics, however, the wave-particle duality in the behavior of light and matter makes this flexibility *necessary* for a proper understanding of quantum mechanics.

We wish to extend these views on learning to the context of quantum physics in a way that would similarly address difficulties students have with changing their classical conceptions of light and matter. We first hypothesize that the intuitively *realist* perspectives of introductory physics students are reinforced by classical physics instruction, and that instruction in quantum physics can lead to measureable changes in student thinking. [Chapter 2] We will find that the highly contextual nature of student conceptions of light and matter are differentially influenced by the myriad ways in which instructors may choose (or choose not) to address interpretive themes in quantum mechanics, and that these instructional choices manifest themselves both explicitly and implicitly in the classroom. [Chapter 3] We further hypothesize that *realist* expectations among novices and experts in quantum physics are a manifestation of *classical ontological attribute inheritance*; in other words, quantum particles (at least initially, and despite evidence to the contrary) *inherit* many of their classical attributes, which can lead to incorrect or contradictory interpretations of quantum phenomena. We will demonstrate that novice quantum physics students exhibit varying degrees of flexibility in the ontological categorization of the behavior of quanta, and present evidence of students not only switching between ontological attributions both within and across contexts, but also creating a *blended* ontological category for quantum entities, simultaneously classifying them as both particle and wave (most consistent with a *pilot-wave* interpretation of quantum mechanics [4]). Moreover, it will be seen that ontological category reassignment among students can occur piecewise, context by context (particularly in cases where instruction is explicit), and that our findings are not reflective of some sudden, wholesale change in student perspectives on the ontological nature of quanta. [Chapter 4]

III. Motivation and Overview of Dissertation Project

A detailed exploration of student perspectives on the physical interpretation of quantum mechanics is necessary, since these perspectives are an aspect of understanding physics, and have implications for how traditional content might be taught. Introductory modern physics courses are of particular interest since they often represent a first opportunity to transition students away from classical epistemologies and ontologies, to ones that are more aligned with those of practicing physicists.

In terms of assessing student difficulties in quantum mechanics, several conceptual surveys have been developed, [44-50] though most are appropriate for advanced undergraduate and beginning graduate students, since they address such advanced topics as the calculation of expectation values, or the time-evolution of quantum states. Because there does not seem to be a canonical curriculum for modern physics courses, the applicability of assessment instruments designed specifically for this kind of student population must be evaluated course-by-course. The Quantum Physics Conceptual Survey (QPCS) [51] is a recent example of an assessment instrument developed for introductory modern physics students. The authors of the QPCS found that students had the most difficulty with six questions they had classified as *interpretive*; for example, the two survey items with the lowest percentage of correct responses (~20% for each) ask whether, "according to the standard (Copenhagen) interpretation of quantum mechanics," light (or an electron) is behaving like a wave or a particle when traveling from source to detector. These authors also found that not only do a significant number of students perform reasonably well on non-interpretative questions while still scoring low on the interpretative items, there were no students who scored high on the interpretative questions but scored low on the non-interpretative ones. As the authors note, this parallels findings from Mazur [52] when comparing student performance on conventional classical physics problems versus ones requiring a solid conceptual understanding. Their results suggest that many introductory modern physics students may grasp how to use the computational tools of quantum mechanics, without a corresponding facility with notions (such as wave-particle duality) that are at odds with their classical intuitions.

Mannila, et al. [53] have previously explored student perspectives on particle-wave duality and the probabilistic nature of quantum mechanics within the context of a double-slit experiment, where a low intensity beam of quanta passes through a two-slit system and gradually forms a fringe pattern on a detecting screen. Their analysis of open-ended written student responses to a series of questions found they were dominated by "semi-classical" or "trajectory-based" ontologies, and that very few students expressed perspectives that were aligned with expert models, or even productive transitional models.[7] These authors also reported many instances of *mixed* student ontologies within that single context of a double-slit experiment, yet the design of their study provided no opportunity to further question students on any apparent inconsistencies. Our studies have

[7] Non-local and/or statistical (probabilistic) perspectives, by their standards.

demonstrated that student perspectives on quantum phenomena can vary significantly by context, [54–56, Chapters 2-4] so that it may not always be possible to make generalizations about student beliefs based on investigations within a single context.

This dissertation concerns itself with a detailed exploration and characterization of student perspectives on the physical interpretation of quantum mechanics, and how these perspectives develop within the context of an introductory modern physics course. [Chapter 2] In doing so, we identify variations in teaching approaches with respect to interpretation, and their associated impacts on student thinking. [Chapters 3 & 4] These studies serve to inform the development of instructional materials designed to positively influence student perspectives on quantum physics. [Chapter 5] Further research conducted during the implementation of these materials in a modern physics course for engineering majors allow for an assessment of their effectiveness in influencing student perspectives, and inform their refinement for future use. [Chapter 6]

Chapter 2: Development of Quantum Perspectives – Initial Studies

The first indication that student perspectives are being significantly influenced through formal instruction came from an analysis of student responses to a particular statement on the Colorado Learning Attitudes about Science Survey (CLASS) [57]: *It is possible for physicists to carefully perform the same experiment and get two very different results that are both correct.* There is a clear trend in how student responses to this statement change over the course of a three-semester introductory sequence of physics courses. In a cross-sectional study of student responses (PHYS1 [classical mechanics], N=2200; PHYS2 [classical electrodynamics], N=1650; PHYS3 [modern physics], N=730) we see a shift first from agreement to disagreement, and then back to agreement with this statement. [Fig. 1.11] At the beginning of instruction in classical mechanics (A), more students will agree (40%) with this statement than disagree (26%); yet the number in agreement decreases significantly (B) following instruction in classical physics (to 30%, $p<0.001$), while an increasing number of students disagree (to 39%, $p<0.001$). This trend then reverses itself over a single semester of modern physics (C), at the end of which a greater percentage of students agree with this statement (46%) than at the beginning of classical physics instruction.

We then analyzed the reasoning provided by approximately 600 students in an optional text box following the multiple choice response, in order to establish if their reasons for agreeing or disagreeing had changed. We find that, among students of introductory classical physics, those who disagree with this statement primarily concern themselves with the idea that there can be only one correct result for any physical measurement, while those in agreement are more conscious of the possibility for random, hidden variables to influence the outcomes of two otherwise identical experiments. Few students invoke quantum phenomena when responding before any formal instruction in modern physics; however, a single semester of modern physics instruction results in a significant increase in the percentage of students who believe that quantum phenomena would allow for two valid (but different) experimental results.

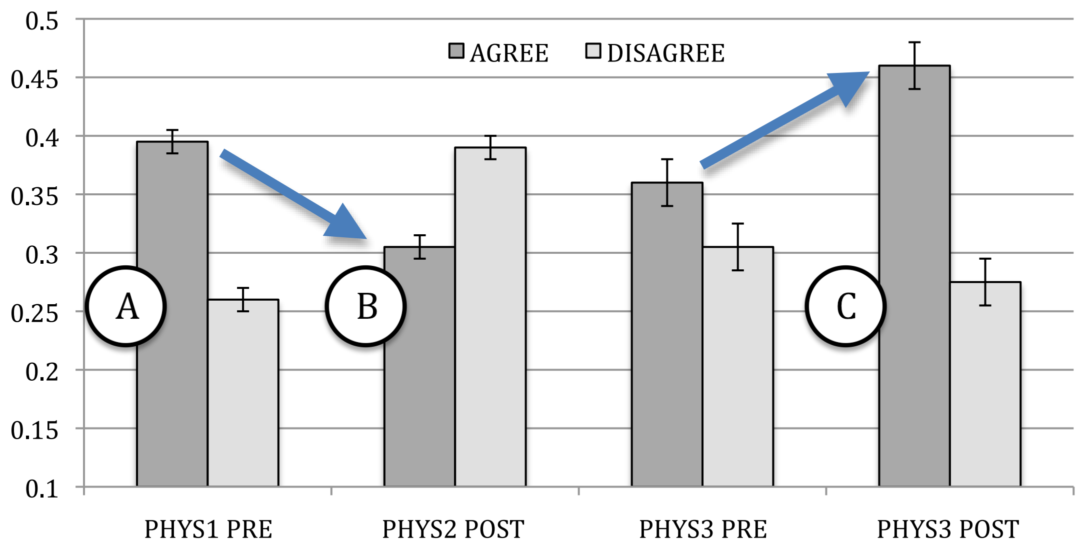


FIG. 1.11. Cross-sectional analysis of student responses to the statement: *It is possible for physicists to carefully perform the same experiment and get two very different results that are both correct* (expressed as a fraction of total responses: PHYS1, N=2200; PHYS2, N=1650; PHYS3, N=730). Error bars represent the standard error on the proportion.

Chapter 3: Quantum Interpretation as Hidden Curriculum – Variations in Instructional Approaches and Associated Student Outcomes

Our efforts to characterize student perspectives on quantum physics were initially limited to the application of coarse labels (discussed below) to student responses to a post-instruction online essay question on interpretations of the double-slit experiment, coupled with responses to a survey statement concerning the existence of an electron's position within an atom. Students from courses that emphasized a *matter-wave* interpretation overwhelmingly preferred a wave description of electrons in the double-slit experiment (each electron passes through both slits and interferes with itself), while responses from courses taught from a *realist/statistical* perspective were dominated by realist interpretations (each electron goes through either one slit or the other, but not both).

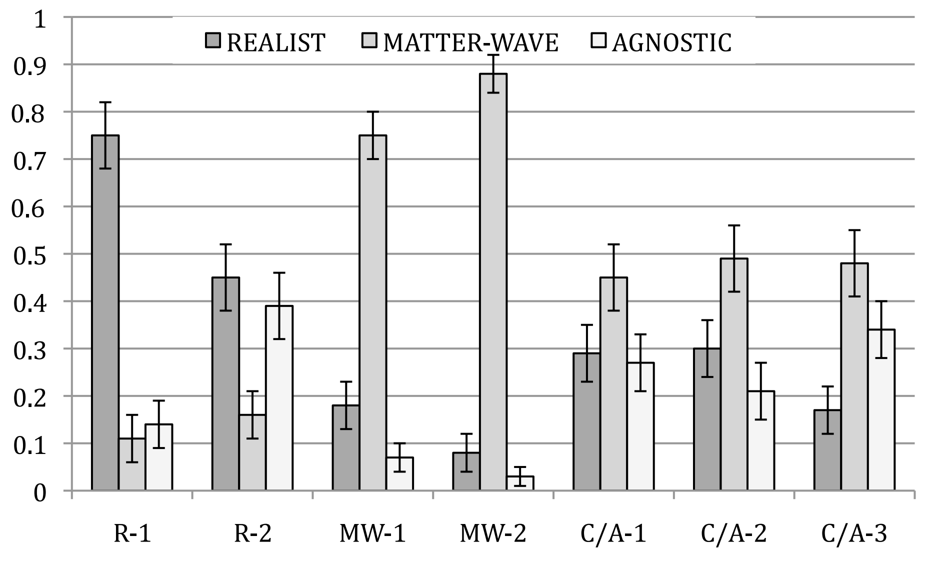


FIG. 1.12. Post-instruction student responses to the double-slit essay question, from seven different modern physics offerings of various instructional approaches [R = *Realist*; MW = *Matter-Wave*; C/A = *Copenhagen/Agnostic*]. Error bars represent the standard error on the proportion; N ~ 50-100 for each course.

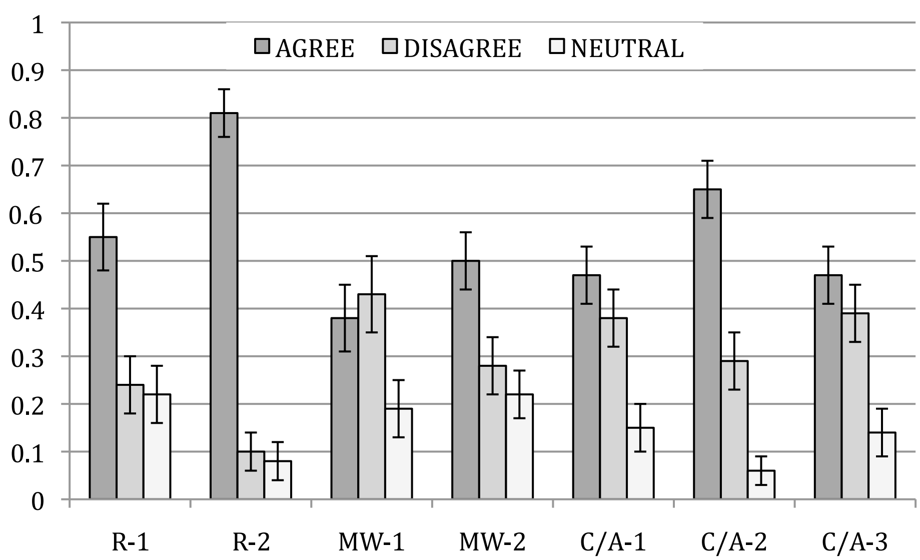


FIG. 1.13. Post-instruction student responses to the statement: *An electron in an atom exists at a definite (but unknown) position at each moment in time*, from seven different modern physics offerings of various instructional approaches [R = *Realist*; MW = *Matter-Wave*; C/A = *Copenhagen/Agnostic*]. Error bars represent the standard error on the proportion; N ~ 50-100 for each course.

Students from courses taught from a *Copenhagen* perspective (or ones that de-emphasized interpretation) offered more varied responses. These latter students were not only more likely to prefer an agnostic stance (quantum mechanics is about predicting the interference pattern, not discussing what happens in between), they were also more likely to align themselves with a realist interpretation. [Fig. 1.12] Of particular interest is how these same students responded to the statement: *An electron in an atom has a definite but unknown position at each moment in time*; [Fig. 1.13] Agreement with this statement would be most consistent with a realist perspective. Students from all of these types of modern physics courses were generally most likely to agree with this statement, including students from courses emphasizing a matter-wave interpretation.

When aggregate student responses from four modern physics offerings are combined so that responses to this statement on atomic electrons are grouped by how those same students responded to the essay question on the double-slit experiment, [Fig. 1.14] we see that students in the (double-slit) *Realist* category were the most consistent, with most preferring realist interpretations in both contexts. However, nearly half of the students who preferred a wave-packet description of electrons in the double-slit experiment would still agree that electrons in atoms exist as localized particles. Only those students who preferred an agnostic stance on the double-slit question were more likely to disagree with the statement than agree, and none of these students felt neutrally about whether atomic electrons are always localized. In addition, a small number of students from all courses (~5%, not shown) chose to agree with both *Matter-Wave* and *Realist* interpretations of the double-slit experiment. These findings indicate a need for more detailed characterizations of student perspectives on quantum phenomena.

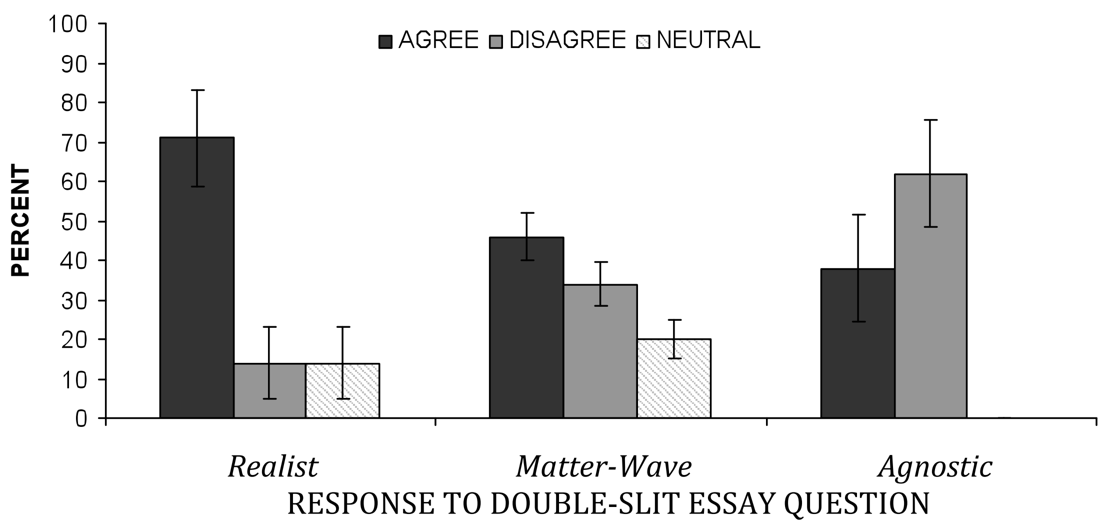


FIG. 1.14. Combined student responses from both PHYS3 courses to the statement: *An electron in an atom has a definite but unknown position at each moment of time*, grouped by how those students responded to the double-slit essay question. Error bars represent the standard error on the proportion (N~60).

Chapter 4: Refined Characterization of Student Perspectives on Quantum Physics

A total of nineteen post-instruction interviews with students from four recent introductory modern physics courses taught at the University of Colorado have demonstrated that, though they may not employ the same formal language as expert physicists, students often invoke concepts and beliefs that parallel those invoked by expert physicists when arguing for their preferred interpretations of quantum mechanics. These parallels allow us to characterize student perspectives on quantum physics in terms of some of the same themes that distinguish these formal interpretations from each other. Of particular significance is the finding that students develop attitudes and opinions regarding these various themes of interpretation, regardless of whether these themes had been explicitly addressed by their instructors in class.

Results from these interviews show that, even when modern physics students have learned about "correct" responses from their instructors (or elsewhere), their classical intuitions may still influence their responses. Similar findings among classical physics students [58, 59] have shown that students most often explained differences between their *personal* and *public* perspectives in terms of responses that made intuitive sense to them (*personal*), versus ones based on their perceptions of scientists' beliefs (*public*), having not yet reconciled that knowledge with their own intuition. The inconsistent responses of some modern physics students may be similarly understood in terms of competing *personal* and *public* perspectives on quantum physics – when responding in interviews or surveys, some students frequently vacillated between what they personally believed and the answer they felt an expert physicist would give, without always explicitly distinguishing between the two.

A significant number of students from our interviews (ten of nineteen) demonstrated a preference for realist interpretations of quantum phenomena; however, only three of these students expressed personal confidence in the correctness of their perspectives, whereas four others differentiated between what made intuitive sense to them (*Realist*) and what they perceived to be correct responses (*Matter-Wave*). In addition to splits between intuition and authority, some of the seemingly contradictory responses from students may also be explained by their preferences for a *mixed* wave-particle ontology (a *pilot-wave* interpretation, wherein quanta are simultaneously *both* particle *and* wave). The realist and nonlocal beliefs of these three students were at odds with how wave-particle duality was addressed in class by their instructors (i.e. quanta are sometimes described by waves, and sometimes as particles, but never both simultaneously). The remaining nine of nineteen students seemed to express fairly consistent views that could be seen as in agreement with the instructional goals of their instructors. In other words, these students seemed to have successfully incorporated probabilistic and nonlocal views of quanta and quantum measurements into their personal perspectives.

Chapter 5: Teaching Quantum Interpretations – Curriculum Development and Implementation

In exploring student perspectives on quantum physics, it seems natural that students should have attitudes regarding some themes of interpretation, in that these stances are reflections of each student's ideas about the very nature of reality, and the role of science in describing it. Is the universe deterministic or inherently probabilistic? When is a particle a particle, and when is it a wave? Is it unscientific to talk about that which can't be measured? A modern physics curriculum aimed at positively influencing student perspectives should provide students with the tools to formulate answers to such questions for themselves, since simply telling students about "scientifically accepted" answers does not seem to impact students at more than a superficial level.

The question remains: In what ways can student perspectives be addressed at a level appropriate for introductory modern physics students, without sacrificing traditional course content and learning goals? Although many instructors may feel that introductory students do not have the requisite sophistication to appreciate matters of interpretation in quantum mechanics, several authors have developed discussions of EPR correlations and Bell inequalities that are appropriate for the introductory level; [60, 61] relevant experimental tests of the foundations of quantum theory [5, 20, 22, 29, 31-35] may be addressed in a non-technical way. [17-19, 21, 61-64] Questions of interpretation may also be framed in terms of *scientific modeling*, an aspect of epistemological sophistication that is often emphasized in physics education research as a goal of instruction. [65] Moreover, a common lament among physics education researchers is that we are losing physics majors in the first years of their studies by only teaching them 19th-century physics in our introductory courses. Similar issues may arise when modern physics instructors limit course content mostly to the state of knowledge at the first half of the last century, or are reluctant to address questions that are clearly of personal and academic interest to students.

A modern physics course that specifically addresses student perspectives might do so within the following topics (among others):

EPR Correlations/Entanglement: Make explicit the assumptions of determinism and locality in the context of classical physics. The notion of atomic spin may be built up from a semi-classical (Bohr-like) atomic model; the limitations of this deterministic model become evident as it leads to predictions in conflict with experimental observation. Issues of measurement, quantum states and state preparation, and interpretation arise naturally. Indeterminacy and non-local aspects of quantum phenomena are demonstrated with simple probability arguments (thought experiments) [60, 61] and experimental evidence. [5, 17-22, 29, 31-36, 61-64] Address implications for quantum information theory (cryptography, computing, etc…). [5]

Single-Quanta and Delayed-Choice Experiments: The experiments of Aspect et al. demonstrate the complementary particle- and wave-like behavior of quanta, [29] providing opportunities to address various aspects of student perspectives on quantum mechanics enumerated in previous studies. [54-56, 65, 69, 70] Delayed-choice experiments [31] demonstrate the limitations of realist/statistical and pilot-wave interpretations. The basics of these experiments requires a simple understanding of atomic spectra and lasers, polarization and polarizers, beam-splitters [interferometry experiments] and photon detectors [photoelectric effect]. Discussion of these experiments can be facilitated by pointing students to non-technical articles. [17-19, 21, 61-64] Address complementarity as a general principle; help students develop an intuition for when interference effects should be visible, and when not.

The Uncertainty Principle: Discussions of the Uncertainty Principle (UP) follow naturally as a mathematical expression of complementarity. The UP can be framed in terms of Fourier decomposition and the properties of wave-packets. It may also be framed in terms of explicit formal interpretations. A realist/statistical interpretation is embodied in Heisenberg's Microscope. [71] A statistical interpretation concerns separate measurements performed on an ensemble of identically prepared system. [72, 73] Matter-wave and Copenhagen interpretations confront issues of indeterminacy in quantum measurement. Order-of-magnitude estimates can be made using simple models and assumptions, indicating a deeper physical meaning behind the UP beyond simple peculiarities of the measurement process. [5]

Such a curriculum has been implemented in the form of an introductory modern physics course for engineers in the Fall 2010 semester at the University of Colorado. Quantitative and qualitative data have been collected in the form of student responses to questions from previously validated instruments such as the CLASS [57], QMCS [49] and QPCS [51], as well as the same survey items and essay questions employed in our previous studies. [54-56] In this chapter, we discuss the guiding principles behind the development of this curriculum, and provide a detailed examination of specific, newly developed course materials designed to meet these goals. [A broader selection of relevant course materials can be found in Appendix C.] In doing so, we address the appropriateness and effectiveness of this curriculum by considering aggregate student responses to a subset of homework, exam, and survey items, as well as actual responses from four select students. We may employ the framework developed in Chapter 4 to characterize the perspectives of these four students as they progress through the course, and compare their incoming reasoning with how they responded at the end of the semester.

Chapter 6: Teaching Quantum Interpretations – Comparative Outcomes and Curriculum Refinement

Results from these data collections may then be compared with previous incarnations of modern physics courses at the University of Colorado where similar data are available. We also examine student responses to specific exam questions and post-instruction content survey items, in an effort to identify which aspects of the new curriculum were most challenging for students, and propose refinements for the sake of potential future implementations and studies. Course materials specific to interpretation will be compiled and archived in a way that allows future instructors to incorporate them into their own curricula.

References (Chapter 1)

1. A. Pais, *Niels Bohr's Times, in Physics, Philosophy and Polity* (Clarendon Press, Oxford, 1991), p. 23.

2. A. Einstein, "Autobiographical Notes" in *Albert Einstein: Philosopher-Scientist*, P. A. Schillp (Ed.) (Open Court Publishing, Peru, Illinois 1949).

3. A. Einstein, B. Podolsky and N. Rosen, Can quantum mechanical description of physical reality be considered complete? *Phys. Rev.* **47**, 777 (1935).

4. D. Bohm and Y. Aharonov, Discussion of Experimental Proof for the Paradox of Einstein, Rosen and Podolsky, *Phys. Rev.* **108**, 1070 (1957).

5. G. Greenstein and A. J. Zajonc, The Quantum Challenge: Modern Research on the Foundations of Quantum Mechanics, 2nd ed. (Jones & Bartlett, Sudbury, MA, 2006).

6. N. Bohr, Can quantum mechanical description of physical reality be considered complete? *Phys. Rev.* **48**, 696 (1935).

7. Quoted in: A. Pais, *Niels Bohr's Times, in Physics, Philosophy and Polity* (Clarendon Press, Oxford, 1991), p. 314.

8. R. G. Newton, *How Physics Confronts Reality: Einstein was correct, but Bohr won the game.* (World Scientific, Singapore, 2009).

9. W. H. Stapp, The Copenhagen Interpretation, *Am. J. Phys.* **40**, 1098 (1972).

10. See, for example: D. J. Griffiths, *Introduction to Quantum Mechanics, 2nd Ed.* (Prentice Hall, Upper Saddle River, New Jersey, 2004), p. 3.

11. N. D. Mermin, What's Wrong with this Pillow? *Phys. Today* **42**, 9 (1989).

12. J. von Neumann, "Measurement and Reversibility" & "The Measuring Process" in *Mathematical Foundations of Quantum Mechanics* (Princeton University Press, Princeton, New Jersey, 1955), pp. 347-445.

13. J. S. Bell, On the Einstein-Podolsky-Rosen paradox, reprinted in *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1987), pp. 14-21.

14. J. S. Bell, "Locality in quantum mechanics: reply to critics," reprinted in *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1987), pp. 63-66.

15. J. F. Clauser, M. A. Horne, A. Shimony and R. A. Holt, Proposed Experiment to Test Local Hidden-Variable Theories, *Phys. Rev. Lett.* **23**, 880 (1969).

16. S. J. Freedman and J. F. Clauser, Experimental test of local hidden-variable theories, *Phys. Rev. Lett.* **28**, 938 (1972).

17. A. L. Robinson, Quantum Mechanics Passes Another Test, *Science* **217**, 435 (1982).

18. A. L. Robinson, Loophole Closed in Quantum Mechanics Test, *Science* **219**, 40 (1983).

19. A. Shimony, The Reality of the Quantum World, *Scientific American* (January 1988, pp. 46-53).

20. W. Tittel, J. Brendel, B. Gisin, T. Herzog, H Zbinden and N. Gisin, Experimental demonstration of quantum correlations over more than 10 km, *Phys. Rev. A* **57** (5), 3229 (1998).

21. A. Watson, Quantum Spookiness Wins, Einstein Loses in Photon Test, *Science* **277**, 481 (1997).

22. A. Aspect, P. Grangier and G. Roger, Experimental Tests of Realistic Local Theories via Bell's Theorem, *Phys. Rev. Letters* **47**, 460 (1981).

23. E. Schrödinger, "The Present Situation in Quantum Mechanics," in *Quantum Theory and Measurement*, J. A. Wheeler and W. H. Zurek (Eds.) (Princeton, NJ, 1983) p. 152. Originally published under the title, "Die gegenwärtige Situation in der Quantenmechanik," *Die Naturwissenschaften*, **23**, 807 (1935); translation into English by J. D. Trimmer, 1980.

24. A. Einstein, in *Letters on Wave Mechanics*, K. Przibram (Ed.) (Philosophical Library, New York, NY, 1986), p. 39.

25. S. Redner, Citation Statistics from 110 Years of Physical Review, *Phys. Today* **58**, 52 (2005).

26. L. E. Ballentine, Resource letter IQM-2: Foundations of quantum mechanics since the Bell inequalities, *Am. J. Phys.* **55**, 785 (1987).

27. A. Aspect, "John Bell and the second quantum revolution," in *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1987), p. xix.

28. P. A. M. Dirac, *The Principles of Quantum Mechanics, 3rd ed.*, (Clarendon Press, Oxford, 1947), p. 9.

29. P. Grangier, G. Roger, and A. Aspect, Experimental Evidence for a Photon Anticorrelation Effect on a Beam Splitter: A New Light on Single-Photon Interferences, *Europhysics Letters* **1**, 173 (1986).

30. J. A. Wheeler, in *Mathematical Foundations of Quantum Mechanics*, A. R. Marlow (Ed.) (Academic Press, New York, NY, 1978), pp. 9-48.

31. T. Hellmuth, H. Walther, A. Zajonc and W. Schleich, Delayed-choice experiments in quantum interference, *Phys. Rev. A* **35**, 2532 (1987).

32. R. Gaehler and A. Zeilinger, Wave-optical experiments with very cold neutrons, *Am. J. Phys.* **59** (4), 316 (1991).

33. O. Carnal and J. Mlynek, Young's Double-Slit Experiment with Atoms: A Simple Atom Interferometer, *Phys. Rev. Letters* **66** (21), 2689 (1991).

34. S. Frabboni, G. C. Gazzadi, and G. Pozzi, Nanofabrication and the realization of Feynman's two-slit experiment, *App. Phys. Letters* **93**, 073108 (2008).

35. A. Tonomura, J. Endo, T. Matsuda, T. Kawasaki and H. Exawa, Demonstration of single-electron buildup of an interference pattern, *Am. J. Phys.* **57**, 117 (1989).

36. A. A. diSessa, "A history of conceptual change research: Threads and fault lines," in *Cambridge Handbook of the Learning Sciences*, K. Sawyer (Ed.) (Cambridge University Press, Cambridge, 2006), pp. 265-281.

37. M. Reiner, J. D. Slotta, M. T. H. Chi and L. B. Resnick, Naïve physics reasoning: A commitment to substance-based conceptions, *Cognition and Instruction* **18**, 1 (2000).

38. M. T. H. Chi, Common sense misconceptions of emergent processes: Why some misconceptions are robust, *Journal of the Learning Sciences* **14**, 161 (2005).

39. J. D. Slotta, In defense of Chi's Ontological Incompatibility Hypothesis, *Journal of the Learning Sciences* **20**, 151 (2011).

40. J. D. Slotta and M. T. H. Chi, The impact of ontology training on conceptual change: Helping students understand the challenging topics in science, *Cognition and Instruction* **24,** 261 (2006).

41. A. Gupta, D. Hammer and E. F. Redish, The case for dynamic models of learners' ontologies in physics, *J. Learning Sciences* **19**, 285 (2010).

42. D. Hammer, A. Gupta and E. F. Redish, On Static and Dynamic Intuitive Ontologies, *J. Learning Sciences* **20**, 163 (2011).

43. D. Hammer, A. Elby, R. E. Scherr and E. F. Redish, "Resources, Framing and Transfer" in *Transfer of Learning*, J. Mestre (Ed.) (Information Age Publishing, 2005) pp. 89-119.

44. E. Cataloglu and R. Robinett, Testing the development of student conceptual and visualization understanding in quantum mechanics through the undergraduate career, *Am. J. Phys.* **70**, 238 (2002).

45. C. Singh, Student understanding of quantum mechanics, *Am. J. Phys.* **69**, 8 (2001).

46. C. Singh, Assessing and improving student understanding of quantum mechanics, *PERC Proceedings 2006* (AIP Press, Melville, NY, 2006).

47. C. Singh, Student understanding of quantum mechanics at the beginning of graduate instruction, *Am. J. Phys.* **76**, 3 (2008).

48. J. Falk, Developing a quantum mechanics concept inventory, unpublished master thesis, Uppsala University, Uppsala, Sweden (2005). Available at: http://johanfalk.net/node/87 (Retrieved January, 2011).

49. S. B. McKagan and C. E. Wieman, Exploring Student Understanding of Energy Through the Quantum Mechanics Conceptual Survey, *PERC Proceedings 2005* (AIP Press, Melville, NY, 2006).

50. S. Goldhaber, S. Pollock, M. Dubson, P. Beale and K. Perkins, Transforming Upper-Division Quantum Mechanics: Learning Goals and Assessment, *PERC Proceedings 2009* (AIP, Melville, NY, 2009).

51. S. Wuttiprom, M. D. Sharma, I. D. Johnston, R. Chitaree and C. Chernchok, Development and Use of a Conceptual Survey in Introductory Quantum Physics, *Int. J. Sci. Educ.* **31 (5)**, 631 (2009).

52. E. Mazur, *Peer instruction: A user's manual* (Prentice Hall, New York, NY, 1997).

53. K. Mannila, I. T. Koponen and J. A. Niskanen, Building a picture of students' conceptions of wave- and particle-like properties of quantum entities, *Euro. J. Phys.* **23**, 45-53 (2002).

54. C. Baily and N. D. Finkelstein, Development of quantum perspectives in modern physics, *Phys. Rev. ST: Physics Education Research* **5**, 010106 (2009).

55. C. Baily and N. D. Finkelstein, Teaching and understanding of quantum interpretations in modern physics courses, *Phys. Rev. ST: Physics Education Research* **6**, 010101 (2010).

56. C. Baily and N. D. Finkelstein, Refined characterization of student perspectives on quantum physics, *Phys. Rev. ST: Physics Education Research* **6**, 020113 (2010).

57. W. K. Adams, K. K. Perkins, N. Podolefsky, M. Dubson, N. D. Finkelstein and C. E. Wieman, A new instrument for measuring student beliefs about physics and learning physics: the Colorado Learning Attitudes about Science Survey, *Phys. Rev. ST: Physics Education Research* **2**, 1, 010101 (2006).

58. T. L. McCaskey, M. H. Dancy and A. Elby, Effects on assessment caused by splits between belief and understanding, *PERC Proceedings 2003* **720**, 37 (AIP, Melville, NY, 2004).

59. T. L. McCaskey and A. Elby, Probing Students' Epistemologies Using Split Tasks, *PERC Proceedings 2004* **790**, 57 (AIP, Melville, NY, 2005).

60. D. F. Styer, *The Strange World of Quantum Mechanics* (Cambridge University Press, Cambridge, 2000).

61. N. D. Mermin, Is the moon there when nobody looks? Reality and the quantum theory, *Phys. Today* **38** (4), 38 (1985).

62. M. Tegmark and J. A. Wheeler, 100 Years of Quantum Mysteries, *Scientific American* (February 2001, pp. 68-75).

63. D. Z. Albert and R. Galchen, A Quantum Threat to Special Relativity, *Scientific American* (March 2009, pp. 32-39).

64. B.-G. Englert, M. O. Scully and H. Walther, The Duality in Matter and Light, *Scientific American* (December 1994, pp. 86-92).

65. S. B. McKagan, K. K. Perkins and C. E. Wieman, Why we should teach the Bohr model and how to teach it effectively, *Phys. Rev. ST: Physics Education Research* **4**, 010103 (2008).

66. E. Redish, J. Saul and R. Steinberg, Student expectations in introductory physics, *Am. J. Phys.* **66**, 212 (1998).

67. R. Hake, Interactive-Engagement Versus Traditional Methods: A Six-Thousand-Student Survey of Mechanics Test Data for Introductory Physics Courses, *Am. J. Phys.* **66** (1), 64 (1998).

68. L. C. McDermott, Oersted Medal Lecture 2001: "Physics Education Research – The Key to Student Learning," *Am. J. Phys.* **69**, 1127 (2001).

69. S. B. McKagan, K. K. Perkins and C. E. Wieman, Reforming a large lecture modern physics course for engineering majors using a PER-based design, *PERC Proceedings 2006* (AIP Press, Melville, NY, 2006).

70. S. B. McKagan, K. K. Perkins, M. Dubson, C. Malley, S. Reid, R. LeMaster and C. E. Wieman, Developing and Researching PhET simulations for Teaching Quantum Mechanics, *Am. J. Phys.* **76**, 406 (2008).

71. W. Heisenberg, "The Physical Content of Quantum Kinematics and Mechanics," in *Quantum Theory and Measurement*, J. A. Wheeler and W. H. Zurek (Eds.) (Princeton, 1983) p. 62. Originally published under the title, "Uber den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik," Zeitschrift fur Physik, **43**, 172 (1927); translation into English by J. A. Wheeler and W. H. Zurek, 1981.

72. L. E. Ballentine, The Statistical Interpretation of Quantum Mechanics, *Rev. Mod. Phys.* **42**, 358 (1970).

73. L. E. Ballentine, Quantum Mechanics: A Modern Development (World Scientific Publishing, Singapore, 1998).

CHAPTER 2

Development of Student Perspectives - Initial Studies

I. Introduction

Our initial investigations into student perspectives seek to document and better understand the changes students undergo as they make the transition from learning classical physics to learning about quantum mechanics. We first analyze student responses to pre- and post-instruction surveys at various stages of an introductory physics sequence in order to demonstrate the development and reinforcement of deterministic perspectives during classical physics instruction, as well as the emergence of probabilistic and nondeterministic perspectives following instruction in modern physics. We also find that a modern physics instructor's choice of learning goals can significantly influence student responses: they are more likely to prefer either a *Realist* or *Quantum* (*matter-wave)* perspective in a context where such a perspective has been explicitly taught. Furthermore, a student's degree of commitment to any particular perspective is not necessarily robust across contexts: students may invoke both *Realist* and *Quantum* perspectives, without always knowing when either of these epistemological and ontological frames is appropriate. These studies serve as motivation for a more detailed exploration of variations in learning goals among modern physics instructors, and the associated impacts on student perspectives. [Chapter 3]

II. Studies

The University of Colorado offers a three-semester sequence of calculus-based introductory physics courses: PHYS1 and PHYS2 are large-lecture courses [1] (N~300-600) in classical mechanics and electrodynamics, respectively; PHYS3 covers a variety of topics from modern physics, and is offered in two sections (N~50-100, each). At the beginning and end of each semester, students from several offerings of each of the above courses were asked to respond to a series of survey questions designed to probe their epistemic and ontological perspectives on physics. The first of these surveys was an online version of the Colorado Learning Attitudes about Science Survey (CLASS), [2] wherein students responded using a 5-point Likert-scale (ranging from strong disagreement to strong agreement) to a series of 42 statements, including:

> **#41:** It is possible for physicists to carefully perform the same experiment and get two very different results that are both correct.

CLASS researchers do not score student responses to this statement as favorable or unfavorable [2] due to a lack of consensus among expert responses[1]. The myriad ambiguities contained in this statement allow for a number of legitimate (but different) interpretations by expert physicists: they may disagree on what it means to conduct the *same* experiment, what qualify as *very different* results, or even what it means for an experimental result to be considered *correct*.

II.A. Student ideas about measurement change over time.

There is a clear trend in how student responses to CLASS #41 change over the course of this introductory sequence. In a cross-sectional study of student responses from the three introductory physics courses (PHYS1, N=2200; PHYS2, N=1650; PHYS3, N=730) we see a shift first from agreement to disagreement, and then back to agreement with this statement. [Fig. 2.1] At the beginning of instruction in classical mechanics (A), more students will agree (40%) with this statement than disagree (26%); yet the number in agreement decreases significantly (B) following instruction in classical physics (to 30%, $p<0.001$), while an increasing number of students disagree (to 39%, $p<0.001$). This trend then reverses itself over a single semester of modern physics (C), at the end of which a greater percentage of students agree with this statement (46%) than prior to classical physics instruction.

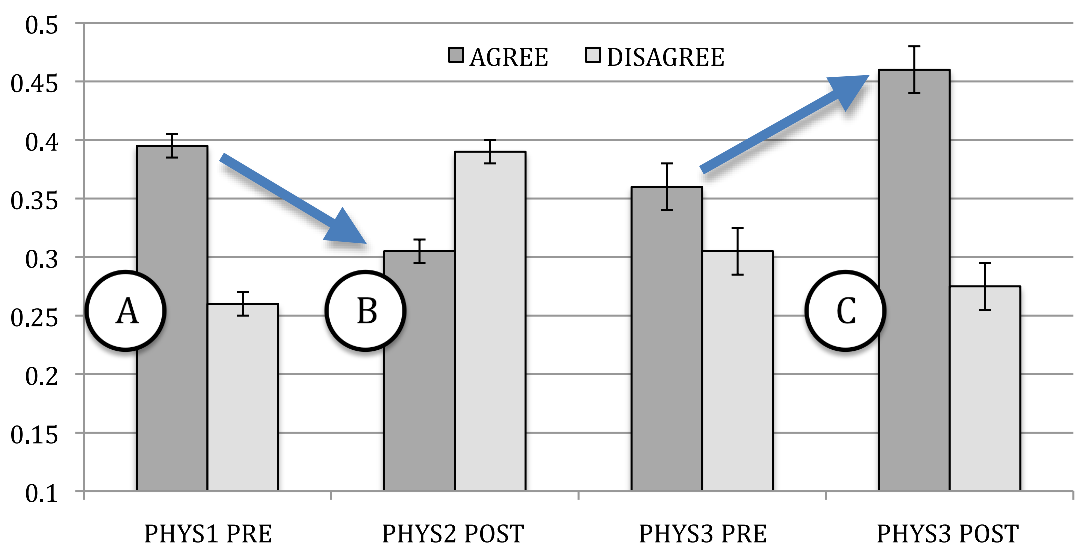


FIG. 2.1. Cross-sectional analysis of student responses to the statement: *It is possible for physicists to carefully perform the same experiment and get two very different results that are both correct* (expressed as a fraction of total responses: PHYS1, N=2200; PHYS2, N=1650; PHYS3, N=730). Error bars represent the standard error on the proportion.

[1] In informal interviews, physics faculty members at the University of Colorado responded approximately 35% Agree, 60% Disagree, and 5% Neutral.

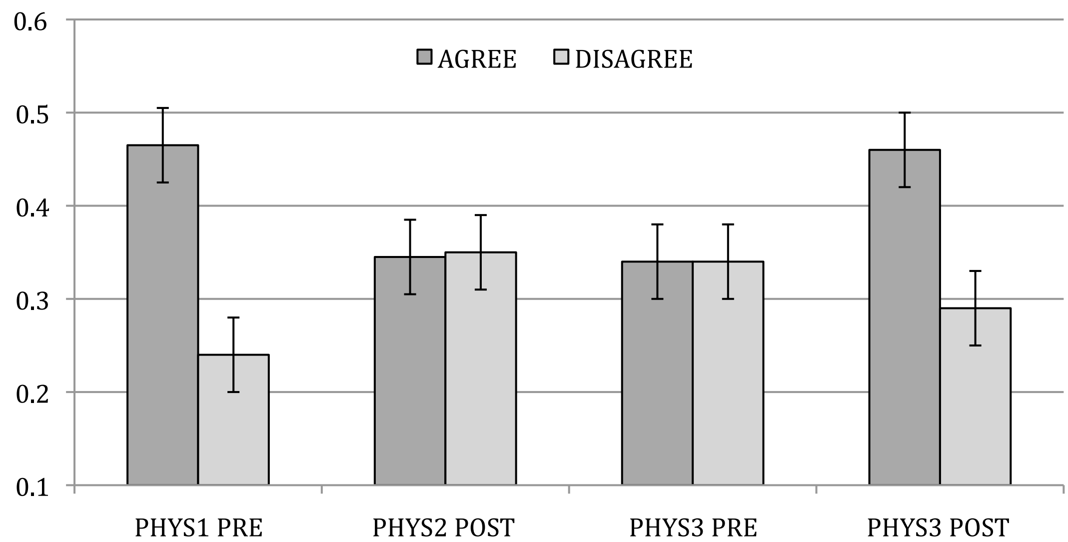


FIG. 2.2. Longitudinal study of student responses to the statement: *It is possible for physicists to carefully perform the same experiment and get two very different results that are both correct* (expressed as a fraction of total responses: N=124). Error bars represent the standard error on the proportion.

In a longitudinal study of 124 students over three semesters, we observe the same trends. [Fig. 2.2]

The distribution of student responses at the end of this introductory sequence is similar to that at the beginning (in terms of agreement versus disagreement); we are naturally interested then in finding out if and how the reasoning invoked by students in defense of their responses changes. We analyzed the reasoning provided by approximately 600 students in an optional text box appended to an online version of the CLASS. These open-ended responses were coded into five categories through an emergent coding scheme. [3] [Table 2.I] The types of reasons offered by modern physics students at the start of instruction was similar to that from students in classical physics courses (pre- and post-instruction), and so the data for both have been combined into a single, pre-quantum instruction group. [Table 2.II]

TABLE 2.I. Categorization of reasoning provided by students in response to the statement: *It is possible for physicists to carefully perform the same experiment and get two very different results that are both correct.*

A	Quantum theory/phenomena
B	Relativity/different frames of reference
C	There can be more than one correct answer to a physics problem. Experimental results are open to interpretation.
D	Experimental/random/human error Hidden variables, chaotic systems
E	There can be only one correct answer to a physics problem. Experimental results should be repeatable.

TABLE 2.II. Distribution of reasoning provided by students before and after instruction in modern physics, in response to the statement: *It is possible for physicists to carefully perform the same experiment and get two very different results that are both correct.* Categories are as given in Table 2.I. Errors are the standard error on the proportion.

CATEGORY	PRE-QM INSTRUCTION (+/-2%)		POST-QM INSTRUCTION (+/-5%)	
	AGREE (N=231)	DISAGREE (N=199)	AGREE (N=41)	DISAGREE (N=26)
A	10%	5%	32%	27%
B	3%	0%	17%	4%
C	28%	6%	10%	8%
D	59%	20%	41%	19%
E	0	69%	0	42%

Our analysis shows that, prior to instruction in modern physics, 59% of those who agreed with the statement offered Category **D** explanations (experimental error, hidden variables); Category E explanations (physics problems have only one correct answer) were preferred by those who disagreed (69%). These results (in conjunction with other studies [4]) allow us to conclude that most introductory classical physics students who disagree with this statement interpret the results of experimental measurements as an approximation of the true (real) value of the quantity being measured; whereas most of those who agree with the statement allow for the possibility of random, hidden factors to influence the outcome of two otherwise identical experiments.

We find that before any formal instruction in modern physics, few students invoke quantum phenomena, despite the fact that a majority of them reported having heard about quantum mechanics in popular venues before enrolling in the course (e.g., books by Greene [5] and Hawking, [6]). However, a single semester of modern physics instruction results in a significant increase in the number of students who believe that quantum physics could allow for two valid, but different, experimental results. Students shift from 13% to 49% in referencing quantum or relativistic reasons for agreeing with the statement. [Table 2.II] Responses from each population were compared with a Chi-Square test and were found to be statistically different ($p<0.001$).

II.B. Instructional choices influence student perspectives.

To see if different types of instruction and learning goals can significantly influence student commitments to any particular perspective, we examined data from two PHYS3 offerings intended for physics majors. Course PHYS3A was taught by a PER instructor who employed in-class, research-based reforms [7], including interactive engagement and computer simulations [8] designed to provide students with a visualization of quantum processes; course PHYS3B was taught the following semester in the form of more traditional lectures. Both modern physics offerings were similar in devoting roughly one-third of the course to special relativity, with the remaining lectures covering the foundations of quantum mechanics and simple applications (as is typical at the University of Colorado). Notable differences in these two courses included the instructional approaches and learning goals of the instructors. Through informal end-of-term interviews and an analysis of course materials, it is clear that each of the instructors held different beliefs about incorporating interpretive aspects of quantum mechanics into a modern physics curriculum. In the context of a double-slit experiment performed with electrons, the instructor for PHYS3A ("Instructor A") explicitly taught that each electron propagates as a delocalized wave while passing through both slits, interferes with itself, and then becomes localized upon detection. Instructor B preferred a more agnostic stance on the physical interpretation of this experiment, and generally did not address such issues:

> "It seems like there's a new book about different interpretations of quantum mechanics coming out every other week, so I see this as something that is still up for debate among physicists. When I talked about the double-slit experiment in class, I used it to show students the need to think beyond F=ma, but I didn't talk about any of that other stuff. [...] We did talk a little about [quantum weirdness] at the very end of the semester, but it was only because we had some time left over and I wanted to give the students something fun to talk about."

Despite Instructor B's self-reported *Agnostic* stance on quantum interpretations, his instructional practices differed in that he explicitly told students that each electron in a double-slit experiment passes through either one slit or the other, but that it is fundamentally impossible to determine which one without destroying the interference pattern (he characterized this *Realist* perspective as the one with which he was "least dissatisfied").

Students from both of these courses were given an end-of-term essay question asking them to argue for or against statements made by three fictional students discussing the Quantum Wave Interference (QWI) PhET simulation's [9] representation of a double-slit experiment with single electrons. [Fig. 2.3] In this simulation, a large circular spot (representing the magnitude of the wave function for a single electron, equivalent to the probability density) (A) emerges from a gun, (B) passes through two slits, and (C) a small dot appears on a detection screen; after a long time (many electrons) an interference pattern develops (not shown).

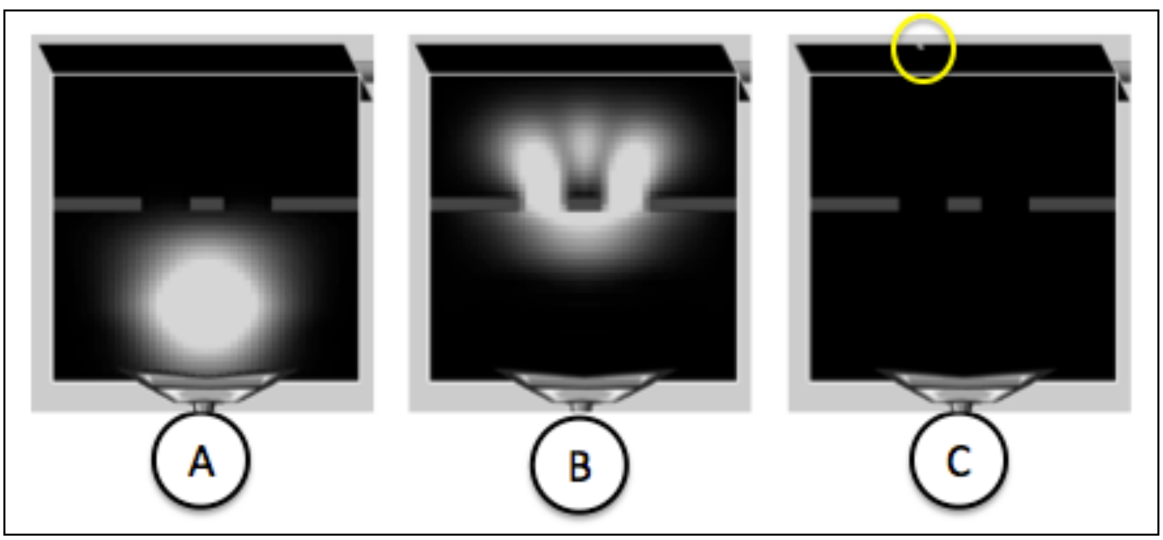


FIG. 2.3. Representation of a double-slit experiment with single electrons in the Quantum Wave Interference PhET simulation; used in the end-of-term essay question.

Each of the following statements (made by a *fictional* student) is meant to represent a potential perspective on how to think of an electron between the time it is emitted and when it is detected at the screen:

> **Student 1**: *That blob represents the probability density, so it tells you the probability of where the electron could have been before it hit the screen. We don't know where it was in that blob, but it must have actually been a tiny particle that was traveling in the direction it ended up, somewhere within that blob.*
>
> **Student 2**: *No, the electron isn't inside the blob, the blob represents the electron! It's not just that we don't know where it is, but that it isn't in any one place. It's really spread out over that large area up until it hits the screen*.
>
> **Student 3**: *Quantum mechanics says we'll never know for certain, so you can't ever say anything at all about where the electron is before it hits the screen.*

In this end-of-term survey question, students were asked to agree or disagree with any (or all) of the fictional students, and to provide evidence in support of their responses, which were then coded according to whether students preferred a *Realist* or a *Matter-Wave* perspective in their argumentation. A random sample of 20 student responses were re-coded by a PER researcher unaffiliated with this project as a test for inter-rater reliability; following discussion of the coding scheme, the two codings were in 100% agreement. The following sample of two student responses is illustrative of the types of responses seen:

> **Student Response (*Realist*):** "We just can't know EXACTLY where the electron is and thus the blob actually represents the probability density of that electron. In the end, only a single dot appears on the screen, thus the electron, wherever it was in the probability density cloud, traveled in its own direction to where it ended up."
>
> **Student Response (*Matter-Wave*)**: "The blob is the electron and an electron is a wave packet that will spread out over time. The electron acts as a wave and will go through both slits and interfere with itself. This is why a distinct interference pattern will show up on the screen after shooting out electrons for a period of time."

The distribution of all responses for the two courses is summarized in Table 2.III (columns do not add to 100% because some students provided a mixed or otherwise unclassifiable response; almost none of the responses favored Student 3). For this essay question, there is a strong bias towards a *Matter-Wave* perspective among PHYS3A students, while students from PHYS3B highly preferred a *Realist* perspective. Virtually no student agreed with fictional Student 3 (which might be consistent with an *Agnostic* perspective); among those who explicitly disagreed with Student 3, most felt that knowing about the probability density was a sufficient form of knowledge about this quantum system.

TABLE 2.III. Student responses to the Quantum Wave Interference essay question from two offerings of PHYS3. Numbers in parentheses represent the standard error on the proportion.

CATEGORY	PHYS3A (%) (N=72)	PHYS3B (%) (N=44)
Realist	18 (5)	75 (7)
Matter-Wave	78 (5)	11 (5)

Students from both PHYS3 courses also responded at the beginning and end of the semester to additional statements appended to an online version of the CLASS for modern physics students, including:

> **QA#2**: An electron in an atom has a definite but unknown position at each moment in time.

It might be expected that students who have learned to view an electron as delocalized until detected in the context of a double-slit experiment would also view it as such in other contexts, such as atoms. Disagreement with this statement on atomic electrons could be consistent with either a *Matter-Wave* or *Copenhagen/Agnostic* perspective, whereas agreement would be more consistent with a *Realist* perspective. While we again observe differences in student responses between the two PHYS3 course offerings [Table 2.IV] there is not the same strong bias toward a single perspective as seen in Table 2.III. Disagreement with this statement among PHYS3A students increased by 22%, and by 13% for PHYS3B students; agreement with this statement decreased by 5% in PHYS3A, while the number of PHYS3B students agreeing with this statement increased by a comparably small amount.

TABLE 2.IV. Student responses to the statement: *An electron in an atom has a definite but unknown position at each moment in time.* Numbers in parentheses represent the standard error on the proportion.

RESPONSE	PHYS3A (%) (N=41)		PHYS 3B (%) (N=36)	
	PRE	POST	PRE	POST
AGREE	44 (8)	39 (8)	48 (8)	54 (8)
NEUTRAL	32 (7)	17 (6)	39 (8)	21 (7)
DISAGREE	22 (6)	44 (8)	10 (5)	23 (7)

II.C Consistency of student perspectives

An important question remains: are there consistencies in student perspectives across domains? The differences in responses from PHYS3A and PHYS3B students are less significant for QA#2 [Table 2.IV] than those seen for the QWI essay question [Table 2.III], but together indicate a possible lack of consistency in their preferred perspectives in different contexts. This inconsistency can be better illustrated by combining matching data for both questions, and then grouping together students from both courses according to how they responded to the QWI essay question. [Table 2.V] In doing so, we see that students who preferred a *Matter-Wave* perspective in the essay question tended to disagree with the notion that atomic electrons exist as localized particles; and the majority of students who preferred a *Realist* perspective in the first case also took a *Realist* stance on the question of atomic electrons. Of particular interest, however, are the students who were not consistent in their responses: 18% of those who disagreed with QA#2, and 33% of those who agreed, offered a response that was inconsistent with their response to the QWI essay question. That is, 18% of students disagreed with the statement on atomic electrons, yet gave a *Realist* response on the interference question; 33% of students were the reverse: taking a *Realist* stance on atomic electrons, but preferring a *Matter-Wave* perspective on the question of electron interference.

TABLE 2.IV. Student responses to the statement: *An electron in an atom has a definite but unknown position at each moment in time,* grouped according to how they responded to the QWI essay question. Numbers in parentheses represent the standard error on the proportion.

QA#2 - POST / QWI	DISAGREE (%)	NEUTRAL (%)	AGREE (%)
Matter-Wave (N=66)	56 (6)	11 (4)	33 (6)
Realist (N=46)	18 (6)	18 (6)	64 (7)

III. Summary and Discussion

The data presented in this chapter serve as evidence in support of three key findings. First, student perspectives with respect to measurement and determinism in the contexts of classical physics and quantum mechanics evolve over time. The distribution of reasoning provided by students in response to the CLASS survey statement indicate that the majority of those who disagree with this statement believe that experimental results should be repeatable, or that there can be only one correct answer to a physics problem. One could easily imagine that students begin their study of classical physics at the university level with a far more deterministic view of science than is evidenced by their initial responses (after all, most students do arrive with some training in classical science). We take the first significant shift in student responses (a decrease in agreement and an increase in disagreement with this statement, as shown in Fig. 2.1) to be indicative of the promotion and reinforcement of a deterministic perspective in students as a result of instruction in classical physics. After a course in modern physics, student responses shift a second time (an increase in agreement and a decrease in disagreement with the survey statement), although the reasoning behind their responses changed. Students of modern physics are instructed that different frames of reference could lead to different experimental results, both of which are correct (special relativity); they are also taught that the quantum-mechanical description of nature is probabilistic, and that the determinism assumed by Newtonian mechanics is no longer valid at the atomic scale. The impact of this type of instruction is reflected in the significant increase in the number of students who invoke relativistic or quantum phenomena as a reason for agreeing with the survey statement.

Second, we observe that how students develop and apply a particular perspective can depend upon the learning goals of their instructors. The results for the Quantum Wave Interference essay question indicate that how students view an electron within the context of a double-slit experiment can be significantly influenced by instruction. Instructor A explicitly taught students that each electron passes through both slits and interferes with itself, and provided students with an in-class visualization of this process via the QWI PhET simulation. The positivistic aspects of the *Copenhagen Interpretation* [10] insist that questions of which slit any particular electron passed through are (at best) ill-posed, and that quantum mechanics concerns itself only with the probabilistic prediction of experimental results. An *Agnostic* stance might say that the question of which slit an electron passed through is irrelevant to the proper application of the mathematical formalism. Although Instructor B reported personally holding an *Agnostic* stance on questions of interpretation in quantum mechanics, he did not teach this perspective explicitly, but rather was explicit in teaching a *Realist* interpretation of the double-slit experiment; this instructional approach is partly reflected in how the majority of PHYS3B students preferred a *Realist* stance on electrons in this context.

Third, we find that many students do not exhibit a consistent perspective on questions of ontology and epistemology across multiple contexts. While the data shown in Table 2.IV do demonstrate some amount of consistency in responses regarding the question of an electron's location, a significant number of students who preferred a *Matter-Wave* interpretation of an electron diffraction experiment would still agree that an electron in an atom has a definite (but unknown) position. We conclude that students will not necessarily develop robust concepts regarding the nature of quanta, which would be consistent with a resources view of student epistemologies and ontologies in physics. [16-19]

Without passing judgment on any particular set of instructional goals, it is worth acknowledging that significant differences in the teaching of modern physics courses do exist (as with upper-division courses in quantum mechanics[11]), and that these learning goals manifest themselves both explicitly and implicitly (intentionally, or not) during the course of instruction. It is in itself a significant finding that, at least in this regard, students are open to adopting their instructor's explicit interpretations of quantum phenomena (though it may be argued in the case of Instructor B that his explicit instruction was already in alignment with the *realist* expectations of his students); there is substantial evidence that students do not necessarily adopt an instructor's views in other contexts. Previous studies of introductory classical physics courses have shown that, with notably few exceptions, [12-14] students tend to shift to more unfavorable (novice-like) beliefs about physics and about the learning of physics [12, 15]. It has been demonstrated, however, that making epistemology an explicit aspect of instruction in introductory physics courses can positively influence this negative trend. [14] The studies presented in this chapter provide further indication that instructors should not take for granted that students will adopt their perspectives on quantum physics unless such learning goals are made explicit in their teaching.

In the end, it seems that a reasonable instructional objective would be for students to apply a particular perspective (deterministic or probabilistic, local or nonlocal) at the appropriate time. If we are to include these goals for our classes, it is important to understand how these messages are sent to our students, and what instructional practices may promote such understandings. [Chapter 3]

References (Chapter 2)

1. S. Pollock and N. D. Finkelstein, Sustaining Change: Instructor Effects in Transformed Large Lecture Courses, *PERC Proceedings 2006* (AIP Press, Melville, NY, 2006).

2. W. K. Adams, K. K. Perkins, N. Podolefsky, M. Dubson, N. D. Finkelstein and C. E. Wieman, A new instrument for measuring student beliefs about physics and learning physics: the Colorado Learning Attitudes about Science Survey, *Phys. Rev. ST: Physics Education Research* **2**, 1, 010101 (2006).

3. J. W. Creswell, *Education Research*, 2nd Ed. (Prentice Hall, Englewood Cliffs, NJ, 2005), pp. 397-398.

4. A. Buffler, S. Allie, F. Lubben & B. Campbell, The development of first year physics students' ideas about measurement in terms of point and set paradigms, *Int. J. Sci. Educ.* **23,** 11 (2001).

5. B. R. Greene, *The Elegant Universe* (Norton, New York, NY, 2003).

6. S. W. Hawking, *A Brief History of Time* (Bantam, New York, NY, 1988).

7. S. B. McKagan, K. K. Perkins and C. E. Wieman, Reforming a large lecture modern physics course for engineering majors using a PER-based design, *PERC Proceedings 2006* (AIP Press, Melville, NY, 2006).

8. http://phet.colorado.edu

9. http://phet.colorado.edu/simulations/sims.php?sim=QWI

10. W. H. Stapp, The Copenhagen Interpretation, *Am. J. Phys.* **40**, 1098 (1972).

11. S. Goldhaber, S. Pollock, M. Dubson, P. Beale, and K. Perkins, Transforming Upper-Division Quantum Mechanics: Learning Goals and Assessment, *PERC Proceedings 2009* (AIP, Melville, NY, 2009).

12. E. Redish, J. Saul and R. Steinberg, Student expectations in introductory physics, *Am. J. Phys.* **66**, 212 (1998).

13. V. K. Otero and K. E. Gray, "Attitudinal gains across multiple universities using the Physics and Everyday Thinking curriculum," *Phys. Rev. ST: Physics Education Research* **4** (1), 020104 (2008).

14. D. Hammer, Student resources for learning introductory physics, *Am. J. Phys.* **68**, S52 (2000).

15. S. Pollock, “No Single Cause: Learning Gains, Student Attitudes, and the Impacts of Multiple Effective Reforms” *PERC Proceedings 2004* (AIP Press, Melville, NY, 2005).

16. D. Hammer, Student resources for learning introductory physics, *Am. J. Phys.* **68**, S52 (2000).

17. D. Hammer, A. Elby, R. E. Scherr and E. F. Redish, “Resources, Framing and Transfer” in *Transfer of Learning*, edited by J. Mestre (Information Age Publishing, 2005) pp. 89-119.

18. A. Gupta, D. Hammer and E. F. Redish, The case for dynamic models of learners’ ontologies in physics, *J. Learning Sciences* **19**, 285 (2010).

19. D. Hammer, A. Gupta and E. F. Redish, On Static and Dynamic Intuitive Ontologies, *J. Learning Sciences* **20**, 163 (2011).

CHAPTER 3

Quantum Interpretation as Hidden Curriculum - Variations in Instructor Practices and Associated Student Outcomes

I. Introduction

In physics education research, the term *hidden curriculum* generally refers to aspects of science and learning about which students develop attitudes and opinions over the course of instruction, but which are primarily only implicitly addressed by instructors. [1] Students may hold varying beliefs regarding the relevance of course content to real-world problems, the coherence of scientific knowledge, or even the purpose of science itself, depending (in part) on the choices and actions of their instructors. Education research has demonstrated that student attitudes regarding such matters tend to remain or become less expert-like when instructors are not explicit in addressing them. [1] In this chapter we present similar findings: the less explicit an instructor is in addressing student perspectives within a given topic area, the greater the likelihood for students (within that specific context) to favor an intuitive, *realist* perspective. In other words, the less the interpretive aspects of quantum mechanics are explicitly addressed by instructors, the more they become part of a hidden curriculum. We explore here how modern physics instructors may (or may not) address this hidden curriculum, and examine the impact of specific instructional approaches on student thinking. Figs. 3.1 & 3.2 (where letters refer to specific instructors and their particular approaches, to be discussed below) illustrate how instructional choices can lead to significantly different student outcomes, as well as the mixed nature of student responses across contexts.

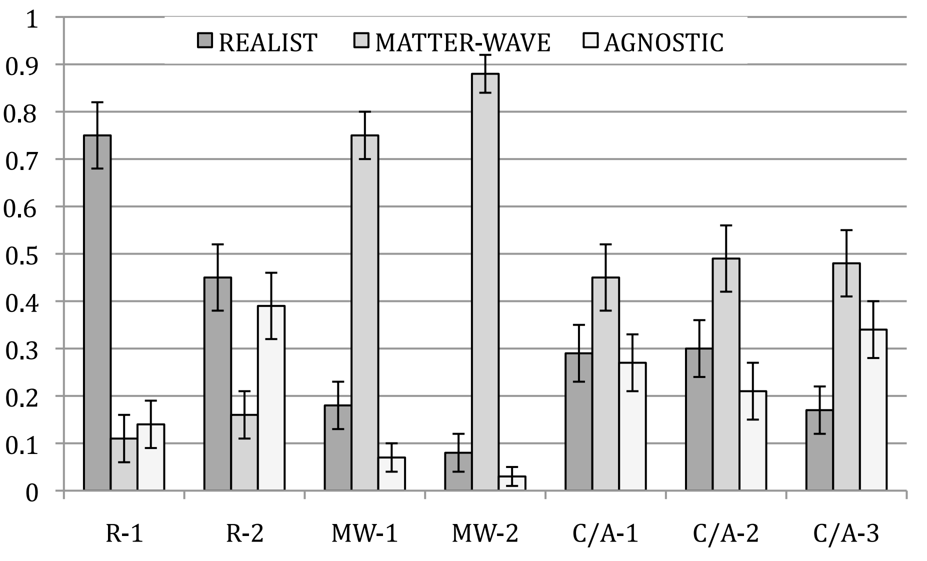


FIG. 3.1. Post-instruction student responses to the double-slit essay question, from seven different modern physics offerings of various instructional approaches [R = *Realist*; MW = *Matter-Wave*; C/A = *Copenhagen/Agnostic*]. Error bars represent the standard error on the proportion; N ~ 50-100 for each course.

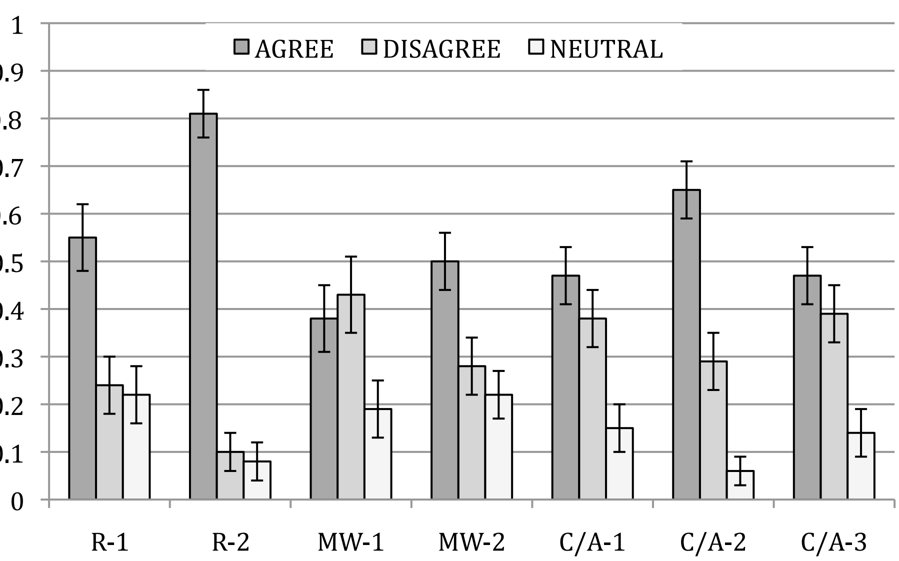


FIG. 3.2. Post-instruction student responses to the statement: *An electron in an atom exists at a definite (but unknown) position at each moment in time*, from seven different modern physics offerings of various instructional approaches [R = *Realist*; MW = *Matter-Wave*; C/A = *Copenhagen/Agnostic*]. Error bars represent the standard error on the proportion; N ~ 50-100 for each course.

II. Instructors approach quantum interpretation differently

This section describes four specific approaches to addressing quantum interpretation in four different modern physics courses recently taught at the University of Colorado, each resulting in significant differences in student thinking by the end of the semester. All four courses were large-lecture (N~100), utilized interactive engagement in class, and devoted the usual proportions of lecture time to special relativity and quantum mechanics. Student responses to the double-slit essay question and statement on atomic electrons described in Chapter 2 are shown in Figs. 3.3 & 3.4, where letters refer to the specific instructors discussed in this section (and their particular approaches to instruction). With respect to the double-slit experiment with electrons, each of these instructors had been explicit in teaching one particular interpretation (*though not explicitly as an interpretation*); student responses in this context were generally reflective of the teaching approaches for each course. [Fig. 3.3]

In two of the four courses (B1 & C) instructors paid considerably less attention to interpretive themes at later stages of the course, as when students learned about the Schrödinger model of hydrogen. Students from all four courses were more likely to agree than disagree with the statement: *An electron in an atom has a definite (but unknown) position at each moment in time*. [Fig. 3.4] What follows is a more detailed discussion of the specific instructional approaches employed in the courses described above, where letters refer to specific instructors, as given in the figure captions.

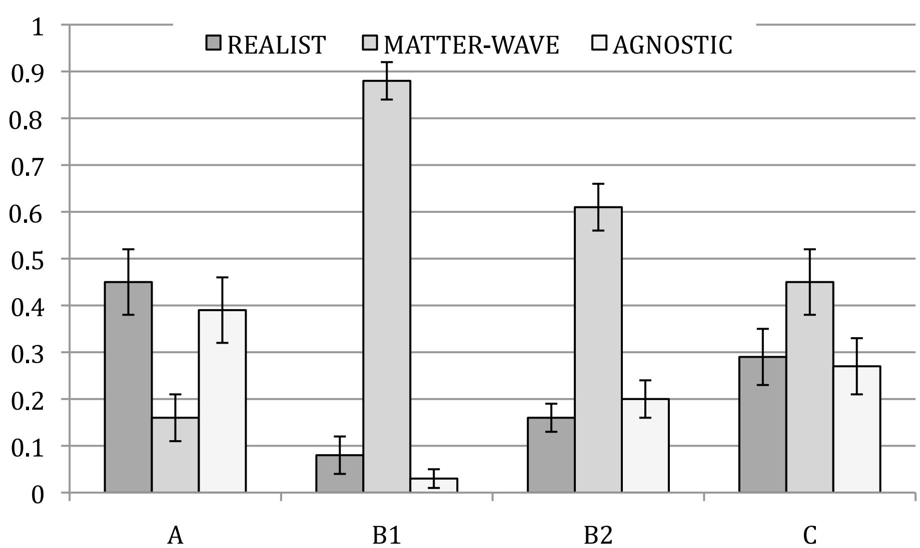


FIG. 3.3. Post-instruction student responses to the double-slit essay question, from four different modern physics offerings of various instructional approaches [A = *Realist/Statistical*; B1 & B2 = *Matter-Wave*; C = *Copenhagen/Agnostic*]. Error bars represent the standard error on the proportion; N ~ 100 for each course.

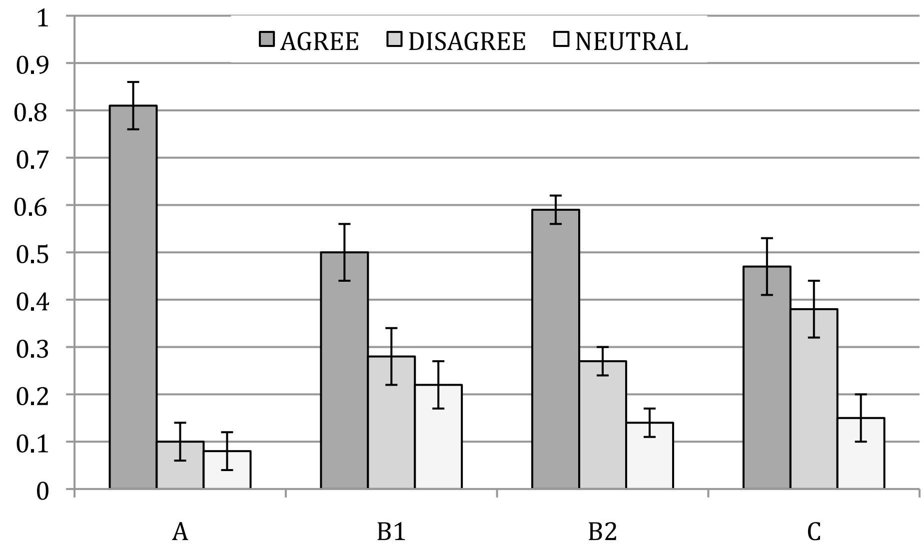


FIG. 3.4. Post-instruction student responses to the statement: *An electron in an atom exists at a definite (but unknown) position at each moment in time*, from four different modern physics offerings of various instructional approaches [A = *Realist/Statistical*; B1 & B2 = *Matter-Wave*; C = *Copenhagen/Agnostic*]. Error bars represent the standard error on the proportion; N ~ 100 for each course.

A. Explicitly teach an interpretation that aligns with student intuition, without discussing alternatives: Instructor A taught this course for engineering majors from a *Realist/Statistical* perspective (though he did not call it such), and explicitly referred to this in class as his own interpretation of quantum phenomena, one that other physicists would not necessarily agree with. Beyond his *Realist* stance on the double-slit experiment, students were explicitly instructed to think of atomic electrons as localized particles, and that energy quantization is the result of their average behavior; there was no discussion of alternatives to the perspective being promoted in class. Student responses from this course in both contexts were in alignment with Instructor A's explicit learning goals: they were the most likely to prefer a *Realist* interpretation of the double-slit experiment [each electron goes through either one slit or the other, but not both], as well as the most likely to agree that atomic electrons exist as localized particles. We believe student responses from this course are reflective not only of this instructor's explicit instruction, but also that this particular kind of interpretation of quantum mechanics is in agreement with intuitively *realist* expectations.

B1. Teach one interpretation (though not explicitly as an interpretation) in some topic areas (particularly at the beginning of the course) and expect students to generalize to other contexts on their own: When first teaching this modern physics course for engineering majors, Instructor B was explicit in modeling single quanta in the double-slit experiment as delocalized waves that pass through both slits simultaneously. He did not frame this discussion in terms of modeling or interpretation, but rather made what he saw as sufficient arguments in favor of this particular interpretation, as he stated in an informal post-instruction interview:

> "This image that [students] have of this [probability] cloud where the electron is localized, it doesn't work in the double-slit experiment. You wouldn't get diffraction. If you don't take into account both slits and the electron as a delocalized particle, then you will not come up with the right observation, and I think that's what counts. The theory should describe the observation appropriately. […] It really shouldn't be a philosophical question just because there are different ways of describing the same thing [i.e. as a wave or a particle]. They seem to disagree, but in the end they actually come up with the right answer."

Students from this *Matter-Wave* course overwhelmingly preferred a wave-packet description of individual electrons [each electron passes through both slits simultaneously and interferes with itself]. However, these students did not seem to generalize this notion of particles as delocalized waves to the context of atoms, where Instructor B was not explicit regarding the ontological nature of electrons, and where a majority still agreed that atomic electrons exist as localized particles. Students were more likely to prefer *Realist* notions in a topic area where Instructor B was not explicit regarding interpretation.

B2. Teach one interpretation (though not explicitly as an interpretation) in some topic areas, combined with a more general discussion of interpretative themes towards the end of the course: Instructor B later taught a second modern physics course for engineering majors in a similar manner, but this time devoted two lectures near the end of the course to interpretive themes in quantum mechanics, including a discussion of the interpretive aspects of the double-slit experiment (but without reference to atomic systems). Student responses were similar to the previous *Matter-Wave* course (B1) on interpretations of the double-slit experiment, but a majority of students still preferred a *Realist* stance on atomic electrons.

C. Teach a Copenhagen/Agnostic perspective, or de-emphasize questions of interpretation: In this modern physics course for physics majors, Instructor C did touch on some interpretive themes during the course, though he ultimately emphasized a perspective that was more pragmatic than philosophical, as when faced with the in-class question of whether particles have a definite but unknown position, or have no definite position until measured:

> "Newton's Laws presume that particles have a well defined position and momentum at all times. Einstein said we can't know the position. Bohr said, philosophically, it has no position. *Most physicists today say: We don't go there. I don't care as long as I can calculate what I need.*" [Emphasis added]

In an end-of-term interview, Instructor C clarified his attitude toward teaching any particular perspective to students in a sophomore-level course:

> "In my opinion, until you have a pretty firm grip on how QM actually works, and how to use the machine to make predictions, so that you can confront the physical measurements with pairs of theories that conflict with each other, there's no basis for ragging on the students about, 'Oh no, the electron, it's all in your head until you measure it.' They don't have the machinery at this point, and so anybody who wants to stand in front of [the class] and pound on the table and say some party line about what's really going on, nevertheless has to recognize that the students have no basis for buying it or not buying it, other than because they're being yelled at."

Student responses from this course to the double-slit essay question were more varied than with the other courses – students were not only more likely to prefer an *Agnostic* stance [quantum mechanics is about predicting the interference pattern, not discussing what happens between], a significant number of students (30%) preferred a *Realist* interpretation – more than with the *Matter-Wave* courses, but less so than with the *Realist/Statistical* course. Nearly half of all students from this course also preferred a *Realist* stance on atomic electrons.

III. Comparing Instructor Practices (A Closer Look)

The goal of understanding the interplay between instructor practices and student perspectives calls for a more detailed comparison of two modern physics courses with similar content and presentation, but different in their approach to interpretive themes in quantum mechanics (Courses B1 & C from Section II, both of which took place in the semester immediately following the studies described in Chapter 2).

III.A. Background on course materials and curriculum similarities.

Each semester, the University of Colorado (CU) offers two versions of its introductory modern physics course; one section is intended for engineering majors (e.g., Course B1), and the other for physics majors (Course C). The curricula for both versions of the course have traditionally been essentially the same, with variations from semester to semester according to instructor preferences. In the fall of 2005, a team from the physics education research (PER) group at CU introduced a transformed curriculum for the engineering course incorporating research-based principles. [2] This included interactive engagement techniques (in-class concept questions, peer instruction, and computer simulations [3]), as well as revised content intended to emphasize reasoning development, model building, and connections to real-world problems. These course transformations, implemented during the FA05-SP06 academic year, were continued in FA06-SP07 by another physics education researcher at CU, who then collaborated in the FA07 semester with a non-PER faculty member to adapt the course materials into a curriculum appropriate for physics majors (by including topics from special relativity).

The course materials [4] for all five of these semesters (which included lecture slides and concept tests) were made available to Instructors B & C, who both reported changing a majority of the lecture slides to some extent (as well as creating new ones). By examining the course syllabi and categorizing the lecture material for each course into ten standard introductory quantum physics topics, we find the general progression of topics in both classes to be essentially the same (the presentation of content was many times practically identical), with slight differences in emphasis. [Table 3.I]

TABLE 3.I. Progression of topics and number of lectures devoted to each topic from the quantum physics portion of both modern physics courses B1 & C.

CODE	**TOPIC**	**# OF LECTURES**	
		B1	**C**
A	Introduction to quantum physics	2	1
B	Photoelectric effect, photons	5	4
C	Atomic spectra, Bohr model	5	3
D	DeBroglie waves/atomic model	1	1
E	Matter waves, interference/diffraction	3	2
F	Wave functions, Schrödinger equation	2	5
G	Potential energy, infinite/finite square well	3	3
H	Tunneling, alpha-decay, STM's	2	4
I	3-D Schrödinger equation, hydrogen atom	4	2
J	Multi-electron atoms, periodic table, solids	3	3

III.B. Differences in instructional approaches.

While the learning environments and progression of topics for both modern physics courses were essentially the same, the two courses differed in sometimes obvious, other times more subtle ways with respect to how each instructor addressed student perspectives and themes of interpretation. An analysis of the instructional materials used in each of the two courses offers a first-pass comparison of the two approaches. When comparing the homework assignments for each course, there were no (or very minimal) opportunities for students to reflect on physical interpretations of quantum phenomena. Similarly, an examination of the midterms and finals from both courses revealed no emphasis on questions of interpretation. The one place that afforded the most faculty/student interaction with respect to interpretation was in the lecture portions of each course, and so we examine how these two instructors specifically addressed interpretation during lecture.

A first analysis of lecture materials entails a coding of lecture slides (which were later posted on the course website). We employ a simple counting scheme by which each slide is assigned a point value of zero or one in each of three categories, according to its relevance to three interpretive themes. [Table 3.II] These three categories (denoted as *Light*, *Matter,* & *Contrasting Perspectives*) were chosen to highlight key lecture slides that were explicit in promoting non-classical perspectives. Since light is classically described as a wave, slides that emphasized its particle-like nature, or explicitly addressed its dual wave-particle characteristics, were assigned a point in the *Light* category; similarly, slides that emphasized the wave nature of matter, or its dual wave/particle characteristics, were given a point in the *Matter* category. Other key slides (*Contrasting Perspectives* category) were those that addressed randomness, indeterminacy, or the probabilistic nature of quantum mechanics; or those that made explicit contrast between quantum results and what would be expected in a classical system. While most of the slides in Table 3.II received only one point in a single category, many slides were relevant to multiple categories, and so the point totals do not represent the total number of relevant slides from each course.

TABLE 3.II. Categorization of lecture slides relevant to promoting non-classical perspectives, with a point total for each category.

THEME	DESCRIPTION OF LECTURE SLIDE	B1	C
Light	Relevant to the dual wave/particle nature of light, or emphasizing its particle-like characteristics	15	9
Matter	Relevant to the dual wave/particle nature of matter, or emphasizing its wave-like characteristics	15	16
Contrasting Perspectives	Relevant to randomness, indeterminacy, or the probabilistic nature of quantum mechanics; explicit contrast between quantum & classical descriptions.	28	22

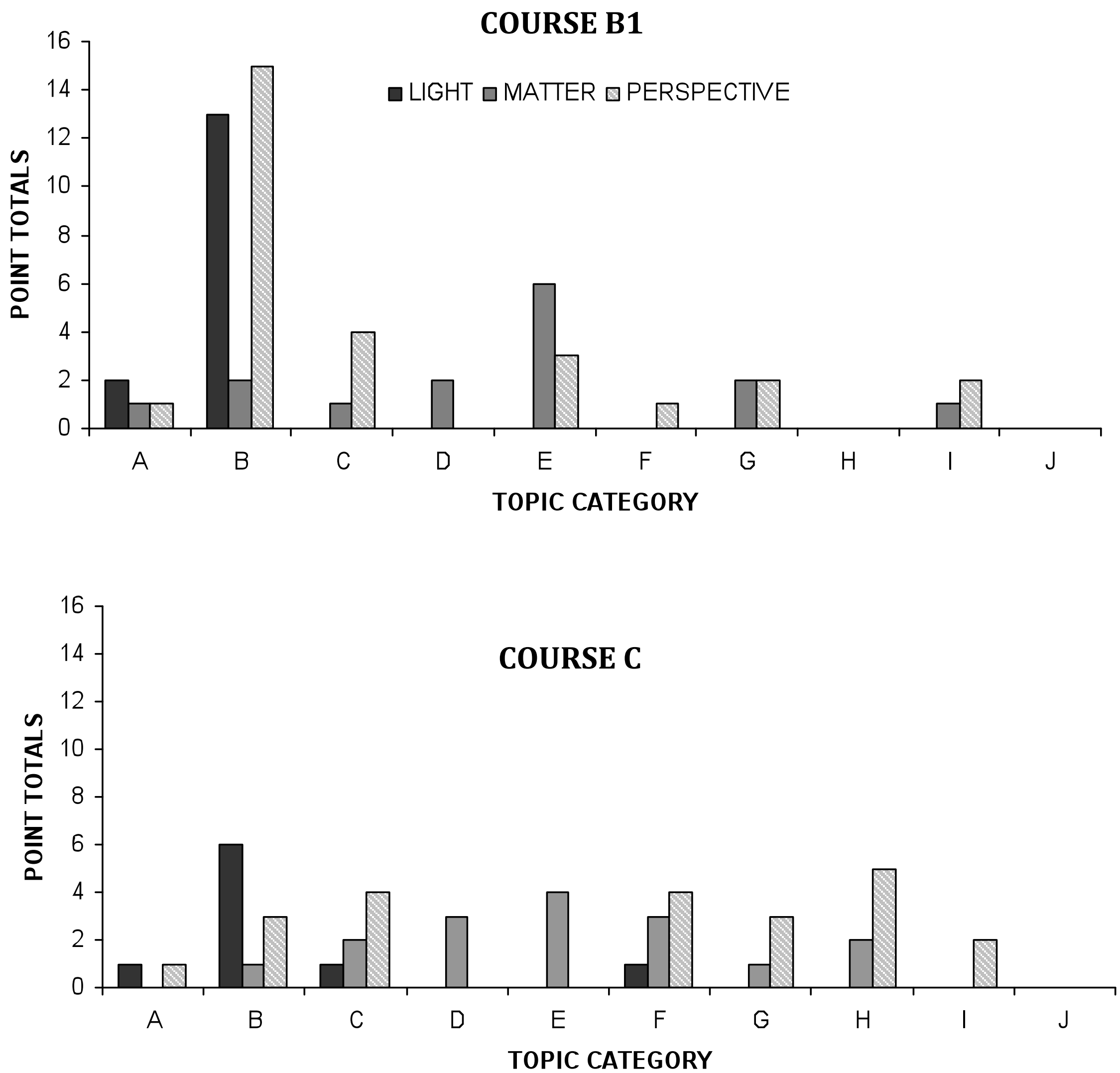


FIG. 3.5. The occurrence of lecture slides for both PHYS3 courses by topic (as describe in Table 3.I), for each of the themes described in Table 3.II.

Course B1 had a greater number of slides that scored in the *Light* and *Contrasting Perspectives* categories, though the graphs in Fig. 3.5 (which group the point totals for each course by topic area, as listed in Table 3.I) show that this difference can be largely attributed to instructor choices at the outset of the quantum physics sections of the two courses, in topic category B (photoelectric effect and photons). That this topic area should stand out in this analysis seems natural if one considers that: i) The photoelectric effect requires a particle description of light; ii) The double-slit experiment with single photons requires both a wave and a particle description of light in order to fully account for experimental observations; and iii) Being the first specific topic beyond the introductory quantum physics lecture(s), it represents an opportunity to frame the content of the course in terms of the need to think beyond classical physics. While both modern physics courses had the greatest point totals in this topic category, B1 devoted a greater portion of lecture time here to addressing themes of indeterminacy and probability (B1 also totaled more points in the *Light* category, though this difference can be largely attributed to Instructor B's brief coverage of lasers, a topic not covered in Course C).

Fig. 3.6 shows the ratio of the point totals for each of the three interpretive themes (from topic area B only) to the total number of slides used during these lectures; the differences between the two courses in terms of the amount of lecture time spent contrasting perspectives is statistically significant ($p=0.001$, by a one-tailed t-test). We note, finally, that in both courses all three of these interpretive themes received considerably less attention at later stages of the course.

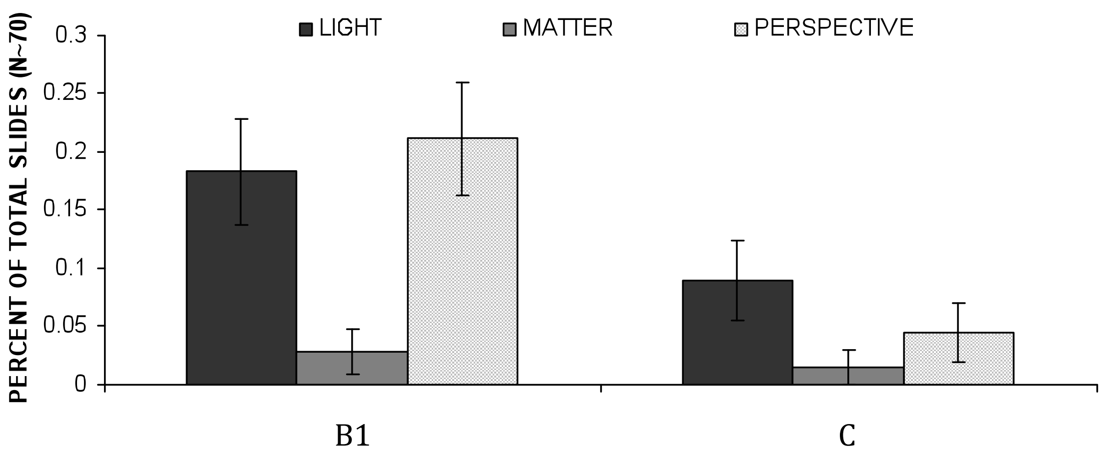


FIG. 3.6. Ratio of point totals from topic area B for each interpretive theme to the total number of slides used during these lectures. Error bars represent the standard error on the proportion.

The lecture slide shown in Fig. 3.7 is one example of how Course B1 differed from Course C in attending to student perspectives during the discussion of photons, by explicitly addressing the likelihood for students to think of quanta as being spatially localized. There were no comparable slides from Course C from this topic category, though this should not be taken to mean that Instructor C failed to address such issues at other times during the semester, or one-on-one with students. We note simply that there were no such explicit messages as part of the artifacts of the course in this topic area (which reflects a value judgment on the part of Instructor C regarding content), and students from Course C who accessed the lecture slides as posted online would have no indication that such ideas were deserving of any particular emphasis.

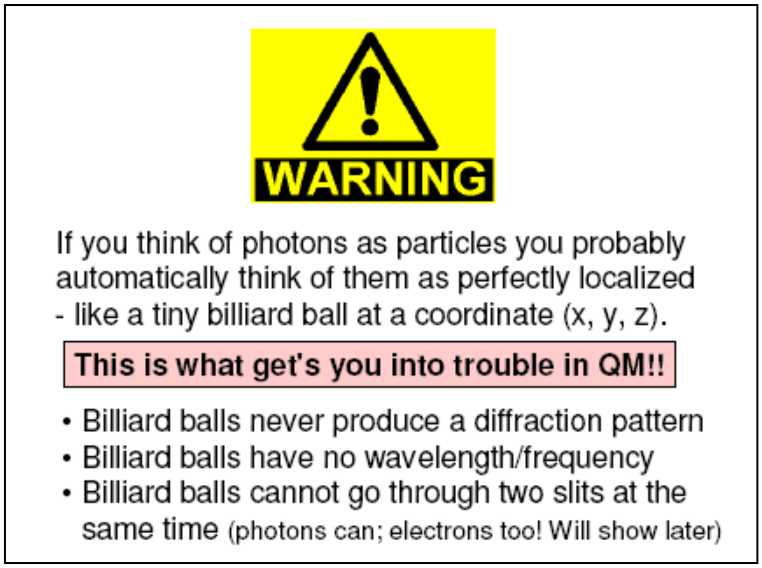


FIG. 3.7. A lecture slide used in Course B1 during the discussion of photons.

While there are coarse differences in how the instructors addressed student perspectives in some topic areas, the instructional approaches sometimes differed in more subtle ways. The two slides shown in Fig. 3.8 are illustrative of how the differences between the two courses could sometimes be less obvious, though still of potential significance. Both slides summarize the results for a system referred to in Course B1 as the *Infinite Square Well*, and by Instructor C as the *Particle in a Box*. At first glance, the two slides are almost identical: each depicts the first-excited state wave function of an electron in a potential well, as well as listing the normalized wave functions and quantized energy levels for this system. Both slides make an explicit contrast between the quantum mechanical description of this system and what would be expected classically, each pointing out that a classical particle can have any energy, whereas an electron confined in a potential well can only have specific energies.

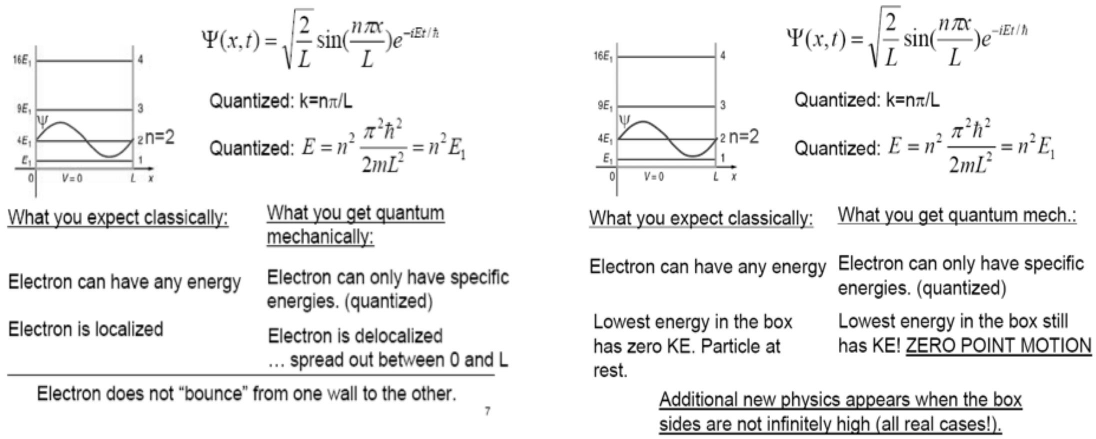


FIG. 3.8. Lecture slide from Course B1 (left, *Infinite Square Well*) and a nearly identical one from Course C (right, *Particle in a Box*).

However, Course B1 differed from Course C by emphasizing a wave model of the electron, delocalized and spread out, stating explicitly that the electron should not be thought of as bouncing back and forth between the two walls of the potential well. Instructor C focused instead on the kinetic energy of the system, pointing out that a classical particle can be at rest, whereas the quantum system has a non-zero ground state energy. It is arguable that Instructor C's choice of language, to speak of a *particle* in a box exhibiting zero-point motion, could implicitly reinforce in students the *realist* notion that in this system a localized electron is bouncing back and forth between two potential barriers. Both of these slides received a point in the *Contrasting Perspectives* category, but only the slide from PHYS3A received a point in the *Matter* category for its emphasis on the wave-like properties of an electron in a potential well.

III.C. The double-slit experiment with single quanta.

As taught in these two courses, the double-slit experiment [Fig. 3.9] consists of a monochromatic beam of quanta that: (1) impinges on two closely spaced slits and diffracts; (2) wavelets spread out behind the slits and (3) interfere in the regions where they overlap; (4) bright fringes appear on the detection screen where the anti-nodal lines intersect.

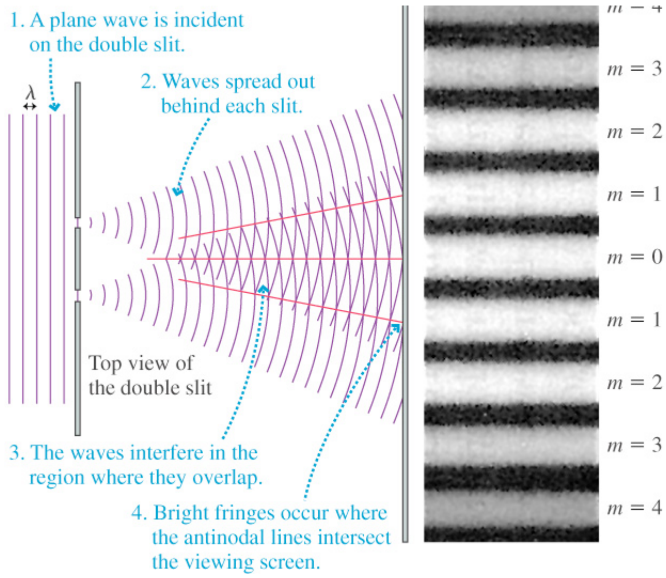


FIG. 3.9. Lecture slide used in both PHYS3 courses describing the double-slit experiment in terms of wave interference.

Both PHYS3 courses also instructed students that the intensity of the beam can be turned down to the point where only single quanta pass through the apparatus at a time; individual quanta are detected as localized particles on the screen, yet an interference pattern still develops over time. A wave description of quanta explains the interference pattern on the detection screen, while a particle description addresses the fact that individual quanta are detected as localized particles; in other words, a single ontological categorization of quanta (particle or wave) is inadequate for explaining all of what's observed in the double-slit experiment. Both instructors addressed during lecture a mathematical description of the interference pattern (how to relate the distance between the slits and the wavelength of the beam to the locations of fringe maxima and minima), and both used the Quantum Wave Interference simulation [5] in class to provide students with a visualization of the process. The approaches taken by the two instructors (B1 & C) with respect to quantum interpretation were as described in Section II; in brief, Instructor B took a *Matter-Wave* approach, while Instructor C was more *Agnostic* in his learning goals.

In the last week of the semester, students from both PHYS3 courses responded to an online survey designed to probe their ontological and epistemological beliefs about quantum mechanics. Students received homework credit for responding to the survey (equivalent to the number of points given for a typical homework problem), and the response rate for both courses was approximately 90%. Students were also told they would only receive full credit for providing thoughtful answers, and the text of the survey itself emphasized in bold type that there were no *right* or *wrong* answers to the questions being asked, but that we were particularly interested in what the students personally believed.

Instructors for both courses vetted the wording of the items on the survey, and interviews conducted after the end of the semester [Chapter 4] indicate that students interpreted the meaning of the questions in a way that was consistent with our intent. [See Appendix A for the evolution of the survey items (SP08-FA10).]

At the time of this study, the wording of the fictional student statements in the double-slit essay question had been changed in order to better reflect the language and argumentation of actual students (crafted in part from actual student responses from the study described in Chapter 2):

> **Student One:** *The probability density is so large because we don't know the true position of the electron. Since only a single dot at a time appears on the detecting screen, the electron must have been a tiny particle, traveling somewhere inside that blob, so that the electron went through one slit or the other on its way to the point where it was detected.*
>
> **Student Two:** *The blob represents the electron itself, since an electron is described by a wave packet that will spread out over time. The electron acts as a wave and will go through both slits and interfere with itself. That's why a distinct interference pattern will show up on the screen after shooting many electrons.*
>
> **Student Three:** *Quantum mechanics is only about predicting the outcomes of measurements, so we really can't know anything about what the electron is doing between being emitted from the gun and being detected on the screen.*

The results for both PHYS3 courses (B1 and C) are shown in Fig. 3.3, where responses are categorized according to which fictional student(s) the respondents agreed with (*Realist*, *Matter-Wave*, or *Agnostic*). While most students chose to agree with only a single statement, there were a few respondents from both courses who chose to agree with both the fictional *Realist* and *Agnostic* students, or with both the *Matter-Wave* and *Agnostic* students; we feel the *Realist* and *Matter-Wave* statements are not individually incompatible with the *Agnostic* statement, since simultaneously agreeing with the latter allowed students to acknowledge that they had no way of actually knowing if their preferred interpretations were correct. The relatively few students (~5%) who responded in this way are grouped together with the other students in the *Realist* or *Matter-Wave* categories, as appropriate.

As might be predicted based on the specific practices of Instructor B, most of his students chose to agree with the *Matter-Wave* statement (the electron is a delocalized wave packet that interferes with itself). The responses from Course C students were more varied: they were nearly four times more likely than B1 students to prefer a *Realist* interpretation; similarly, they were half as likely to favor the wave-packet description. More specifically, 29% of Course C students chose to agree with the *Realist* statement of Student One, and 27% of them agreed with the *Copenhagen/Agnostic* stance of Student Three, while only a combined 11% of students from Course B chose either of these responses.

III.D. (In)consistency of student responses.

As seen in Fig. 3.5, both PHYS3 courses paid less explicit attention to student perspectives at later stages of instruction, as when covering the Schrödinger model of hydrogen. In lecture slides, both courses described an electron in the Schrödinger atomic model as a "cloud of probability surrounding the nucleus whose wave function is a solution of the Schrodinger equation," without further elaboration with respect to interpretation. We are interested in knowing if how students came to think of quanta in the context of the double-slit experiment would be relevant to how they thought of atomic electrons, particularly when they hadn't been given the same kind of explicit instruction in this topic area as with the double-slit experiment or the infinite square well.

In addition to the essay question, students responded (and provided reasoning) to the pre/post online survey statement regarding the position of atomic electrons; the following student quotes are illustrative of the reasoning offered by students in support of their responses:

> **AGREE**: "The probability cloud is like a graph method. It tells us where we are most likely to find the electron, but the electron is always a point-particle somewhere in the cloud."
>
> **DISAGREE**: "The electron is delocalized until we interfere with the system. It is distributed throughout the region where its wave function is non-zero. An electron only has a definite position when we make a measurement and collapse the wave function."

At the end of instruction, B1 students were just as likely to agree with the statement on atomic electrons as students from Course C, [Fig. 3.10] despite the emphasis given in Course B1 to thinking of an electron as delocalized in other contexts. Both courses showed a modest (and statistically insignificant) decrease in *Realist* responses to this statement between pre- and post-instruction, yet students from both courses were still more likely to agree than disagree with this statement in the end.

If responses from both courses to the statement on atomic electrons are grouped by how those same students responded to the double-slit essay question [Fig. 3.11] we see that 70% of students who preferred a *Realist* interpretation in the essay question took a stance on atomic electrons that would also be consistent with *realist* expectations. And while students who preferred a wave-packet description in the essay question were more likely than *Realist* category students to disagree with the statement on atomic electrons, 46% of those students still agreed that an electron in an atom has a definite position at all times. Only in the case of students who preferred the *Agnostic* statement did a majority disagree with this statement, and no students from this group responded neutrally.

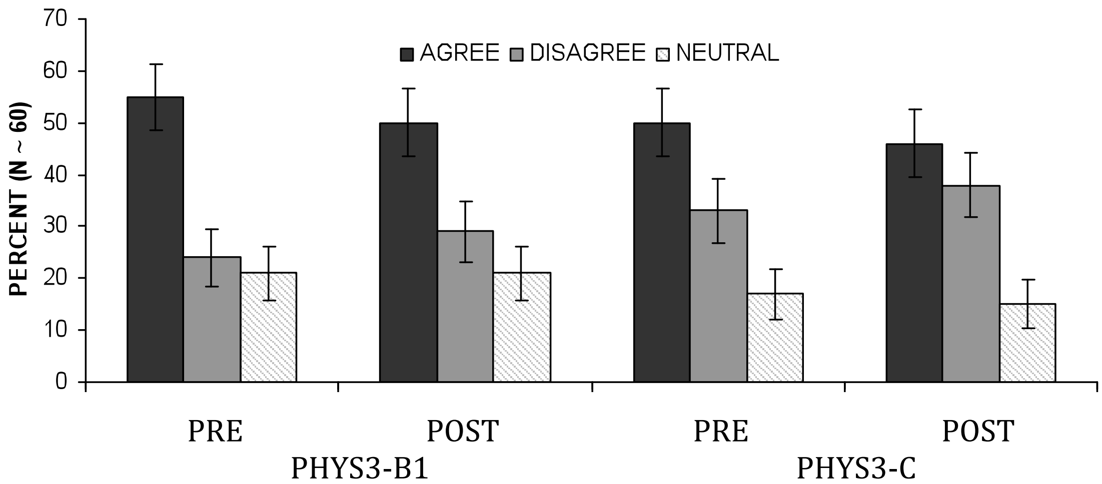


FIG. 3.10. Pre/post student responses from both PHYS3 courses to the statement: *An electron in an atom has a definite but unknown position at each moment of time.* Error bars represent the standard error on the proportion (N~60).

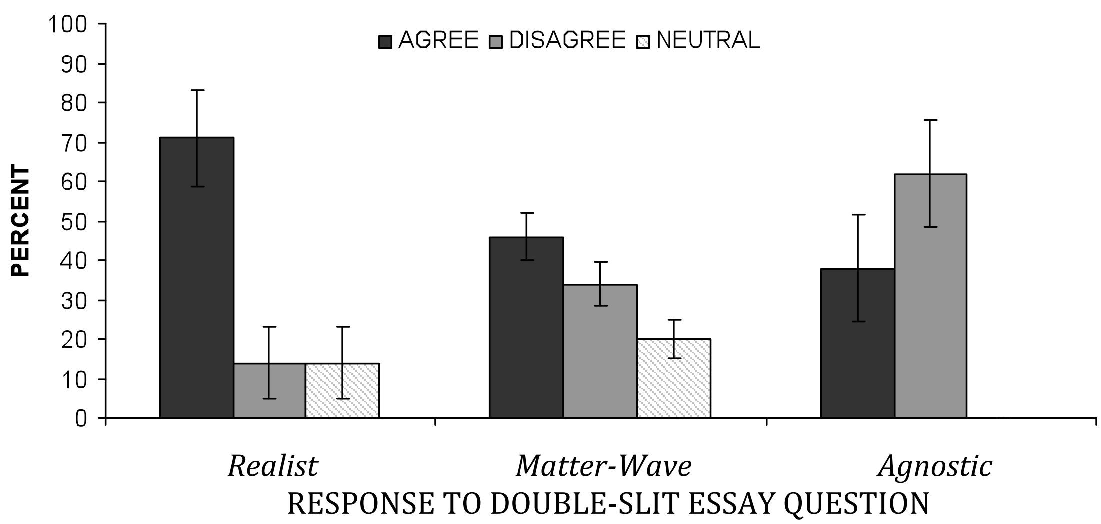


FIG. 3.11. Combined student responses from both PHYS3 courses to the statement: *An electron in an atom has a definite but unknown position at each moment of time*, grouped by how those students responded to the double-slit essay question. Error bars represent the standard error on the proportion (N~60).

IV. Summary and Discussion

Modern physics instructors differ not only in their personal perspectives regarding the physical interpretation of quantum mechanics, but also in their decisions to teach (or not teach) about quantum interpretations in their introductory courses. In this chapter, we have documented significant instructor effects in terms of how students respond to post-instruction surveys; we have also examined in detail two different approaches to addressing interpretative themes in two introductory modern physics courses with similar content.

When comparing these two courses in detail, Instructor B's more explicit approach to teaching a *matter-wave* interpretation of the double-slit experiment had a significant impact on how students said they thought of electrons within that specific context. Instructor C's less explicit and more *Agnostic* instructional approach is reflected in the greater variation of student responses to the essay question; not only were Course C students more likely than B1 students to prefer an *Agnostic* stance (which would be in alignment with Instructor C's instructional approach), these students were also more likely to align themselves with a *Realist* interpretation. In addition, the emphasis given in Course B toward thinking of electrons as delocalized in the double-slit experiment and the infinite square well had no discernible impact on student responses in areas where instruction was less explicit. Both courses were similar in their treatment of the Schrödinger atomic model, and student responses from both courses regarding the existence of an electron's position in an atom were not significantly different, with the majority of students from both courses favoring a *Realist* perspective in this specific context.

We may investigate the consistency in how students apply perspectives across contexts by comparing responses to the double-slit essay question with a statement regarding the position of an electron in an atom. We find that most every student who preferred a *Realist* interpretation of the double-slit experiment also took a *Realist* stance on the question of whether an electron in an atom has a definite position. On the other hand, almost half of the students who preferred the wave-packet description of a single electron in the double-slit experiment would still agree with particle-like descriptions of atomic electrons. Such responses evidence the greater likelihood for students to favor *Realist* perspectives in topic areas where instruction is less explicit, and suggest that instructors who wish to promote any particular perspective in quantum physics should do so explicitly across a range of topics, rather than assuming it to be sufficient to address student perspectives primarily at the outset.

These findings also indicate that, just as with topics in classical physics, [6-14] naïve intuition (being congruent with *realist* expectations) can serve as a barrier to conceptual understanding in quantum physics. A major difference between the intuitive barriers in classical physics and in quantum physics lies in the nature of the questions, both ontological (when is a particle a particle, and when is it a wave?) and epistemic (what is the difference between classical ignorance and fundamental uncertainty?). End-of-semester comments from Instructor C support the notion that students who preferred a *Realist* interpretation of the double-slit experiment were not doing so from a simple lack of understanding:

> “Some of the students who I considered to be the most engaged went with [the *Realist* statement. They said]: ‘…the electron is a real thing; it’s got to be in there somehow. I know that’s not what you told us, but that’s what I’m thinking…’ I thought that was just great; it was sort of honest. They were willing to recognize that that’s not what we’re saying, but they’re grappling with that’s how it’s got to be anyways.”

Furthermore, one-on-one interviews conducted with students from these two courses following the end of the semester [Chapter 4] showed that those who had favored a *Realist* perspective in the interference essay question were still able to correctly describe from memory the particulars of the double-slit experiment.

It is also worth noting that the two instructors considered in our detailed comparative study, while sometimes explicit in teaching *an* interpretation of quantum mechanics, were not explicit in teaching these interpretations *as interpretations*. In other words, they did not teach quantum mechanics from an axiomatic standpoint, did not explicitly teach the *Copenhagen Interpretation* (or any other formal interpretation); nor did they frame their interpretations in terms of *modeling,* or *nature of science* (NOS) issues. Instead, instructors for both courses addressed questions of interpretation as they arose within the contexts of specific topics, without making the physical interpretation of the wave function (beyond its probabilistic interpretation, à la Born [15]) into a major topic unto itself. The sense in which quantum interpretation is *hidden* in modern physics curricula becomes apparent when considering how students may default to intuitive *realist* expectations in topic areas where instructors are less explicit; and in recognizing that interpretive aspects of quantum physics tend to remain unaddressed in a way that is meaningful to students.

The studies considered in this chapter suggest that instructors should be aware of the potential impact they may have on student thinking as a consequence of their instructional choices – instructors who spend less time explicitly attending to student knowledge and intuition are less likely to transition students away from inappropriately *realist* perspectives. These studies have also indicated that students may favor a variety of perspectives in a way that may seem contradictory to expert physicists, indicating the need for a deeper exploration into the contextual aspects of student perspectives in quantum physics. [Chapter 4]

References (Chapter 3)

1. E. Redish, J. Saul and R. Steinberg, Student expectations in introductory physics, *Am. J. Phys.* **66**, 212 (1998).

2. S. B. McKagan, K. K. Perkins and C. E. Wieman, Reforming a large lecture modern physics course for engineering majors using a PER-based design, *PERC Proceedings 2006* (AIP Press, Melville, NY, 2006).

3. http://phet.colorado.edu

4. Available at http://www.colorado.edu/physics/EducationIssues/ modern/ (password protected, retrieved January, 2011).

5. http://phet.colorado.edu/simulations/sims.php?sim=QWI

6. A. A. diSessa, "A history of conceptual change research: Threads and fault lines," in *Cambridge Handbook of the Learning Sciences*, K. Sawyer (Ed.) (Cambridge University Press, Cambridge, 2006), pp. 265-281.

7. M. Reiner, J. D. Slotta, M. T. H. Chi and L. B. Resnick, Naïve physics reasoning: A commitment to substance-based conceptions, *Cognition and Instruction* **18**, 1 (2000).

8. M. T. H. Chi, Common sense misconceptions of emergent processes: Why some misconceptions are robust, *Journal of the Learning Sciences* **14**, 161 (2005).

9. J. D. Slotta, In defense of Chi's Ontological Incompatibility Hypothesis, *Journal of the Learning Sciences* **20**, 151 (2011).

10. J. D. Slotta and M. T. H. Chi, The impact of ontology training on conceptual change: Helping students understand the challenging topics in science, *Cognition and Instruction* **24,** 261 (2006).

11. D. Hammer, Student resources for learning introductory physics, *Am. J. Phys.* **68**, S52 (2000).

12. D. Hammer, A. Elby, R. E. Scherr and E. F. Redish, "Resources, Framing and Transfer" in *Transfer of Learning*, edited by J. Mestre (Information Age Publishing, 2005) pp. 89-119.

13. A. Gupta, D. Hammer and E. F. Redish, The case for dynamic models of learners' ontologies in physics, *J. Learning Sciences* **19**, 285 (2010).

14. D. Hammer, A. Gupta and E. F. Redish, On Static and Dynamic Intuitive Ontologies, *J. Learning Sciences* **20**, 163 (2011).

10. W. H. Stapp, The Copenhagen Interpretation, *Am. J. Phys.* **40**, 1098 (1972).

CHAPTER 4

Refined Characterizations of Student Perspectives on Quantum Physics

I. Introduction

We have thus far seen how *realist* perspectives among modern physics students may translate into specific beliefs about quantum phenomena; e.g., particles are always localized in space, or that probabilistic descriptions of quantum measurements are the result of classical ignorance. We engage here in a more detailed exploration of student perspectives on quantum physics through a number of one-on-one student interviews. The resulting implications for modern physics instruction are particularly significant in that the learning goals for such courses typically include transitioning students away from classical epistemologies and ontologies, to ones that are more aligned with the beliefs of practicing physicists.

Still, it is not always clear exactly what expert physicists believe regarding the physical interpretation of quantum mechanics. [1] A recent survey [2] of quantum physics instructors at the University of Colorado and elsewhere (all of whom use quantum mechanics in their research) found that 30% of them interpreted the wave function as being physically real, while nearly half considered it to contain information only. The remaining respondents held some kind of mixed view on the physical interpretation of the wave function, or saw little distinction between the two choices. And only half of those who expressed a clear preference (matter-wave or information-wave) did so with confidence, being of the opinion that the other view was probably wrong. We find that students also develop attitudes and opinions regarding the reality of the wave function, as well as other interpretive themes from quantum mechanics.

The efforts described in Chapters 2 & 3 at characterizing student perspectives on quantum physics were limited to the application of three coarse labels (*Realist*, *Matter-Wave*, *Agnostic*) which are useful, but in light of the results of these studies, seem limited in terms of capturing the many nuances of student responses, and in particular understanding why students seem to exhibit contradictory perspectives between and within contexts. In this chapter, we therefore address the following:

1) How might our classification scheme be refined to better describe the nuances of student perspectives on interpretive themes in quantum physics?

2) For what reasons do students exhibit mixed perspectives within and across contexts?

From a total of 19 post-instruction interviews with students from four recent introductory modern physics courses at the University of Colorado we find that, though they may not employ the same formal language as expert physicists, students often invoke concepts and beliefs that parallel those invoked by expert physicists when arguing for their preferred interpretations of quantum mechanics. These parallels allow us to characterize student perspectives on quantum physics in terms of some of the same themes that distinguish these formal interpretations from each other. Of particular significance is the finding that students do indeed develop attitudes and opinions regarding a variety of interpretive themes in quantum mechanics, regardless of whether these themes had been explicitly addressed by their instructors. The mixed or seemingly contradictory student responses may be better understood in that: (A) some students prefer a mixed wave-particle ontology (a *pilot-wave* interpretation, wherein quanta are simultaneously *both* particle *and* wave); and (B) students are most likely to vacillate in their responses when what makes intuitive sense to them is not in agreement with what they perceive as a scientifically accepted response.

II. Interview participants and course characteristics

We sought to recruit five students from each of the four modern physics offerings at the University of Colorado from a single academic year (immediately following the studies described in Chapter 2) to participate in an hour-long post-instruction interview. A mass email was sent to all students enrolled in these courses, offering a nominal sum of fifteen dollars in exchange for their participation; students were not informed ahead of time about the nature of the interview questions, only that we would be discussing some ideas from modern physics. There was no real opportunity to select among students since volunteers were sometimes scarce, and so there was no attempt to make the cohort representative of all students from those courses. A total of 19 students were interviewed from these four courses [Table 4.I], either in the last week of the semester or after the course had ended. Interview participants from the courses for physics majors were all physics or engineering physics majors, plus one astronomy major; those from the courses for engineers were all engineering majors (but not engineering physics), plus one mathematics major. The average final course grade for all 19 students was 3.4 (out of 4.0, where overall course averages fall in the 2.0–3.0 range), indicating that participants were generally better than average students, as might be expected for a group of volunteers. Interviews followed the protocol as given in Appendix B. It should be emphasized that our characterizations of instructional approaches in Table 4.I and elsewhere in this chapter come from analyses of course materials and practices, and are not necessarily reflective of each instructor's personal perspective on quantum mechanics, but rather of how that instructor addressed interpretive themes in class.

TABLE 4.I Summary of four courses from which students were recruited for interviews, including a characterization of each instructor's stance on interpretive themes, as taught in that course; instructor labels correspond to those given in Figs. 3.1 & 3.2.

INSTRUCTOR	STUDENT POPULATION	INTERPRETIVE APPROACH	STUDENTS INTERVIEWED
MW-1	Engineering	*Matter-Wave*	3
C/A-2		*Copenhagen*	5
C/A-1	Physics	*Copenhagen/Agnostic*	6
C/A-3			5

The instructor labels given in Table 4.I correspond to those given in Figs. 3.1 & 3.2 (here, the labels MW-1 and C/A-1 correspond to Instructors/Courses B1 & C, respectively, as described in Chapter 3). The labels used for describing instructional approaches have been described earlier, but can be best illustrated by how each instructor addressed the double-slit experiment with single quanta. Instructor MW-1 (B1 in Chapter 3) was explicit in promoting a wave model of individual quanta as they propagate through both slits, interfere with themselves, and then become localized upon detection. Instructor C/A-2 told students that a *quantum mechanical wave of probability* passes through both slits, but that which-path questions change the circumstances of the experiment, making them ill-posed at best. While similar to C/A-2, Instructors C/A-1 (Instructor C in Chapter 3) and C/A-3 ultimately placed more emphasis on calculation (predicting features of the interference pattern) than matters of interpretation.

For the 19 students interviewed for the present studies, there were no discernible connections between a specific instructional approach and the preferred perspectives of the students interviewed from that course, likely due to the limited number of participants. Therefore, discussion in this chapter of specific instructional approaches will be limited to the brief characterizations given above, and a few specific statements below concerning the influence of an instructional approach on that student's individual responses.

III. Refined characterizations of student perspectives

As will be demonstrated below, we find it useful to consider student perspectives in quantum physics in terms of concepts associated with some of the more common (i.e., less exotic) formal interpretations of quantum mechanics. In doing so, we do not mean to imply that student perspectives are as coherent or sophisticated as any formal interpretation (although other research [2] suggests that expert perspectives on quantum physics may be similarly tentative). In fact, our results can best be understood within a theoretical framework that views student perspectives (including the process of *ontological attribution*) as cognitive frameworks that are dynamic emergent processes (as opposed to fixed or static cognitive structures), that are contextually sensitive, and that sometimes simultaneously blend ontological attributions that belong to classically distinct categories. [See Refs. 3-5, as well as Chapter 1, Section II.] Nor do we assume that any one label is necessarily sufficient for describing the nuanced and sometimes inconsistent perspectives exhibited by any particular student; or even that the development of student perspectives on quantum physics follows along the lines of historical developments.

We do, however, find that some formal interpretations of quantum mechanics can be distinguished from each other in terms of a few key themes, and that students do have beliefs or ideas concerning these themes of interpretation, regardless of whether these themes had been explicitly addressed by their instructors. In other words, we have observed that many introductory modern physics students, when formulating a stance on these interpretive themes, employ some of the same epistemological tools used by expert physicists, and will sometimes invoke similar experimental results and intuitive notions of particles and waves as motivation for their preferred interpretations of quantum phenomena. An analysis of all 19 interview transcripts revealed student beliefs and attitudes (of varying degrees of sophistication) concerning the following three interpretive questions:

1) Is the position of a particle objectively real, or indeterminate and observation dependent? [Existence or non-existence of certain hidden variables.]

2) Is the wave function a mathematical tool that encodes probabilities [information-wave], or is it physically real [matter-wave]?

3) Does the *collapse of the wave function* (or *reduction of the state*) represent a physical process, or simply a change in knowledge of the observer?

III.A. Discussion of formal interpretations

We present here a brief summary of some key features of several formal interpretations of quantum mechanics, in terms of the three interpretive themes given in Section II. [Table 4.II] Many aspects of these formal interpretations have been previously discussed in greater detail, [Chapter 1] and it should be emphasized that it would be impossible for these short summaries to be comprehensive, but are offered as working definitions for the sake of clarity when associating these labels with the expressed beliefs of individual students.

***Realist/Statistical*:** From either a *Realist* or S*tatistical* perspective, the physical properties of a system are objectively real and independent of experimental observation (observations reveal reality, not create it). The state vector encodes probabilities for the outcomes of measurements performed on an ensemble of similarly prepared systems, but cannot provide a complete description of individual systems. The wave function is not physically real; the collapse of the wave function represents a change in the observer's knowledge of the system, and not a physical change brought about by the act of measurement.

***Copenhagen*:** The probabilistic nature of quantum measurements is a reflection of the inherently probabilistic behavior of quantum entities; in general, the properties of a system are indeterminate until measured. The wave function is not a literal representation of a physical system, and the *collapse of the wave function* corresponds to a change in knowledge of the observer, though it does represent a physical transition from an indeterminate state to one where certain properties of the state become well defined.

***Matter-Wave*:** Similar to the *Copenhagen Interpretation* with respect to indeterminacy and the non-existence of hidden variables, but also ascribes physical reality to the wave function. Though not described by the Schrödinger equation, the *collapse of the wave function* represents a physical process induced by measurement.

Pilot-Wave: From this perspective, quanta are simultaneously both particle and wave: localized particles follow trajectories determined by a physically real quantum wave. In the double-slit experiment, an electron is all at once both a particle that goes through only one slit, and a wave that passes through both slits and interferes with itself. In this context[1], the position of a particle is objectively real and predetermined based on unknowable initial conditions, so that the reduction of the state represents a change in knowledge of the observer.

[1] Nonlocal features come into play when other quantum effects (e.g., entanglement) are to be accounted for, in which case the *collapse of the wave function* must be seen as a (non-local) physical process.

TABLE 4.II. Summary of our characterizations of four formal interpretations of quantum mechanics, in terms of three interpretive themes (described in Section II). The *Agnostic* perspective is not a formal interpretation in itself, but is included for completeness.

INTERPRETATION	HIDDEN VARIABLES?	INFO- OR MATTER-WAVE?	COLLAPSING WAVE FUNCTION?
Realist/Statistical	YES/AGNOSTIC	INFO	KNOWLEDGE
Copenhagen	NO	INFO	PHYSICAL
Matter-Wave	NO	MATTER	PHYSICAL
Pilot-Wave	YES	MATTER	KNOWLEDGE
Agnostic	AGNOSTIC	AGNOSTIC	AGNOSTIC

Agnostic: Though not a formal interpretation in itself, we distinguish between this stance and the positivistic aspects of the *Copenhagen Interpretation* (declining to speculate on the unobservable). The *Agnostic* perspective accounts for multiple interpretations of quantum mechanics and their ontological implications, but takes no definite stance on which might correspond to the best description of reality. The utility of quantum mechanics is generally favored over questions of interpretation.

III.B. Students express beliefs that parallel those of expert proponents

We have hypothesized that the perspectives of many modern physics students on quantum phenomena are significantly influenced by the commonplace (and intuitive) notion of particles as localized in space. In classical physics, as in colloquial usage, the word particle generally connotes some small object, so it should not be surprising that students who have learned about particles primarily within the context of classical physics should persist in thinking of them as microscopic analogs to macroscopic objects when learning about quantum physics. This would be an example of *classical attribute inheritance*, in the sense that students may explicitly attribute to quantum particles *all* of their classical analogs, including a localized position (student codes are as given below in Table 4.III):

> "I guess an electron has to [always be at] a definite point. It is a particle, we've found it has mass and it has these intrinsic qualities, like the charge it has, so it will have a definite position, but due to uncertainty it will be a position that is unknown." [STUDENT QR2]

This statement reveals not only one student's belief in localized massive particles, it also suggests a stance on the uncertainty associated with a particle's position: its objectively real value will be unknown until revealed by measurement.

This student (and others with similar attitudes) reported interpreting the probability density for an atomic electron as strictly a mathematical tool used only for describing the probable locations for where that electron might be found once measured; probabilistic descriptions of such measurements were therefore seen as a reflection of *classical ignorance* concerning the true state of that particle just prior to measurement. We thus see how an intuitive notion of particles as localized objects can influence what physical meaning students ascribe to both the wave function and the probabilistic nature of quantum mechanics.

In a similar vein, another student explicitly objected to the idea that wave-packets could represent single particles. Here, this student is discussing the Quantum Wave Interference [6] (QWI) simulation's depiction of a wave-packet's propagation through both of two slits on its way to detection:

> "One electron can't go through both slits at the same time because electrons have mass. Wouldn't it violate conservation of mass and charge if [the electron] were split into two like it shows in the [QWI] simulation?" [STUDENT R1]

Such objections are reminiscent of those made by L. Ballentine (a major proponent of the *Statistical Interpretation* of quantum mechanics [7, 8]) when discussing a thought experiment in which an incident wave packet is divided by a semi-reflecting barrier into two distinct transmitted and reflected wave packets. The reflected and transmitted waves are then directed toward a pair of detectors connected to a coincidence counter. Ballentine argues:

> "Suppose that the wave packet *is* the particle. Then since each packet is divided in half, [...] the two detectors will always be simultaneously triggered by the two portions of the divided wave packet." [Ref. 8, p. 101, emphasis in original]

In this thought experiment (and in practice [9]), single quanta trigger either one detector or the other (and not both simultaneously); Ballentine therefore concludes that, while the wave function may have nonzero amplitude in two spatially separated regions, it cannot be interpreted as describing individual particles, since individual particles are never found in two places at once. In making this argument, Ballentine has implicitly assumed that the *collapse of the wave function* (or *reduction of the state*) represents a change in knowledge of the observer, and not an actual physical process induced by measurement.

In his own book on quantum mechanics, Dirac [10] considers the same type of thought experiment as Ballentine, but provides a radically different explanation:

> "The result of [the detection] must be either the whole photon or nothing at all. Thus the photon must change suddenly from being partly in one beam and partly in the other to being entirely in one of the beams." [Ref. 10, p. 9]

As counterintuitive as this interpretation may be, we find that a number of modern physics students report having accepted such ideas, and have incorporated them into their descriptions of quanta:

> "[T]he electron, until it's measured, until you try to figure out where it is, the electron is playing out all the possibilities of where it could go. Once you measure where it is, that collapses its wave function [and it] loses its properties as a wave and becomes particle in nature." [STUDENT Q3]

Students within this category all explicitly exhibited this kind of flexibility in their ontological descriptions of the behavior of electrons. Other students, like Ballentine, find these types of explanations unsatisfying:

> "[A] single electron is detected at the far screen, and I feel like that really can't be explained for the wave-packet, by one specific detection in a small place like that, if you say [the wave-packet] is the electron. That's really the only discrepancy I have with that: What happens when it hits the screen?" [STUDENT QR2]

Indeed, the question of what happens when individual quanta are detected in a double-slit experiment has played a significant role for some physicists in motivating their perspectives on quantum phenomena, as with Ballentine:

> "[I]t is possible to detect the arrival of individual electrons, and to see the diffraction pattern emerge as a statistical pattern made up of many small spots. *Evidently, quantum particles are indeed particles*, but particles whose behavior is very different from what classical physics would have led us to expect." [Ref. 8, p. 4, emphasis added]

This statement exemplifies a degree of ontological inflexibility in expert thinking: Ballentine is assuming that the detection of electrons as localized particles implies they exist as localized particles at all times. J. S. Bell has also invoked the double-slit experiment when discussing interpretation, but in this particular case as motivation for a pilot-wave interpretation, as proposed by Bohm and others [11]:

> "Is it not clear from the smallness of the scintillation on the screen that we have to do with a particle? And is it not clear, from the diffraction and interference patterns, that the motion of the particle is directed by a wave?" [Ref. 12, p. 191]

This student's discussion of the double-slit experiment echoes sentiments expressed by both Ballentine and Bell – by employing similar argumentation, he reaches similar conclusions:

> "For me, saying that the [wave] represents the electron isn't accurate because an electron, after it's measured on that screen, is a point-particle, you see a distinct interference pattern after shooting many electrons, but you still see one electron hit the screen individually. [...] I do agree that the electron acts as a wave because that's obviously what causes the pattern; if it didn't interfere with itself, or create a wavelike function, then you wouldn't see the patterns on the screen also." [STUDENT R3]

Historically, and in our classrooms today, different physicists have offered different interpretations of quantum diffraction experiments. For Ballentine, diffraction patterns form as a consequence of the quantized momentum transfer between localized particles and the diffracting medium. [Ref. 8, p. 136] These patterns are more commonly explained in terms of wave interference, but for some, the wave is guiding the trajectory of a localized particle, while others would claim that each particle interferes with itself as a delocalized wave until becoming localized upon detection. At the same time, a number of *both* expert and student physicists find it unscientific to speculate on that which cannot be experimentally observed:

> "I understand why people would think [the electron] has to exist between here and where it impacts, and it does, but the necessity of [thinking of it] between here and where it impacts as an actual concept like a particle or a wave, I don't see much of the point. We're not going to observe what it is between here and there, so it doesn't seem like a statement for science to make. It seems right now to be entirely unobservable." [STUDENT C2]

The refusal to speculate on unobservable processes is a key feature of the orthodox *Copenhagen Interpretation* of quantum mechanics, which seems to be favored by a majority of practicing physicists, if only for the fact that it allows them to apply the mathematical tools of the theory without having to worry about what's "really" going on (as embodied in the popular phrase: *Shut Up and Calculate!* [12], and the sentiments expressed by Instructor C's [C/A-1, in this chapter] in-class comments from Chapter 3).

We also find it necessary to distinguish between the agnostic or positivistic aspects of an instructional approach, and the agnosticism of those who are aware of multiple interpretations, but are unsure as to which offers the best description of reality:

> "For now, for me, the electron is the wave function. But whether the electron is distributed among the wave function, and when you do an experiment, it sucks into one point, or whether it is indeed one particle at a point, statistically the average, I don't know." [STUDENT QA1]

III.C. Categorization and summary of student responses

We summarize here a categorization of individual students in terms of the interpretive themes discussed above, grouped by overall perspective, as discussed in Section III.A; [Table 4.III] in this section, the label *Quantum* (Q) is used as shorthand for a *Matter-Wave* perspective, for brevity and for consistency with prior published research. [13] A discussion of key findings and commonalities among students within individual categories follows.

TABLE 4.III. Summary of individual student interview responses with respect to three interpretive themes (as described in Section III), grouped by overall perspective. The label *Quantum* (Q) is used as shorthand for a *Matter-Wave* perspective.

STUDENT PERSPECTIVE	CODE	HIDDEN VARIABLES?	INFO OR MATTER WAVE?	COLLAPSING WAVE FUNCTION?
Realist	**R1**	YES	INFO	KNOWLEDGE
	R2	YES	INFO	KNOWLEDGE
	R3	YES	INFO	KNOWLEDGE
Split Quantum/ Realist	**QR1**	NO/YES	MATTER/INFO	PHYSICAL
	QR2	NO/YES	MATTER/INFO	KNOWLEDGE
	QR3	NO/YES	MATTER/INFO	KNOWLEDGE
	QR4	NO/YES	MATTER/INFO	AGNOSTIC
Pilot-Wave	**P1**	YES	MATTER	KNOWLEDGE
	P2	YES/ AGNOSTIC	MATTER/ AGNOSTIC	KNOWLEDGE/ AGNOSTIC
	P3	YES	MATTER	KNOWLEDGE
Quantum (Matter-Wave)	**Q1**	NO	MATTER	KNOWLEDGE
	Q2	NO	MATTER	PHYSICAL/AGNOSTIC
	Q3	NO	MATTER	PHYSICAL
	Q4	NO	MATTER	PHYSICAL
	Q5	NO	MATTER	PHYSICAL/AGNOSTIC
Quantum/ Agnostic	**QA1**	NO/ AGNOSTIC	MATTER/ AGNOSTIC	PHYSICAL/ AGNOSTIC
Copenhagen	**C1**	NO	INFO	PHYSICAL
	C2	NO	INFO/AGNOSTIC	PHYSICAL/AGNOSTIC
	C3	NO/ AGNOSTIC	INFO	AGNOSTIC

We first note that many student responses agreed well with our characterizations of the formal interpretations, while other students provided one or more responses that were not entirely consistent with those characterizations; in the few such cases, a category was assigned based on what would be most consistent with the overall responses from that student. A second, independent physics education researcher coded a subset of five transcribed interviews (all students who were not quoted in this chapter), both by interpretive theme and by overall interpretation, with an initial inter-rater reliability of 93% on individual stances on the interpretive themes, and 100% on overall perspective; following discussion, there was 100% agreement between both coders.

All of the students in the *Split* category were explicit in distinguishing between what made intuitive sense to them (*Realist*) and what they perceived to be a correct response (*Quantum*). Other students offered opinions on specific themes when asked to take a stance, but chose to ultimately remain agnostic for lack of sufficient information (as indicated by the *XX/Agnostic* entries in the interpretive themes columns of Table 4.III). This agnostic characterization of individual responses differs from the overall stance of Student QA1, who said he preferred a *Quantum* interpretation, but expressed a sophisticated overall agnosticism on the legitimacy of a contrasting *Statistical* interpretation.

Realist Category: All three of these students considered probability waves to be mathematical tools used only to describe the probable outcomes of measurements. These students all objected to the idea that a wave packet could represent a single particle, and said they always consider an electron to be a localized object traveling somewhere inside the probability wave describing the system. These students were not classified as holding a *Statistical* perspective because they were explicit in their stance on electrons as localized particles (as opposed to agnostic), and did not have sufficient content knowledge (e.g., consequences of Bell's Theorem [14]) to appreciate why an agnostic stance on hidden variables might be necessary. All three of these students specifically objected to the notion of *wave function collapse*, calling it too counterintuitive or too unphysical to be a correct description of reality. These students all claimed to be aware of at least one alternative to their *Realist* interpretations, but said they hadn't yet been convinced by instructor arguments that their preferred perspective was incorrect.

Split Quantum/Realist Category: While the *Realist Category* students all expressed a measure of confidence in their perspectives on quantum physics (even when those perspectives differed from what they had heard in class), the four students in this *Split Quantum-Realist* category were, by the end of the interview, explicit in differentiating between what made intuitive sense to them, and what they considered to be a correct response. In example, Student QR1 first agreed that an electron in an atom always exists at a definite point, and continued with this line of thinking, both when first describing the double-slit experiment, and again as he began reading the *Realist* statement of Student One from the double-slit essay question:

STUDENT QR1: I would agree with what Student One is saying, that the electron is traveling somewhere inside that probability density blob, and it is a tiny particle. The problem here that I see is that the electron went through one slit or the other. [PAUSE] So, now I'm disagreeing with myself. OK, my intuition is fighting me right now. I said earlier that there should be one point in here that is the electron, and it goes through here and hits the screen, but I also know that I've been told that the electron goes through both slits and that's what gives you the interference pattern. Interesting. [LONG PAUSE] OK, somehow I feel like the answer is going to be that this probability density, it is the electron, and that can go through both slits, and then when it's observed with this screen, the probability density wave collapses, and then only exists at one point. But at the same time I feel that there should be a single particle, and that somehow a single, finite particle exists in this wave, and will either travel through one slit or the other. Why would a single particle be affected by a slit? That I don't have an answer to, other than that it's the wave that's actually being propagated, the wave is the electron.

INTERVIEWER: OK. It seems like you're talking about two different ideas. One is that the electron is a point somewhere inside this wave, and the other is saying the electron is the wave. Do you feel those two ideas conflict in any way?

QR1: Yeah, they do, because one says there is a finite particle at all times, and the other says that there's not, there is just this probability density, and I think the answer will turn out to be that the electron is the probability density, and that's contrary to what I said earlier. But I don't see how it could be the other way, with a finite particle. I don't see how you could get an interference pattern here with the electron being a finite particle the whole time.

INT: OK. What about [the *Matter-Wave*] statement?

QR1: [BEGINS READING] So, that goes off of what I was just saying. [READS] So, I agree with everything up to here, the electron acts as a wave and will go through both slits and interfere with itself, I believe that's true. And that's why an interference pattern develops after shooting many electrons; I guess I agree with that too, because when the blob gets to the screen, it can't just still have a probability density that would look like an interference pattern by itself. It's going to have one finite location. But after multiple electrons, multiple blobs have passed through, they will collectively form an interference pattern. So I would agree with Student Two.

INT: So you're agreeing with Student Two. And did you say that you disagree with Student One, or do you just have reservations about what they're saying?

> **QR1:** Intuitively, I kind of agree with Student One, but I think I have reservations. I don't think, Student One, that they're right.
>
> **INT:** But it appeals to you, what they're saying?
>
> **QR1:** Based upon lecture, and upon those who have greater knowledge of physics than me, I would say that this [second] statement agrees more with that than the initial situation.
>
> **INT:** So you say Student One's statement disagrees more with what you've heard in class?
>
> **QR1:** Yes. But not more with what I envision. This [first] one kind of depicts more of my rational depiction, all that I can wrap myself around and understand, and the second one is more of what I've been told, but don't completely understand. I've been told it's right, so…

This excerpt serves two purposes. It first explicitly demonstrates how students may change as needed between ontological attributions in their descriptions of electrons in order to explain observed phenomena (electrons as particles in order to explain localized detections, electrons as waves in order to explain interference). It also underscores the need to distinguish between the *personal* and the *public* [15] perspectives of students on quantum physics: these students differed from their *Realist* category counterparts in that they explicitly differentiated between what made intuitive sense to them, and responses they perceived as being correct. This finding parallels studies by McCaskey et al., [16, 17] where students were asked to respond twice to the Force Concept Inventory, [18] first as they personally believed, and then as they felt a scientist would respond. These authors found that most every student *split* on at least one survey item, indicating a difference between their personal beliefs and their perceptions of scientists' beliefs. Following a series of validation interviews, these authors reported that students most often explained their personal responses in terms of what made intuitive sense to them, and that split responses reflected how students had learned a correct response from instruction, without having reconciled that knowledge with their own intuition. Similar studies probing the attitudes and beliefs of introductory classical physics students have demonstrated similar results. [19]

Regarding the public perspectives of modern physics students, we would also point out that students will not necessarily identify an authoritative stance based on specific knowledge of what expert physicists believe. Not only may their perceptions of what scientists believe be inaccurate, students may also employ undesirable epistemological strategies learned from their experiences in the classroom:

> "This [*Quantum* statement of Student Two] is more of a complex definition, I think. […] Probably initially I would be confused by this statement if I hadn't taken this course, but I might be like the public and think the most complicated answer, that must be the right one. Because a lot of times—it's even happened with the [concept] questions in class—where I think: *That's got to be the answer*. But then I'll be like: No, that would be too easy, it's got to be something else. Sometimes that [strategy] can prove correct or incorrect." [STUDENT QR4]

Pilot-Wave Category: The responses from these three students indicated an ontology that blends attributes from both classical particles and waves. These students indicated a belief that wave-particle duality implies that quanta must be thought of as simultaneously *both* particle *and* wave. The following student explained the fringe pattern in the double-slit experiment in terms of constructive and destructive interference, and acknowledged that the experiment had been used in class to demonstrate the wave characteristics of quanta, but had his own ideas about the source of interference for localized particles:

> "It seems like the probable paths for the electron to follow interact with themselves, but the electron itself follows just one of those paths. It's like the electron rides on a track, like a train rides on a rail, but those rails or tracks go through both slits, and the possible paths for the electrons to follow interfere with themselves, create the interference pattern, but the physical electron just rides on the tracks, it picks one. Or maybe switches paths, if two of them cross. I don't know, it seems that the electron has to be on one of those tracks, but the tracks themselves cause the interference pattern." [STUDENT P3]

Of particular interest is the way in which this same student demonstrated how his realist (albeit nonlocal) perspective can be employed as an epistemological tool:

> "As [the electron is] traveling it's going to be somewhere in this [probability density] as it moves along until it's actually detected. And if it was here [INDICATES POINT NEAR DETECTING SCREEN] then it must have been here at one point in time [INDICATES SECOND POINT NEAR THE FIRST] and if it was here, then it had to be here at one point in time, all the way back to here [TRACES LINE BACK TO NEAR BOTH SLITS] in which case there's only two places it could be. So yes, I think it went through one slit or the other." [STUDENT P3]

As another example of the ontological flexibility exhibited in novice thinking, one of these students explained that, while it is necessary to think of an electron in the double-slit experiment as both wave and particle, it was unnecessary to employ a wave description for atomic electrons since, in his mind, there were no wave effects to be accounted for:

> "When I was thinking about [an electron] in an atom, there's really no reason that you have to think about it as a wave, in the fact that it's not really interacting with anything. In [the double-slit] experiment, yes I like to think of it as also a wave, because this is kind of the key experiment of quantum mechanics, to describe this [wave] phenomenon, and so for that reason it is more effective to think of it as both." [STUDENT P1]

With this excerpt, we call attention to the fact that sometimes students employ different models (ontological attribution assignments) in different contexts, without necessarily looking for or requiring internal consistency among them.

Quantum (**Matter-Wave**) **Category:** These five students were consistent in providing responses that indicated a *matter-wave* ontology:

> "I don't think of [the electron] as orbiting the nucleus because it doesn't, it just exists in that region of space. It exists in a volume element that defines the probability of finding the electron in that space […] and that's really what the electron is: a smeared out volume of charge." [STUDENT Q2]

All of these students described unobserved quanta strictly in terms of waves, and discussed the *collapse of the wave function* as a physical process where wave-like quanta suddenly exhibit particle-like properties. According to these students, their personal perspectives on quantum mechanics were in complete agreement with their perceptions of expert beliefs.

Quantum/Agnostic Category: We find it necessary to distinguish this one student from those in the strictly *Quantum* category because, while the *Quantum* category students had all expressed confidence in their *matter-wave* interpretations, this student expressed a degree of sophisticated uncertainty in his own views:

> "The way I think of an electron, I cannot ascribe to it any definite position, definite but unknown position. I mean, it may be that way, but I think that somehow the electron is represented by the wave function, which is just a probability, and if we want to localize it then we lose some of the information. So whether this is true or not is something of a philosophical question. I wish I knew, or understood it, but I don't. For now, for me, the electron is the wave function, so whether the electron is distributed among the wave function, and when you do an experiment, it sucks into one point, or whether it is indeed one particle at a point, statistically the average, I don't know." [STUDENT QA1]

Copenhagen Category: These three students were similar to the *Quantum Category* students in terms of the nonexistence of hidden variables, but saw probability waves as containing information only, rather than representing the actual physical state of a particle. As with student C2 (quoted previously in Section III.B) each of these students stated explicitly that it is unscientific to discuss that which can't be measured or observed. These three students said they considered electrons to be neither wave nor particle; that such concepts were in fact different models for describing the behavior of quanta under different circumstances. These students expressed what we consider to be a moderately sophisticated perspective on both the necessity and the desirability of switching between ontological categorizations.

It should also be noted in our studies that formal instruction is not the only source of information or influence for students regarding quantum physics, as with this student, who explained how his own personal solipsistic philosophy influenced his beliefs about quantum mechanics, and vice-versa:

> **STUDENT C3:** This is more of a philosophical point for me, but if we can't know something, there's no difference between it not existing and us not knowing it. So, for our purposes, it's more useful to say, if we can't know it, where the electron is, then it doesn't have a definite position. […] I believe, so long as we don't measure it, then an electron doesn't have a definite position.
>
> **INTERVIEWER:** What happens when we measure it?
>
> **C3:** Well, we find a position then**…** Then it does.
>
> **INT:** The position we find, is that where the particle was the moment before we measured it?
>
> **C3:** No. We can't know that. So, when we make a measurement, there's the particle. When we look away, the particle goes away. And I sort of felt this way before having learned about quantum mechanics. And it just solidified in my mind that there's no difference between me not knowing it, and it not existing.

In the class-wide online surveys, a majority of students from all of the four courses discussed here reported having previously heard about quantum mechanics in popular venues (e.g., books by Hawking [20] or Greene [21]) before enrolling in the course.

IV. Summary and Discussion

Our more detailed characterization of the perspectives of modern physics students improves upon our previous efforts by addressing the contextual sensitivity of those perspectives, through an exploration of their expressed beliefs about quantum physics across three key interpretive themes. We find that, as a form of sense making, students develop a variety of ideas and opinions regarding the physical interpretation of quantum mechanics, in spite of how their instructors explicitly addressed matters of interpretation in class.

As with past studies, we find that a significant number of students from our interviews (10 of 19) expressed a preference for *realist* interpretations of quantum phenomena. However, the nature of these students' *realist* perspectives were not necessarily of the character we had anticipated from the results of earlier studies. Only three of these students consistently preferred *realist* interpretations of quantum phenomena, while simultaneously expressing confidence in the correctness of their perspectives; whereas four others differentiated between what made intuitive sense to them, and what they perceived to be correct responses. Their particular kind of switching between ontological framings may be best understood in terms of their competing *personal* and *public* perspectives [15] on quantum physics – when responding during interviews, these students frequently vacillated between what they personally believed and the answer they felt an expert physicist would give, without always articulating a difference between the two without prompting. This finding has implications for future research into the ontologies of quantum physics students, who may not always respond to such questions as they actually believe, but rather provide the responses that best mimic their instructors. Such issues are of particular significance with regard to matters of interpretation in quantum mechanics, where the beliefs of practicing physicists are at such variance with each other, which may confuse student perceptions.

The *Realist* beliefs of three other students were of a decidedly nonlocal character: localized quantum entities follow trajectories determined by the interaction of nonlocal quantum waves with the environment. None of these three students claimed to be aware of any formal *pilot-wave* interpretation, and their beliefs in quanta as simultaneously wave and particle were at odds with how wave-particle duality was addressed in class by their instructors (i.e., quanta are sometimes described by waves, and sometimes as particles, but never both simultaneously). The remaining nine of 19 students expressed fairly consistent views that could be seen as in agreement with the (implicit) learning goals of their instructors, whether Quantum or Copenhagen. In other words, these students seemed to have successfully incorporated probabilistic and nonlocal views of quanta and quantum measurements into their personal perspectives, and/or agreed that scientists should restrict discussions to that which can be measured and verified. While these findings are somewhat at odds with previous research into quantum ontologies, which have concluded that student perspectives are rarely in alignment with expert or productive transitional models, we emphasize that the relatively few students who participated in our interviews were generally better-than-average students, and were not representative of an entire class. Ultimately, we believe the value of these findings lies in the demonstration and documentation of a variety of student beliefs regarding quantum phenomena, and not a determination of the relative prevalence of any specific beliefs.

Of equal importance is the demonstration of students employing multiple parallel ontologies, or dynamic ontologies that are flexible and adaptive, each according to their immediate cognitive needs. In one specific case, Student QR1 initially described an electron as a particle localized in space, but wavered in his commitment to this description when he encountered the notion that each electron must have travelled through only one slit on its way to the detecting screen. After a moment of introspection, he concluded that a wave description was necessary in order to explain the observed interference pattern, for he had no explanation as to why a localized particle would be affected by the presence of a slit. He then explicitly stated that the "correct" way of looking at the situation is to equate the electron with the wave itself, which necessitated a corresponding belief in a physically collapsing wave function. Student QR1 was aware of the logical inconsistency in his two competing perspectives, but was able to articulate a need for maintaining both, one in correspondence with his intuition, and one in congruence with what he perceived as an authoritative stance, and which also led to an interpretation of the double-slit experiment that was consistent with observations. We can easily imagine this student's reasoning during the interview briefly recapitulated some of the thought processes he engaged in when first encountering this topic, as he initially seemed unaware of any need to think of electrons as anything other than localized particles, but immediately reconsidered his stance when confronted with an observation that he could only explain in terms of wave interference.

We have also seen how *classical attribute inheritance* will guide the thinking of both experts and novices, through the explicit statements of Ballentine, along with those from Students R1 and QR2: localized detections imply a continuously localized existence, particles are *by definition* localized in space; laws of mass and charge conservation preclude the possibility for particles to be spatially delocalized. These types of epistemological and ontological *resources* are not necessarily wrong in and of themselves, and may be of productive use in classical descriptions of matter, but have enormous implications for what kind of physical meaning students attach to the otherwise mathematically algorithmic process of deriving wave functions and calculating expectation values; and their activation in the context of quantum phenomena may lead students to interpretations that seem paradoxical or are inconsistent with observations.

The demonstration of student flexibility in assigning ontological attributes, switching back and forth (and sometimes blending) them as needed, does more than just explain the contextual sensitivity of student responses; it provides strong evidence of the dynamical nature of the ontologies employed by students when reasoning about quantum phenomena. The students falling into the strictly *Realist* category were the ones showing the least flexibility in their use of ontologies (and even these students were aware of alternative explanations, but hadn't yet bought into them). All of the other students demonstrated varying degrees of flexibility in their use of parallel ontologies: some distinguished between intuitive and normative ontologies; some perceived switches between ontological attributes as reflective of physical transitions; others blended attributes from classically distinct categories, or assigned them separately, all according to their cognitive needs of the moment.

These results are most consistent with the dynamic view of novice and expert ontologies discussed here and in Chapter 1, and are difficult to reconcile with the static, parallel ontologies promoted by Slotta and Chi. First, quantum mechanics describes the *behavior* of light and matter in terms of classically distinct ontological characteristics, and so a rigid (robust) assignment of ontological attributes is not possible for a complete description of electrons and photons. Nor do scientists agree on a normative view of the ontological nature of quanta, and instructors understandably vary in their choices of how to broach this topic in their introductory courses, sometimes fearful of opening a *Pandora's Box* of student questions with no easy answers.

Second, we observe that students frequently modify their patterns of ontological attribution assignment piecewise, both within and across multiple contexts. This type of gradual transition in student thinking cannot be plausibly explained in terms of rigid, parallel ontologies that are developed over the course of instruction, and which then replace the original, intuitive ontologies, unless one were to believe that students develop a whole multitude of parallel ontologies, each specific to the variety of situations they've encountered. In the end, Slotta has conceded that the disagreement between these two opposing views may ultimately be a matter of the degree of ontological flexibility and blending exhibited in both novice and expert thinking, [22] and both sides have made strong arguments in favor of their views on learning and cognition in the context of classical physics; their disparities become all the more apparent, however, in the context of quantum mechanics.

We also find it significant that most every student expressed distaste for deterministic ideas in the context of quantum phenomena, although it had been anticipated that *Realist Category* students might favor such notions. Not only did most every student say they were unfamiliar with the word *determinism* within the context of physics, practically every student believed either that any description of the behavior of quantum particles should be inherently probabilistic, or that the Heisenberg uncertainty principle places a fundamental limit on human knowledge of quantum systems, or a combination of both stances. A superficial analysis showed that the *Realist* and the *Split Quantum/Realist* students were more likely than other students to invoke the uncertainty principle when discussing notions of determinism; the remaining students were more likely to state that the behavior of quantum particles (or the nature of the universe) is inherently probabilistic. These responses indicate a need for a more detailed exploration of the uncertainty principle as an epistemological tool for quantum physics students.

These interviews have demonstrated how matters of interpretation are of both personal and academic interest to students, and modern physics instructors should recognize the potential impact on student thinking when choosing to de-emphasize interpretation in an introductory course. Not only do students develop their own ideas regarding the physical meaning behind quantum mechanics, they also develop attitudes (right or wrong) about the positivistic or agnostic stances of their instructors:

> "It seems that there's this dogma among physicists, that you can't ask that question: *What is it doing between point A and point B? You can't ask that!* And I think that the only way we'll be able to make profound progress is by asking those questions. It doesn't make sense that somebody would say, don't ask that, or you can't ask that. I think somehow they're shutting down free seeking of knowledge. But I don't know enough about quantum mechanics. Maybe when I get more understanding of quantum mechanics, I too will be saying: *You can't ask that!* But as a naïve student it sounds like a bad attitude to have about physics." [STUDENT P3]

Although many instructors may argue that introductory students do not have the requisite sophistication to appreciate matters of interpretation in quantum mechanics, we note that several authors have already developed discussions of EPR correlations and Bell inequalities that are appropriate for the introductory level. [23, 24] Questions of interpretation may also be addressed in terms of *scientific modeling*, an aspect of epistemological sophistication that is often emphasized in physics education research as a goal of instruction, as well as in terms of *nature of science* issues. [25] In the end, we argue that modern physics instructors should concern themselves with matters of interpretation, if only because their students concern themselves with these matters, and as educators we should be concerned with what our students believe about physics and the nature of practicing physics. Modern physics instructors who aim to transition students away from classical epistemologies and ontologies may employ our framework for understanding and interpreting the myriad combinations of student ideas concerning the nature of quantum mechanics and its description of the natural world. Such insight may allow us to target instructional interventions that will positively influence student perspectives, and strengthen their abilities to make interpretations of physical phenomena, and to understand the limitations and bounds of these interpretations.

References [Chapter 4]

1. N. G. van Kampen, The Scandal of Quantum Mechanics, Am. J. Phys **76**, 989 (2008).

2. M. Dubson, S. Goldhaber, S. Pollock and K. Perkins, Faculty Disagreement about the Teaching of Quantum Mechanics, *PERC Proceedings 2009* (AIP, Melville, NY, 2009).

3. G. J. Posner, K. A. Strike, P. W. Hewson and W. A. Gertzog, Accommodation of a scientific conception: Toward a theory of conceptual change, *Sci. Educ.* **66**, 211 (1982).

4. A. Gupta, D. Hammer and E. F. Redish, The case for dynamic models of learners' ontologies in physics, *J. Learning Sciences* **19**, 285 (2010).

5. D. Hammer, A. Elby, R. E. Scherr and E. F. Redish, "Resources, Framing and Transfer" in *Transfer of Learning*, edited by J. Mestre (Information Age Publishing, 2005) pp. 89-119.

6. http://phet.colorado.edu/simulations/sims.php?sim=QWI

7. L. E. Ballentine, The Statistical Interpretation of Quantum Mechanics, *Rev. Mod. Phys.* **42**, 358 (1970).

8. L. E. Ballentine, Quantum Mechanics: A Modern Development (World Scientific Publishing, Singapore, 1998).

9. P. Grangier, G. Roger, and A. Aspect, Experimental Evidence for a Photon Anticorrelation Effect on a Beam Splitter: A New Light on Single-Photon Interferences, *Europhysics Letters* **1**, 173 (1986).

10. P. A. M. Dirac, *The Principles of Quantum Mechanics, 3rd ed.*, (Clarendon Press, Oxford, 1947).

11. D. Bohm and B. J. Hiley, *The Undivided Universe* (Routledge, New York, NY, 1995).

12. N. D. Mermin, What's Wrong with this Pillow? *Phys. Today* **42**, 9 (1989).

13. C. Baily and N. D. Finkelstein, Refined characterization of student perspectives on quantum physics, *Phys. Rev. ST: Physics Education Research* **5**, 020113 (2010).

14. J. S. Bell, Speakable and Unspeakable in Quantum Mechanics (Cambridge University Press, Cambridge, 1987).

15. L. Lising and A. Elby, The impact of epistemology on learning: A case study from introductory physics, *Am. J. Phys.* **73**, 372 (2005).

16. T. L. McCaskey, M. H. Dancy and A. Elby, Effects on assessment caused by splits between belief and understanding, *PERC Proceedings 2003* **720**, 37 (AIP, Melville, NY, 2004).

17. T. L. McCaskey and A. Elby, Probing Students' Epistemologies Using Split Tasks, *PERC Proceedings 2004* **790**, 57 (AIP, Melville, NY, 2005).

18. D. Hestenes, M. Wills, and G. Swackhamer, *Physics Teacher* **30**, 141 (1992).

19. K. E. Gray, W. K. Adams, C. E. Wieman and K. K. Perkins, Students know what physicists believe, but they don't agree: A study using the CLASS survey, *Phys. Rev. ST: Physics Education Research* **4**, 020106 (2008).

20. S. W. Hawking, *A Brief History of Time* (Bantam, New York, NY, 1988).

21. B. R. Greene, *The Elegant Universe* (Norton, New York, NY, 2003).

22. J. D. Slotta, In defense of Chi's Ontological Incompatibility Hypothesis, *Journal of the Learning Sciences* **20**, 151 (2011).

23. D. F. Styer, *The Strange World of Quantum Mechanics* (Cambridge University Press, Cambridge, 2000).

24. http://www.colorado.edu/physics/EducationIssues/modern/

25. S. B. McKagan, K. K. Perkins and C. E. Wieman, Why we should teach the Bohr model and how to teach it effectively, *Phys. Rev. ST: Physics Education Research* **4**, 010103 (2008).

CHAPTER 5

Teaching Quantum Interpretations – Curriculum Development and Implementation

"The tao that can be told is not the eternal Tao. The name that can be named is not the eternal Name." – Lao-tzu, Tao Te Ching

I. Introduction

We wish to address one final question: Can the interpretive aspects of quantum mechanics be addressed at a level that is appropriate and meaningful for introductory modern physics students, without sacrificing traditional course content and learning goals? In fact, it would be hoped that an additional focus on interpretive topics (indeterminacy, the uncertainty principle, wave-particle duality, and the superposition of quantum states) would provide students with tools that would augment their overall understanding of traditional topics (quantum tunneling, atomic models); that discussions of the application of quantum mechanics could subsequently be framed in terms of language that has previously been unavailable to past instructors; and that students may develop more internal consistency in their interpretation of quantum phenomena.

The remainder of this dissertation will concern itself with the development of a modern physics curriculum designed to target these aspects of student thinking, and its recent implementation (Fall 2010) at the University of Colorado in the form of an introductory course for engineering majors. In this chapter, we discuss the guiding principles behind the development of this curriculum, and provide a detailed examination of specific, newly developed course materials designed to meet these goals. [A broader selection of relevant course materials can be found in Appendix C.] In doing so, we address the appropriateness and effectiveness of this curriculum by considering aggregate student responses to a subset of homework, exam, and survey items, as well as actual responses from four select students. [Appendix D contains a larger subset of complete responses from these particular four students.]

II. Curriculum Development and Implementation

It must be *strongly emphasized* from the outset that it is our aim to *improve* upon an already-existing body of work, which has seen contributions from over a dozen physics education researchers and modern physics instructors at the University of Colorado. As was the case for many of the modern physics offerings discussed in these studies, a substantial portion of the course materials we used should be credited to the original work of S. B. McKagan, K. K. Perkins, and C. E. Wieman. Their original course transformations, [1] which served as the basis for our course, incorporated a number of principles learned from physics education research, which include, but are not limited to:

1. Students' attitudes toward science tend to become less expert-like unless instructors are explicit in addressing student beliefs. [2, 3] The original course transformations were explicit in addressing scientific method and logical deduction; experimental evidence and real-world applications; and the uses and limitations of models. [4]

2. Interactive engagement during lecture can lead to higher learning gains than traditional lectures, [5] and can be useful in eliciting known student misconceptions. [6] Concept tests (clicker questions) provide real-time feedback from students, allowing instructors to gauge student understanding, as well as target common misconceptions. Peer discussion during concept tests gives students an opportunity to articulate their knowledge and engage in scientific argumentation in a low-stakes environment. Weekly collaborative homework sessions offer similar benefits for both students and instructors.

3. In order for students to best gain conceptual understanding and reasoning skills, all aspects of the course (including lecture, homework, and exams) should emphasize conceptual understanding alongside numerical problem solving. [1]

4. Interactive simulations used in and outside of the classroom can be useful in helping students to build models and intuition about quantum physics, by providing visual representations of abstract concepts and unobservable processes. [7]

We have argued [Chapter 3] that interpretive themes in quantum mechanics are an often *hidden* aspect of modern physics instruction, according to three criteria: A) These issues are frequently superficially addressed, and in a way that is not meaningful for students beyond the specific contexts in which they arise; B) Students often develop their own ideas regarding these interpretive themes, even when instructors do not adequately attend to them; and C) Those beliefs tend to be more novice-like (intuitively realist) in contexts where instruction is less explicit. We therefore chose to directly confront the kinds of realist beliefs and attitudes that are common to introductory modern physics students, as informed by our own research into quantum perspectives. Our aim was not only to make students consciously aware of their own (often intuitive and tacit) beliefs, but also for them to acquire the necessary language and conceptual inventory to identify and articulate those beliefs (we are reminded that, even at post-instruction, most of the students in our interviews were not familiar with the word *determinism* in the context of physics, though they had certainly developed opinions about it).

We also chose to make the interpretation of quantum physics a course topic unto itself, primarily framing our discussions in terms of the historical back-and-forth between Albert Einstein and Niels Bohr. And though we decided to be explicit in promoting a matter-wave interpretation of quantum mechanics, our ultimate goal was for students to be able to distinguish between competing perspectives, to have the requisite tools for evaluating their advantages and limitations, and to be able to apply this knowledge in novel situations. In short, instead of trying to tell students what they should and shouldn't believe about quantum physics, we chose to engage them in an explicit, extended argument (with us and amongst themselves) against *Local Realism*. This argument was *extended* in two senses: 1) We were able to augment a number of standard topics (e.g., the uncertainty principle, atomic models) with discussions of interpretive themes; and 2) We introduced several entirely new topics (e.g., delayed-choice experiments) that created additional opportunities for students to explore the sometimes fluid boundaries between scientific interpretation and theory.

The entirety of our research has indicated that wave-particle duality is a particularly challenging topic for students, and wholly relevant to their beliefs regarding the physical meaning of quantum mechanics. Whether emphasized or not, *every* modern physics instructor considered in these studies made mention of the fact that double-slit experiments could be performed with single quanta, which are detected as localized particles, but which together form an interference pattern over time. This phenomenon was often (though not universally) demonstrated in class using the Quantum Wave Interference PhET simulation, [8] as seen in the post-instruction attitude surveys. Due to the distance scales involved, a true double-slit experiment was until recently only a thought experiment, crafted as a demonstration of principle; actual experiments had demonstrated the diffraction of electrons through periodic lattices (essentially, a many-slit experiment). [9] We sought in this course to emphasize connections between theory, interpretation, and experimental evidence, and so augmented these discussions with presentations on experimental realizations of these *Gedanken* experiments. In 2008, Frabboni, et al. employed nanofabrication techniques in the creation of a double-slit opening on a scale of tens of nanometers, which they then used to demonstrate electron diffraction, as well as the absence of interference after covering just one of the two slits (they also present in their paper STM images of the double-slits, formed by an ion beam in a gold foil, with both slits open and with one slit covered). [10] Tonomura, et al. have produced a movie that literally demonstrates single-electron detection and the gradual buildup of a fringe pattern. [11, 12] Students from prior courses were often skeptical as to whether such an experiment (where only a single electron passes through the apparatus at a time) could be done in practice – in this way, they can observe the phenomenon with their own eyes.

In addressing the tendency for students to interpret wave-particle duality as implying that quanta may act simultaneously as both particle and wave, we devoted additional class time to a presentation of the single-photon experiments discussed in the first chapter, which are essentially isomorphic to the double-slit arrangement (the double-slit and the beam splitters play analogous roles). One of the guiding principles in the design of this curriculum was to avoid as much as possible the expectation for students to accept our assertions as a matter of faith. Rather than describing what the experimentalists had meant to demonstrate, and then simply asserting that they had been successful, we presented students with the actual reported data, which required the use of statistical arguments, and thereby afforded further opportunity to highlight the role of probability in quantum mechanics. These single-photon experiments demonstrate for students the dualistic nature of photons, and provide strong evidence against realist interpretations, but only if the details and results of the experiments are accessible to them, and so we omitted from our presentation extraneous technical details, while still focusing on the very process of designing the experiment and creating an adequate photon source. Devoting an entire class period to these experiments afforded us the time to walk students through each of the three experiments, and for them to debate the implications of each, while creating further opportunities to distinguish between a collection of data points, and an interpretation of their meaning.

Just as importantly, these experiments call for an explicit discussion of the need for ontological flexibility (without naming it as such) in the description of quanta, from which we may easily segue into a comparison of competing interpretations. Bohr has offered up *Complementarity* as a guide to making sense of this dualistic behavior (note that we refrain here from digressing into a full explication of the Copenhagen Interpretation for our students), but this interpretation can come across as more a philosophical sidestepping of the *measurement problem*, than its scientific resolution. Dirac's matter-wave interpretation allows for a consistent description of the behavior of photons at the beam splitters, but the physical collapse of the wave function is not described by any equation, and accepting it as physically real requires a fairly large leap of faith in itself. Moreover, these discussions allow for the explicit development of quantum epistemological tools [two paths = interference; one path = no interference] that may facilitate student understanding, and which may be applied to novel situations.

Before presenting and evaluating any newly developed course materials, some general comments on the structure of the course in which they were used are in order. As with other modern physics courses described here, our course spanned a 15-week academic semester, and consisted of large lectures ($N \sim 100$) meeting three times per week, together with weekly online and written homework assignments, and twice-weekly problem-solving sessions staffed by the instructors. Course transformations for this semester occurred primarily during Weeks 6-8, spanning a total of nine lectures. [13] Instruction was collaborative, with two lead co-instructors (one of them the author, the other a PER faculty member associated with our prior investigations into quantum perspectives), along with two undergraduate learning assistants, [14] who helped facilitate student discussion during lecture. As with the original course transformations, we omitted topics from special relativity in order to win time for the introduction of new material, without eating into the usual time at the end of the course devoted to applications.

We selected Knight's *Physics for Scientists and Engineers* [15] as a textbook (mostly for its readability), but the lectures did not follow the textbook very closely (if at all), and it was necessary to provide students with outside reading materials for many of the new topics (e.g., single-photon experiments [16] and Local Realism [17]); these *Scientific American* articles were chosen for their non-technical, but scientifically correct, treatment of interpretive ideas and foundational experiments in quantum mechanics. An online discussion board was created to provide students with a forum to anonymously ask questions about the readings, and to provide answers to each other; following these discussions granted us ample opportunity to assess how students were responding to many of the new ideas we were introducing.[1] A total of 13 weekly homework assignments consisted of online submissions and written, long-answer problems; there was a broad mixture of conceptual and calculation problems, both requiring short-essay, multiple-choice, and numerical answers. There were a total of three midterm exams (held outside of class) and the course ended with a cumulative final exam. In lieu of a long answer section on the final exam, students were asked to write a 2-3 page (minimum) final essay on a topic from quantum mechanics of their choosing, or to write a personal reflection on their experience of learning about quantum mechanics in our class (an option chosen by ~40% of students). As opposed to a formal term paper, this assignment was meant to give students the opportunity to explore an aspect of quantum mechanics that was of personal interest to them. The almost universally positive nature of the feedback provided by students in their personal reflections is evidence for the popularity and effectiveness of our transformed curriculum, and its practical implementation.

The progression of topics may be broken into three main parts: classical and semi-classical physics; the development of quantum theory; and its application to physical systems). A complete explication and analysis of the entirety of this new curriculum and associated course materials would be beyond the scope of this dissertation, and so we conclude this section with a summary overview of the progression of topics covered in this class. The remaining sections of this chapter will address specific lecture, homework and exam materials, alongside aggregate and individual student responses from the Fall 2010 semester.

PART I – Classical and Semi-Classical Physics (Weeks 1-5, Lectures 1-12): Introduction to the course and the philosophy behind its structure. Review relevant mathematics (complex exponentials, differential equations, wave equations); review classical electricity and magnetism, Maxwell's equations and how they lead to a wave description of light. [Lectures 1-3] Cover properties of waves (superposition, interference); address the wave properties of light through Young's double-slit experiment and Michelson interferometers. Introduce polarization and polarizing filters in anticipation of future topics concerning photon detection. [Lecture 4]

[1] Students were asked to make a contribution to the discussion board each week of the latter half of the course as part of their homework assignment, but no efforts were made to verify their participation, and students were free to put as little or as much effort as they liked into their postings.

Discuss photoelectric effect experiment in terms of classical wave predictions, contrasted with a particle description of light. Photomultiplier tubes are introduced as an application of the photoelectric effect, but also so as to not be unfamiliar to students when they arise in the future. An emphasis on the physical meaning of the work function foreshadows applications of the Schrödinger equation to square well potentials. [Lectures 4-5] Review potential energy curves and explicitly relate them to models of physical systems. Discuss modeling in physics, and lead discussions on the differences between observation, interpretation, and theory. [Lectures 6-7] Relate spectral lines (Balmer series) to atomic energy levels via the energy-frequency relationship established in the photoelectric effect, and use them to make inferences about quantized atomic energy levels. Emphasize the differences between photon absorption (an all-or-nothing process) and collisional excitation of atoms (discharge tubes). [Lectures 8-9] Apply knowledge of photon absorption and emission processes to the construction of lasers. Compare and contrast wave and particle descriptions of light, and address their ranges of applicability. Relate wave intensity to the probability for photon detection in the context of a single-photon double-slit experiment (simulated). [Lectures 10-11] Review for the first exam. [Lecture 12]

PART II – Development of Quantum Theory (Weeks 5-8, Lectures 13-24): Review potential and kinetic energy of electrons in a Coulomb potential, then introduce the semi-classical Bohr model of hydrogen. Discuss the ad-hoc mixture of classical and quantum rules, along with the strengths and weaknesses of the model. Introduce de Broglie waves and his atomic model as an explanation for quantized energy levels. [Lectures 13-14] Review the behavior of magnets in response to homogeneous and inhomogeneous magnetic fields; employ a Bohr-like model for atomic magnetic moments, and explicitly address classical expectations for their behavior in a Stern-Gerlach type apparatus.[2] [Lecture 15] Use repeated spin-projection measurements to introduce ideas of: quantization of atomic spin (two-state systems); definite versus indefinite states; state preparation; and probabilistic descriptions of measurement outcomes. Digress briefly to cover classical probability, statistical distributions, and the calculation of expectation values. [Lectures 16-17] Offer multiple interpretations of repeated spin measurements for future evaluation, and discuss the differences between classical ignorance and quantum uncertainty. Introduce *entanglement* in the context of distant, correlated atomic spin measurements, and relate to topics in quantum cryptography. Make explicit definitions of *hidden variables*, *locality*, *completeness* and *Local Realism*, followed by a discussion of the EPR argument and its implications for the nature of quantum superpositions. Use the notion of *instruction sets* as a first pass deterministic model, and reveal its limitations in the face of observation.[3] [Lectures 18-19] Use the single-photon experiments by Aspect, et al. as an argument against simultaneous wave and particle descriptions of photons. Invoke *Complementarity*

---

[2] Much of the lecture and homework material on magnetic moments and repeated spin measurements was inspired by D. F. Styer. [18]

[3] The "Local Reality Machine" argument is due to N. D. Mermin. [17]

and other interpretive stances in the establishment of quantum epistemological tools. [Lectures 20-21] Relate conclusions drawn from single-photon experiments to an understanding of the double-slit experiment performed with single electrons. Plane wave descriptions of single particles lead to more generalized notions of quantum wave functions and their probabilistic interpretation. Introduce the Heisenberg uncertainty principle, its mathematical expression, and various interpretations of its physical meaning. [Lectures 22-23] Review for second exam. [Lecture 24]

PART III – Applications of Quantum Mechanics (Weeks 9-15, Lectures 25-44): Motivate the Schrödinger equation through analogies with electromagnetic waves and solve for free particles in terms of plane waves. [Lectures 25-26] Introduce square well potentials (infinite and finite) and use them to model electrons in wires. [Lectures 27-28] Frame discussions of quantum tunneling as a consequence of the wave behavior of matter, then apply tunneling to scanning tunneling microscopes, and a description of alpha-decay. [Lectures 29-31] Apply the Schrödinger equation to an electron in a 3-D Coulomb potential and develop the Schrödinger model of hydrogen. Generalize to multi-electron atoms and account for the periodicity of elements. [Lectures 32-35] Review for the third exam. [Lecture 36] Explain molecular bonding and conduction banding in terms of the superposition of atomic potentials and electron wave functions. [Lectures 37-39] Apply these concepts to the theory of transistors and diodes. [Lecture 40] Finish with a foray into radioactivity, nuclear energy, and nuclear weapons (at student request) [Lectures 41-42] Review for the final exam. [Lectures 43-44]

II.A. Assessing Incoming Student Perspectives and Conceptual Understanding

Developing pre-instruction content surveys for modern physics students is more difficult than assessing incoming student beliefs about classical physics, for several reasons. First, it is expected that introductory students with little knowledge of Newtonian mechanics will have already developed intuitions (right or wrong) through their everyday experiences about the motion of macroscopic objects; in contrast, our everyday experiences with applied quantum physics (e.g. computers) provide little insight into the rules governing the behavior of quantum entities. Second, many of the learning goals for modern physics courses concern topics, such as quantum tunneling, that are entirely foreign to introductory students; and so, for example, it is practically meaningless to discuss incoming student responses to questions regarding deBroglie wavelengths and transmission probabilities, since the distributions of responses are often statistically indistinguishable from guessing.[4] Third, the broad variation in learning goals

[4] For example, an (unpublished) analysis by this author of pre-instruction QMCS scores from several modern physics courses showed them to be normally distributed about an average consistent with random guessing.

among modern physics instructors indicates a lack of consensus in the physics community regarding canonical course content, making it difficult to develop general assessment instruments that would be appropriate for a range of course offerings and student populations.

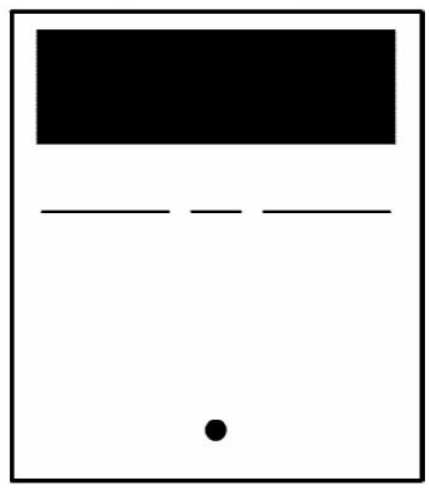
We therefore constructed a content survey (administered in the first week of the semester) that would be appropriate for the specific learning goals of this course, by culling questions from a variety of previously validated assessment instruments, [19-21] and then limiting pre-instruction items to ones where it could be reasonably expected that students would have specific reasons for responding as they do beyond random guessing (i.e., prior content knowledge or intuitive expectations). So, for example, even if students have never heard of a double-slit experiment performed with electrons, their intuitive notions of particles might still lead them expect a pattern that would be consistent with their expectations for macroscopic particles in an analogous situation (these questions taken from the QPCS; [21] student responses are given in Table 5.I):

The following questions refer to the following three experiments:

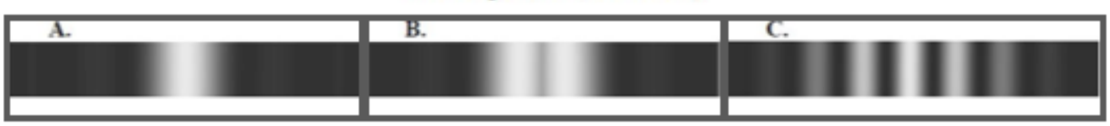
In one experiment electrons pass through a double-slit as they travel from a source to a detecting screen. In a second experiment light passes through a double-slit as it travels from a source to a photographic plate. In a third experiment marbles pass through two slit-like openings as they travel from a source to an array of collecting bins, side-by-side.

The right-hand figure diagrams the experimental setup, and the figures below show roughly the possible patterns that could be detected on the various screens.

Top view of experimental set-up (not to scale)



Possible patterns (not to scale)



A through C represent some patterns which might be observed. If you think none is appropriate, answer D. Which pattern would you expect to observe when…

6. ...*marbles* pass through the double opening?

7. ...*electrons* pass through the double slit?

TABLE 5.I. Pre- and post-instruction student responses (in percent) to items 6 & 7 from the content survey used in the modern physics course from Fall 2010. The standard error on the proportion for all cases was ~5% (Pre: N=110; Post: N=88). Students shift from expecting *similar* behavior for marbles and electrons, to expecting *different* behavior.

PRE (N=110)	**A**	**B**	**C**	**D**
Marbles	15%	60%	21%	5%
Electrons	14%	51%	35%	1%
POST (N=88)	**A**	**B**	**C**	**D**
Marbles	9%	86%	2%	2%
Electrons	0%	12%	88%	0%

We note first that, prior to instruction, the most popular response to both items was the same (B), indicating that most students expected *similar* behavior for both electrons and marbles in similar situations. These responses are consistent with our hypothesis that incoming students have particle-like expectations for the behavior of all matter. These items saw dramatic shifts in post-instruction student responses, indicating that most students expected *different* behavior for macroscopic marbles and microscopic electrons by the end of the course. The class average on the pre-instruction content survey was 46% (+/- 2%), and the average for post-instruction items common to both surveys was 80% (+/- 3%), for a normalized gain of 0.63. [See Appendix C for a complete list of pre- and post-instruction items from the content survey, with an item-by-item summary of student responses.]

As part of their first homework assignment, students were also asked to complete the same online attitudes survey administered in other courses. We summarize below the distribution of pre-instruction student responses (in terms of agree/neutral/disagree) for the entire class, along with the full responses of four select students. These four students (denoted as A, B, C & D) were not selected in order to be representative of any one group of students; their responses instead serve to demonstrate typical pre/post differences in student reasoning, even when overall responses to survey items (agreement or disagreement) had not changed. Their specific homework submissions and exam responses will later serve to address the question of whether topics that are new to the curriculum are accessible to students. Closely following these four students also allows for a more detailed exploration of the curriculum's influence on some of the aspects of student thinking that had been targeted, without making unnecessary extrapolations to the entire class population. Together, these two types of pre-instruction data will allow us to establish a baseline on incoming student perspectives.

1. It is possible for physicists to carefully perform the same measurement and get two very different results that are both correct.

PRE	Agree	Neutral	Disagree
Class (N=94)	0.65	0.13	0.22

Student A: **(Agree)** I feel that no matter how much technology advances or how much we learn, we can never fully understand how the world works and in many cases, we use outcomes of experiments to look at phenomena in different ways that may or may not be entirely correct in the real world. For instance, looking at the behavior of light as both a particle and wave. So, yes, I believe that an experiment came be conducted twice with different outcomes.

Student B: **(Agree)** I don't know of any examples, but the fact that quantum physics has some things that seem counter-intuitive and contradict classical physics, it seems that this could be a possibility.

Student C: **(Strongly Agree)** What the two physicists are measuring could be highly unstable and sensitive to multiple external stimulus.

Student D: **(Strongly Agree)** It is possible for identical measurements to produce different results if that which is being measured can exist in more than one state at the same time. Thus, one would not know whether the subject of the measurement is the object in one state or the other. Interpreting this question differently, one could comment on the fact that the very act of measuring itself introduces new elements into a system, and thus actually changes the outcome of the measurement.

Overall class responses are consistent with prior results, with a strong majority of students agreeing with this statement, though it should be cautioned that students vary greatly in the reasoning behind their responses, as seen in Chapter 2. Students A, B & D have all invoked quantum phenomena in their agreement with this statement, with varying degrees of sophistication. Student D speaks of quantum superposition and the physical influence of observation; Student A notes that light may be described as both particle and wave; Student B simply states his impression that quantum mechanics will challenge his intuition, so perhaps this statement might be true. Student C's reasoning is more consistent with the idea that chaotic, hidden variables may randomly influence the outcomes of similar measurements – an attitude commonly seen in pre-instruction responses.

2. The probabilistic nature of quantum mechanics is mostly due to physical limitations of our measurement instruments.

PRE	Agree	Neutral	Disagree
Class (N=94)	**0.46**	**0.32**	**0.22**

Student A: **(Neutral)** I really don't know enough about quantum theory to make a guess on that. However, even our most basic assumptions about the world have sometimes proven to be incorrect and quantum seems to involve so much theory that we can never really be sure if it actually functions the way physicists think it does or if we are coming up with theories that just fit what we find without even seeing the entire picture.

Student B: **(Strongly Agree)** I believe that in the future, we would be able to make more accurate and exact assertions due to technological advances and would not need to rely on probability.

Student C: **(Neutral)** I don't know what quantum mechanics is yet.

Student D: **(Strongly Disagree)** The probabilistic nature of quantum mechanics is a fundamental property of the system. For example: it is impossible to define (not just measure) the position and momentum of an electron at the same instant in time (Heisenberg's uncertainty principle). Thus, the uncertainty exists outside of the instruments used to try to measure those properties. (I would really, really like to learn the math behind these statements!)

Responses here were more varied than with the first statement, though agreement amongst the class is moderately favored; the individual responses range from strong agreement to strong disagreement. The two neutral responses from Students A & C indicate a similar tentativeness due to a lack of knowledge about quantum mechanics; Students A & B both echo a common perception that knowledge in science is itself tentative, and that profound progress (technological or theoretical) often upends previously held beliefs. In contrast, Student D identifies quantum uncertainty as fundamentally different from experimental uncertainty, explicitly stating there are limits not only on the precision of simultaneous measurements, but also on simultaneous quantum descriptions of incompatible observables (position and momentum, specifically).

3. When not being observed, an electron in an atom still exists at a definite (but unknown) position at each moment in time.

PRE	Agree	Neutral	Disagree
Class (N=94)	0.72	0.09	0.19

Student A: **(Strongly Agree)** An electron is a fundamental piece of an atom, though it moves extremely fast, so at any point in time, yes it does occupy a position being that it is matter.

Student B: **(Strongly Agree)** An electron is a particle, and every particle has a definite position at each moment in time.

Student C: **(Agree)** Because I have been told this since 9th grade.

Student D: **(Agree)** An electron occupies a single definite position at any given point in time. It is only our measurement (and thus knowledge) of that position at any given point in time that is subject to the Heisenberg uncertainty principle, where either the position or the momentum of the electron may be measured to a high level of precision, but not both.

As expected, a strong majority of incoming students chose to respond in a manner that would be consistent with realist expectations; all four of our individual students were in agreement that atomic electrons should exist as localized particles. The reasoning invoked by Students A & B is consistent with our hypothesis of *classical attribute inheritance* – electrons, as a form of matter, have the same properties as macroscopic particles, including a localized position at all times; Student A further implies that the uncertainty in an electron's position can be attributed to its swift, chaotic motion about the nucleus – similar to the hidden-variable style reasoning of Student C in response to the first survey item. Here, Student C makes an appeal to authority: the idea of localized electrons conforms to what he has been told in school since (presumably) first learning about the structure of atoms. Most interestingly, Student D is explicit in asserting the realist belief that electrons always exist as localized particles; he claims it is our simultaneous *knowledge* of incompatible observables that is constrained by the uncertainty principle.

4. I think quantum mechanics is an interesting subject.

PRE	Agree	Neutral	Disagree
Class (N=94)	0.85	0.13	0.02

Student A: **(Strongly Agree)** From the examples I have heard and some of the theory, I think quantum mechanic is very interesting.

Student B: **(Strongly Agree)** I think that I'm going to learn that what I would think is correct is actually completely incorrect. Plus, it just sounds cool.

Student C: **(Neutral)** I don't know yet.

Student D: **(Strongly Agree)** Quantum mechanics fascinates me precisely because it is so counterintuitive. I want to challenge my perception of the world, and there are few better ways to do that than QM. It is also interesting to me because I am much more used to physics on very large, indeed cosmic scales. It is especially interesting to see how the world of the unimaginably tiny and the world of the unimaginably large interact...

5. I have heard about quantum mechanics through popular venues (books, films, websites, etc...)

PRE	Agree	Neutral	Disagree
Class (N=94)	0.61	0.19	0.20

Student A: **(Strongly Agree)** [BLANK]

Student B: **(Strongly Disagree)** I'm completely out of the "physics loop" and hope to get more into it in this class!

Student C: **(Agree)** I read part of the book In Search Of Schrodinger's Cat by John Gribbin

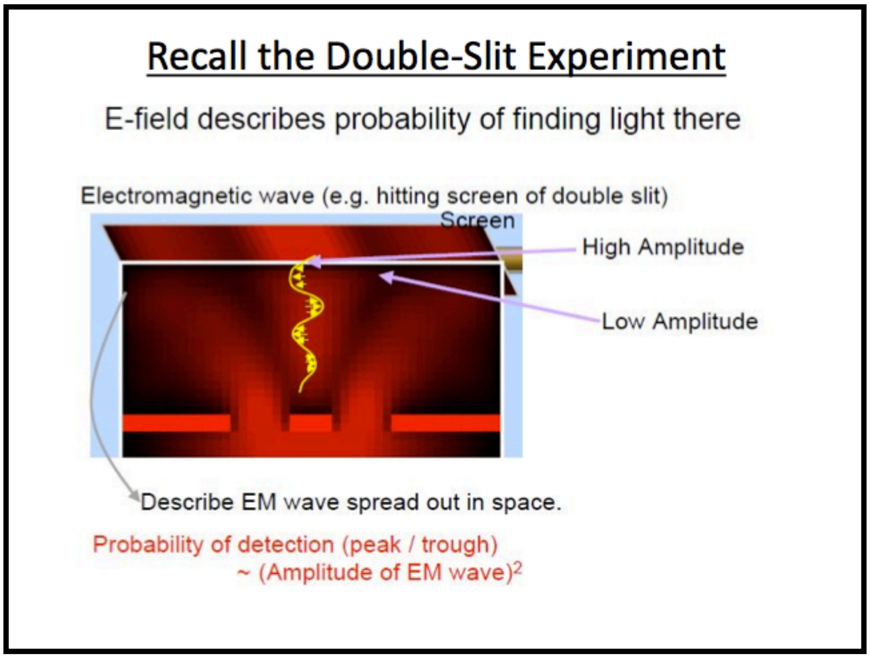
Student D: **(Agree)** In high school, I got a taster of quantum mechanics through generalized physics books, but nothing more in depth. Beyond that, my knowledge of quantum mechanics is limited, and comes primarily from several online lectures by MIT (through itunes U) and several from the University of Madras (posted on youtube).

The reported incoming interest in quantum mechanics for these students is somewhat higher (85%) than is usually seen in a course for engineering majors (~75%; and comparable with typical incoming attitudes among physics majors; see Chapter 6). Because we have no other reason to believe that students from this semester would be any different from previous populations for this course, we can only speculate that this is what resulted from all four members of the instruction team hyping the excitement of quantum physics on the first day of lecture. And as with previous introductory modern physics courses, a majority of students reported having heard *something* about quantum mechanics before enrolling in the course, which underscores the fact that incoming students are not entirely blank slates when it comes to quantum physics, and will certainly bring *some* preconceived notions into the course – incoming students will have impressions about the nature of quantum mechanics, positive or negative.

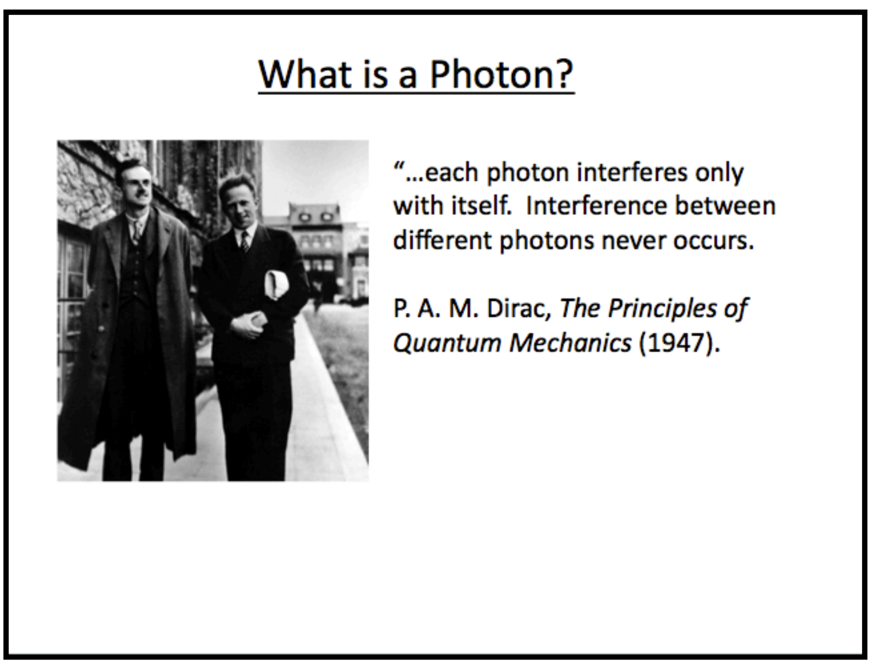
With these considerations in mind, it seems reasonable to conclude that this particular group of students held incoming attitudes and beliefs that were typical of similar student populations (as measured by these specific assessments), and to assert that any aggregate student outcomes associated with the implementation of this curriculum should not be attributed to there being anything unique about this particular class. We have no means of objectively assessing just how representative Students A – D are of the overall student population, but it is our subjective opinion (based on the experience of studying a wide variety of modern physics offerings over the span of several academic years) that Students A, B & C represent several points of view that are common among incoming engineering students. It is also our subjective assessment that Student D holds a relatively sophisticated view on quantum mechanics for an incoming student, but one that could be categorized as *Realist/Statistical* in light of his explicit belief in the localized nature of electrons, and his assertion that the uncertainty principle constrains simultaneous *knowledge* of incompatible observables.

II.B. Lecture Materials

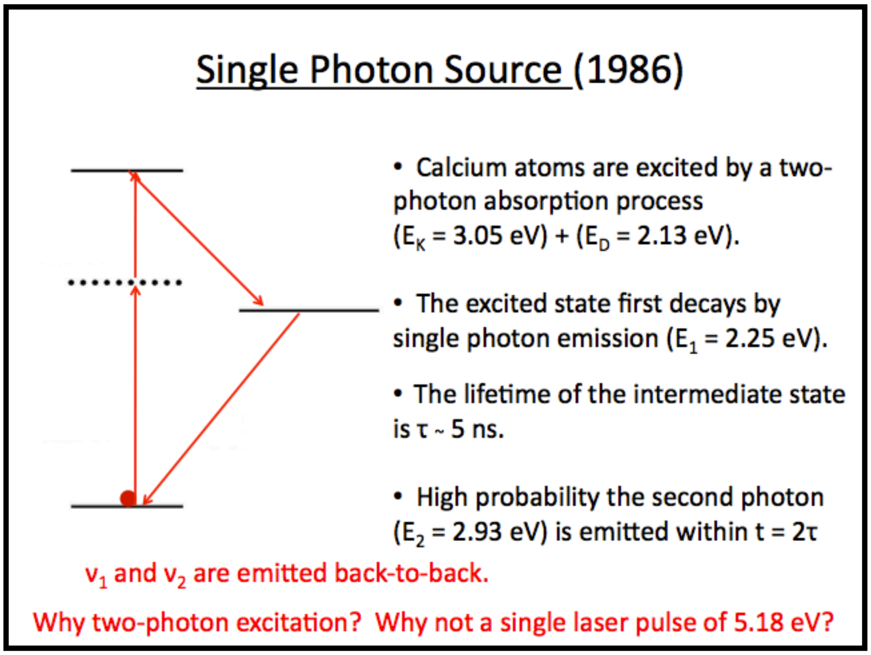
In their end-of-term reflective essays, the topics most frequently cited by students as having influenced their perspectives on quantum physics were the single-quanta experiments with light and/or matter, and so we focus our attention here on one lecture (#20) primarily devoted to the experiments performed by Aspect, et al. (as described in Chapter 1). Topics from immediately prior to this lecture included: hidden variables, Local Realism, and indeterminacy in quantum mechanics. [Lectures 18-19] Our primary objectives for this lecture were for students to understand how two similar experimental setups can lead to dramatically different observations; to highlight the differences between observation and inference (interpretation of experimental facts); and to provide experimental evidence that contradicts the simultaneous attribution of particle and wave characteristics to photons.



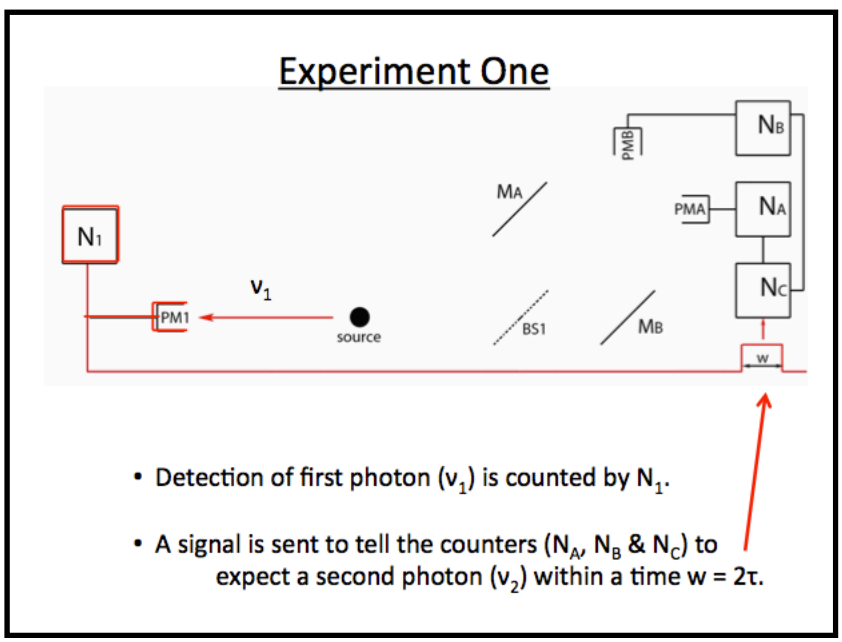
L20.S01. Students are reminded that the double-slit experiment can be performed with single photons, which are detected individually. Wave intensity is associated with the probability for detection, which is greater in locations where there is constructive interference.



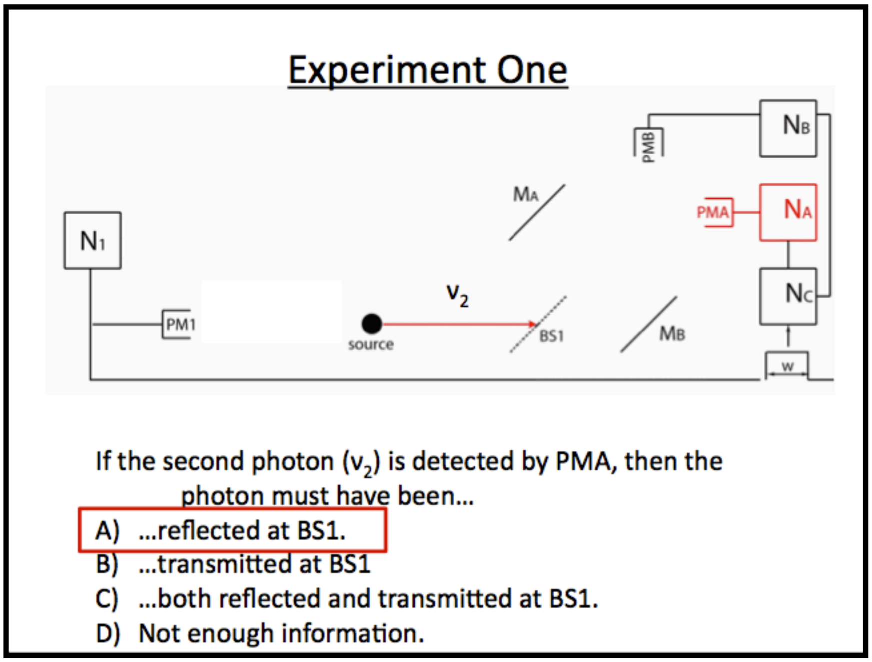
L20.S02. Dirac offered his interpretation of these kinds of experiments long before they could be realized: each photon must pass through both slits as a delocalized wave and interfere with itself; interference with other photons does not occur.



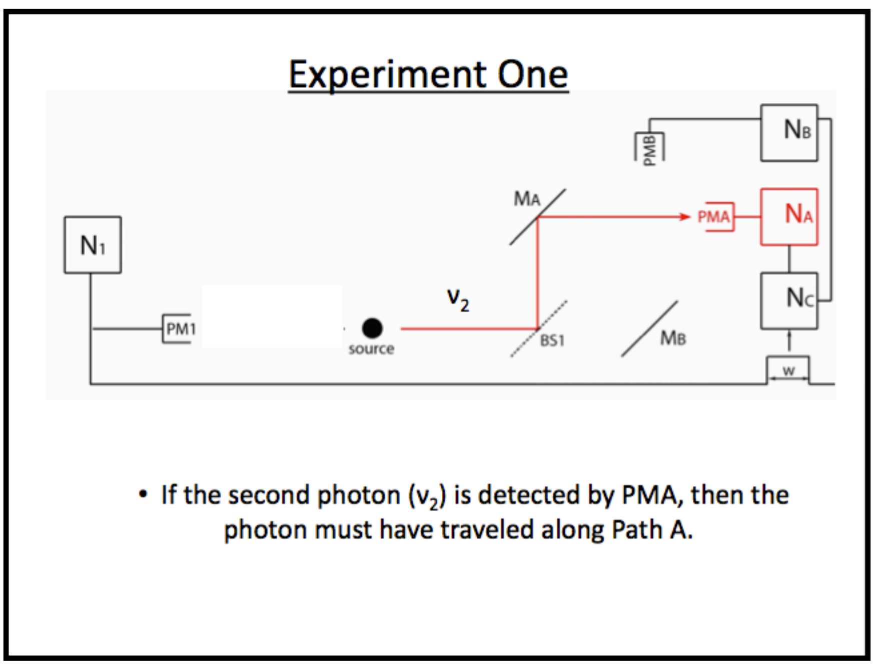
L20.S03. A "single-photon source" was employed by Aspect in 1986 to explore the wave-particle duality of photons. The two-step excitation process greatly reduces the intensity of the source, where the goal is to detect only specific photons: ones emitted in a two-step, back-to-back de-excitation process.



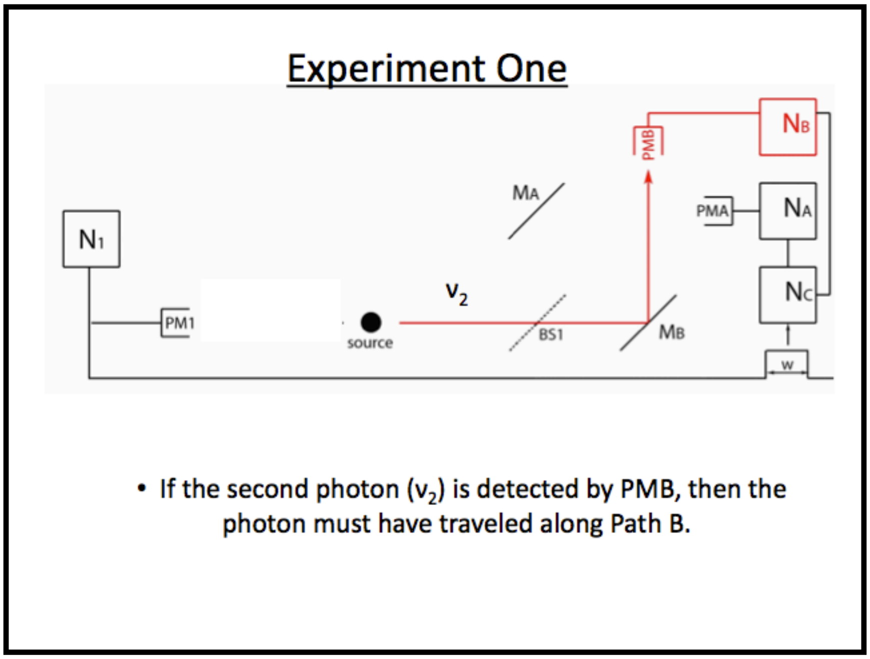
L20.S04. Detection of the first photon (ν_1) in PM1 signals the counters to await the detection of the second photon (ν_2). The gate is open for a time equal to twice the lifetime of the intermediate state, making it highly probable that a second photon was emitted during that time period.



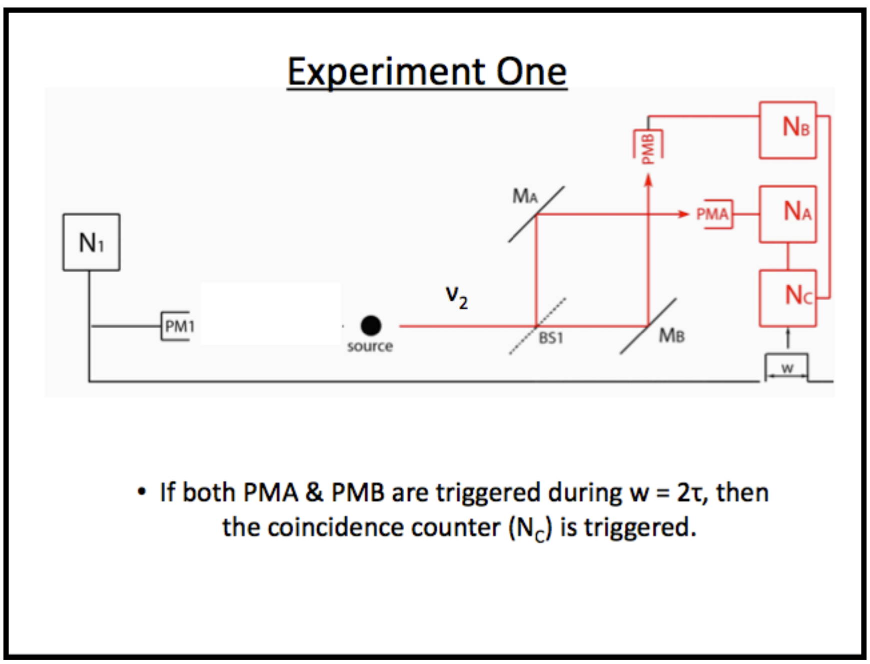
L20.S06. With a little discussion, students quickly converged on (A). The greatest student confusion arose from the schematic nature of the diagram, which implies there is open space between BS1 and the two photomultipliers, which might allow for a photon reflected at BS1 to reach PMB. This question helps check that students understand the purpose of each element of the experimental setup (beamsplitter, mirror, detector, counter).



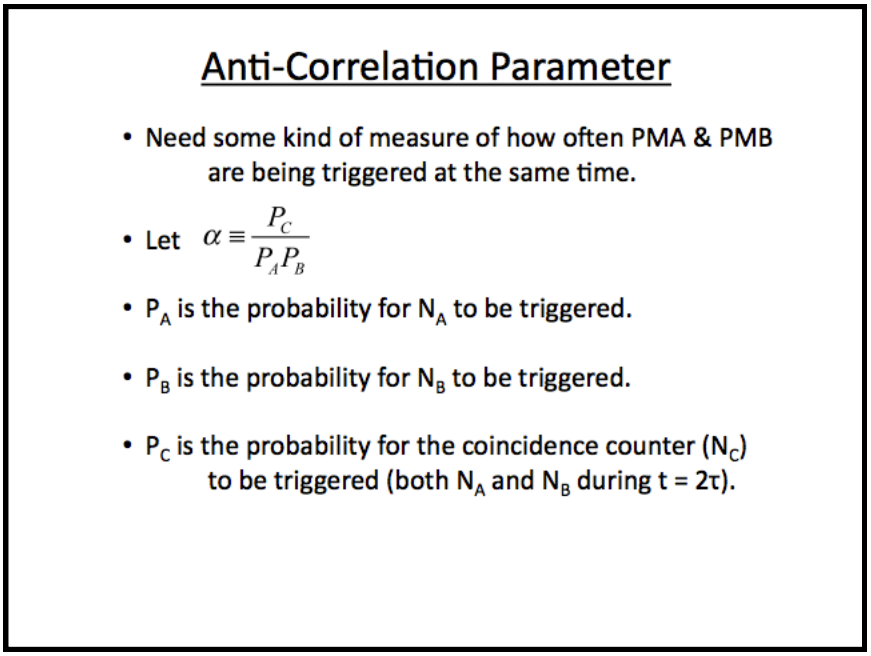
L20.S08. Following the previous concept test and subsequent discussion, it should now be clear there is only one path by which a photon might reach PMA: it must have traveled along Path A, by reflection at BS1, and reflection again at M_A.



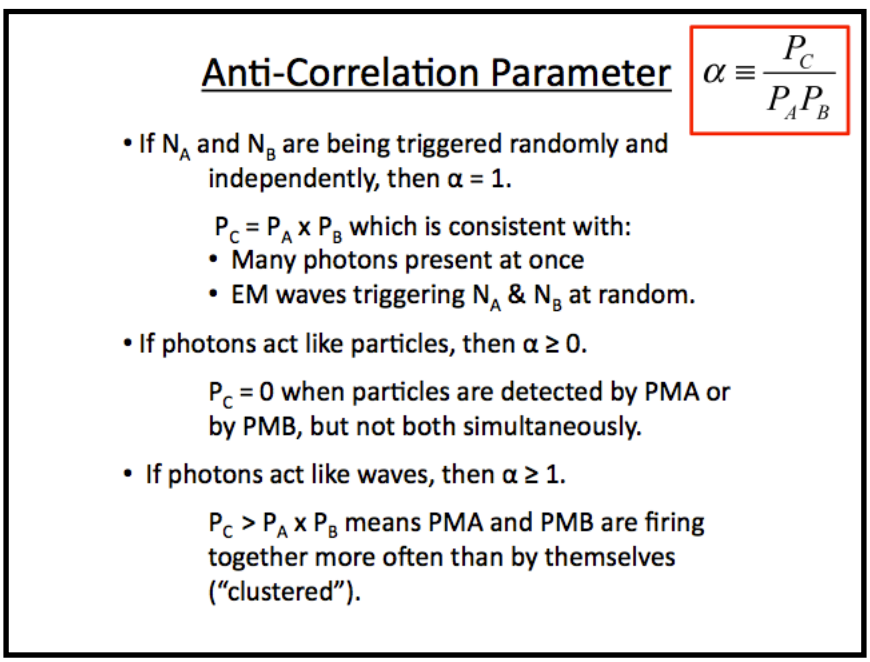
L20.S09. The same is true for a detection in PMB: the photon can only have traveled via Path B, by transmission at BS1, and reflection at M_B.



L20.S10. It is still possible to record a detection in both photomultipliers during the short time the gate is open – when this happens, the coincidence counter (N_C) is triggered. How often this happens has implications for how we interpret the behavior of photons.

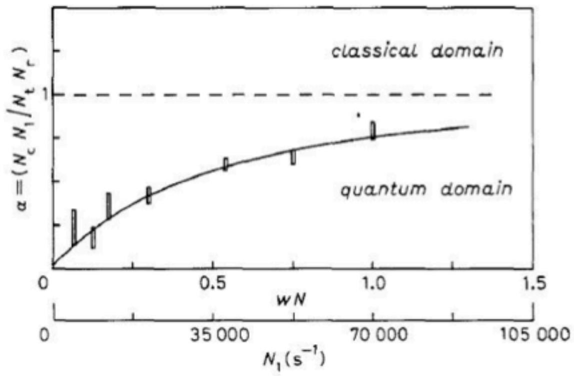


L20.S11. We first require some kind of statistical measure of how often the two photomultipliers are firing together versus firing separately. This can be defined in terms of a ratio of the counting rates per unit time for each of the three counters, or equivalently, in terms of the probability for each of the counters to be triggered during the short time the gate is open.



L20.S12. If the detectors were to fire together more often than not (implying that the photon energy is coherently split at BS1 and deposited equally in both detectors – wave behavior), then α should be ≥ 1. It will be less than one if the detectors tend to fire independently (implying each detection corresponds to a single photon following a single path – particle behavior).

Experiment One



EM Waves → α ≥ 1   Quantum Particles → α ≥ 0

Photons take ***either*** **Path A** ***or*** **Path B**, but not both!!

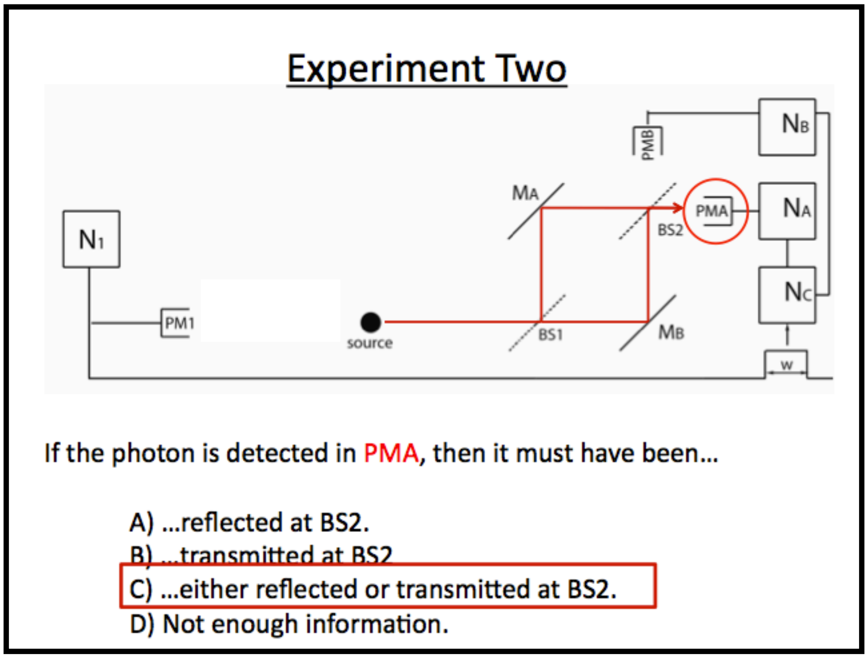
If photons are particles, why don't we ***always*** measure α = 0?

L20.S13. At all intensities (but particularly at low counting rates), the two photomultipliers fire independently more often than not. Since only a single path leads to either of the two detectors, we interpret these results as indicating that each photon is either reflected or transmitted at BS1, but not both.

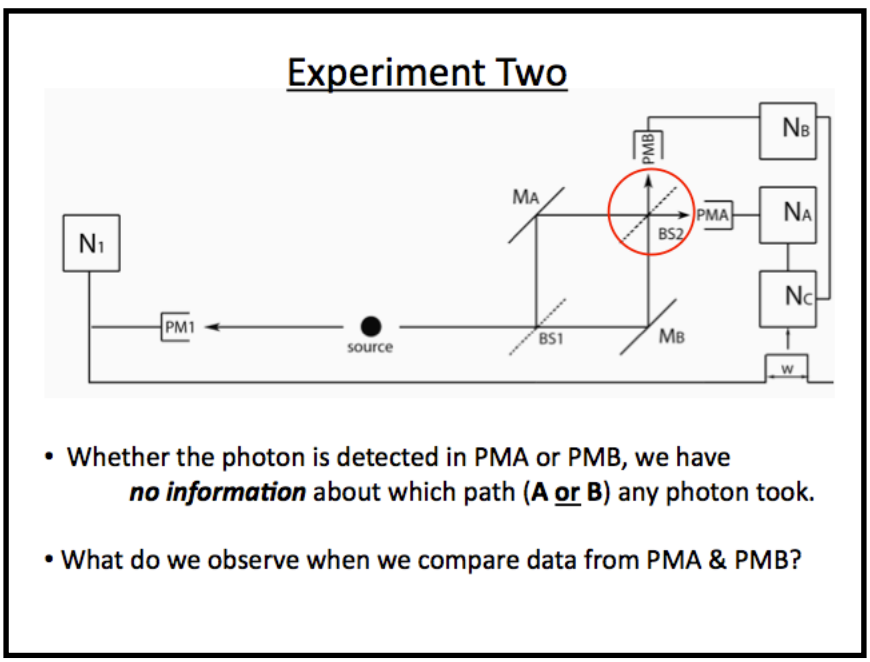
Experiment Two

- Use same single-photon source, but now insert a beam splitter. (BS2)
- Run experiment as before…

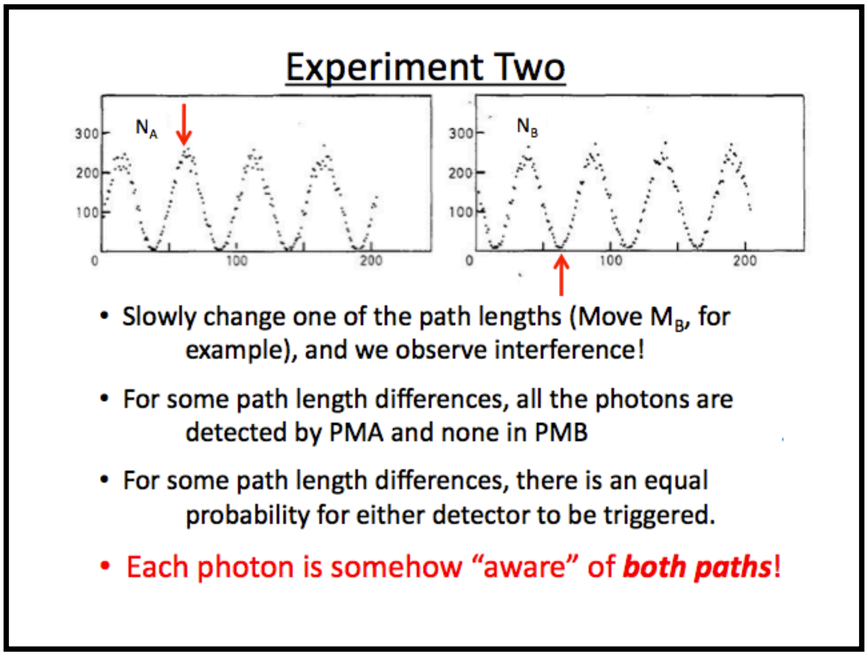
L20.S14. The experiment is run again as before, except that now a second beam splitter (BS2) is inserted into the path. It is impossible to determine which-path information through a detection in either one of the photomultipliers.



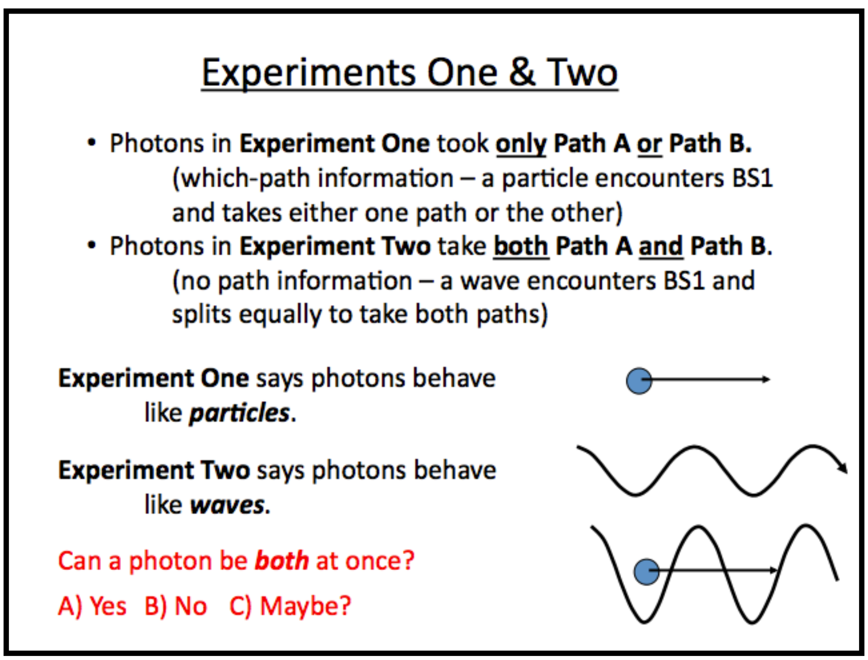
L20.S15. With the second beam splitter in place, there are now multiple paths a photon could take to be detected in a given photomultiplier. Students were quick to converge on (C) as the correct answer, with less discussion than was required for the first concept test.



L20.S16. Detection in either of the photomultipliers yields no information about which path a photon must have taken to get there. With multiple possible paths, interference effects are expected, though not of a kind previously encountered by students. In this case, interference is observed by comparing the counting rates in the two detectors.



L20.S17. According to quantum mechanics, the counting rates in the two detectors are oppositely modulated according to the difference in path lengths between A & B. Photons that had only taken Path A should not be affected by any changes made to Path B, yet their behavior at BS2 is determined entirely by the relative lengths of **both paths**.



L20.S18. An explicit connection is made between the interpretation of a photon's behavior at BS1 and the which-path information available to the experimenter. There was no favored response to this moderately rhetorical clicker question, which was meant more to get students thinking and talking about the validity of our interpretations, and to prime them for the delayed-choice experiment.

The "Conspiracy" Theory

How can the photon "know" whether we are conducting Experiment One or Experiment Two when it encounters BS1?

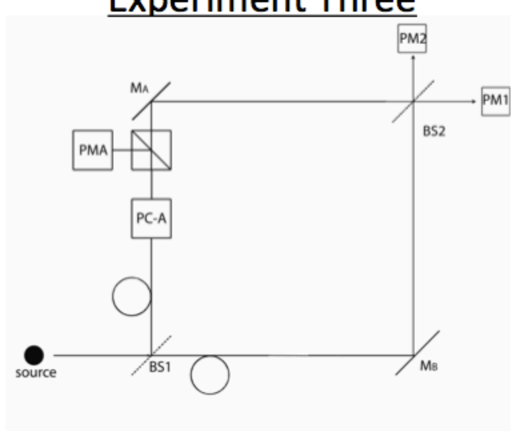
Perhaps each photon "senses" the entire experimental apparatus and always behaves accordingly.

Can we "trick" a photon into acting like a particle when it should act like a wave, or the other way around?

Suppose we let the photon enter the apparatus when the second beam splitter is absent (particles take one path or the other), but then insert the beam splitter at the last moment.

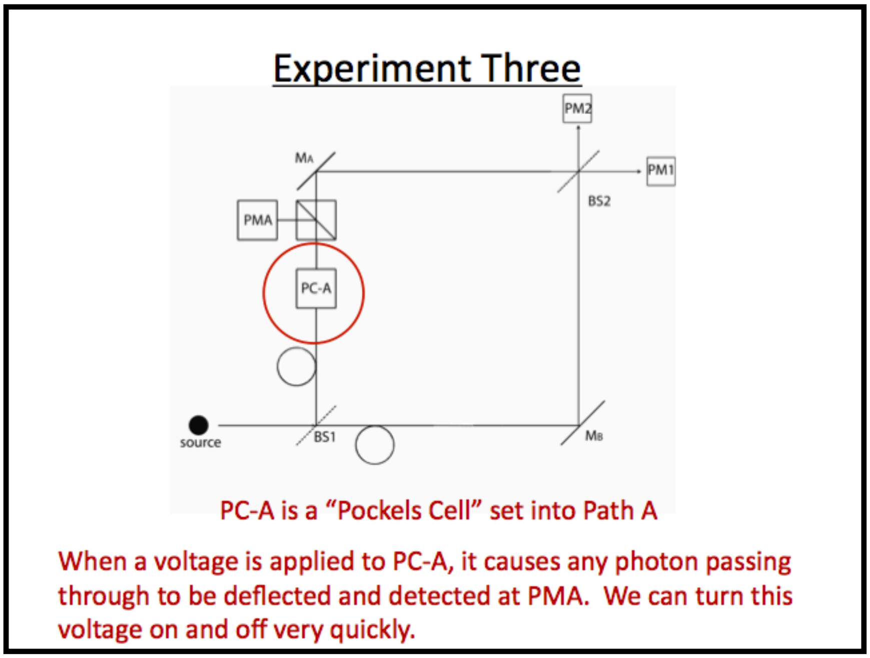
L20.S19. The question is now whether we can make a change in the experimental apparatus after the photon has encountered the first beam splitter; in such a way that we go from conducting Exp. 1 to Exp. 2 (or vice-versa) after the photon has already "decided" how to behave when it encounters BS1.

Experiment Three

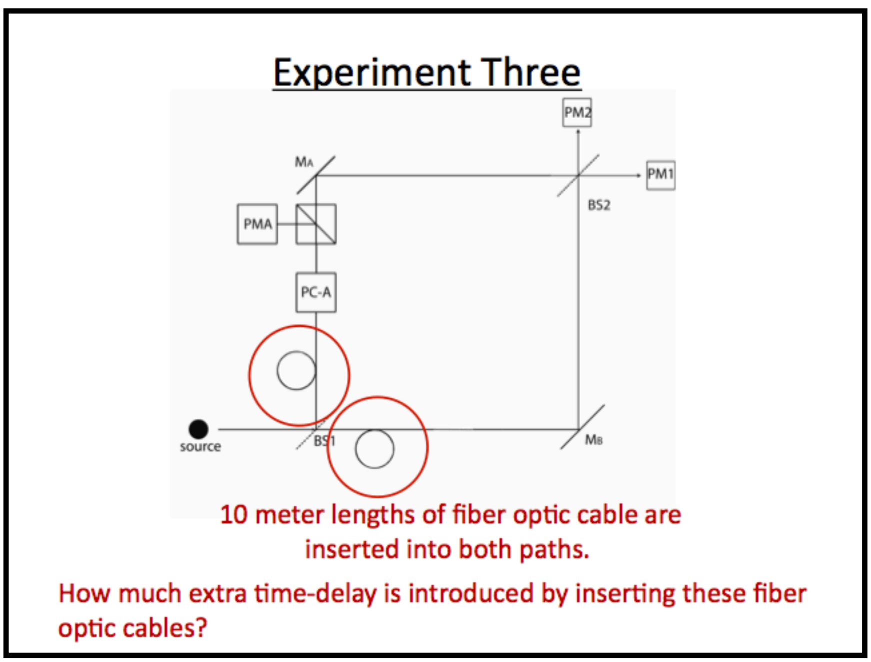


Impossible to physically remove actual beam splitter at the necessary speed, but this type of experimental setup is equivalent to what we just described.

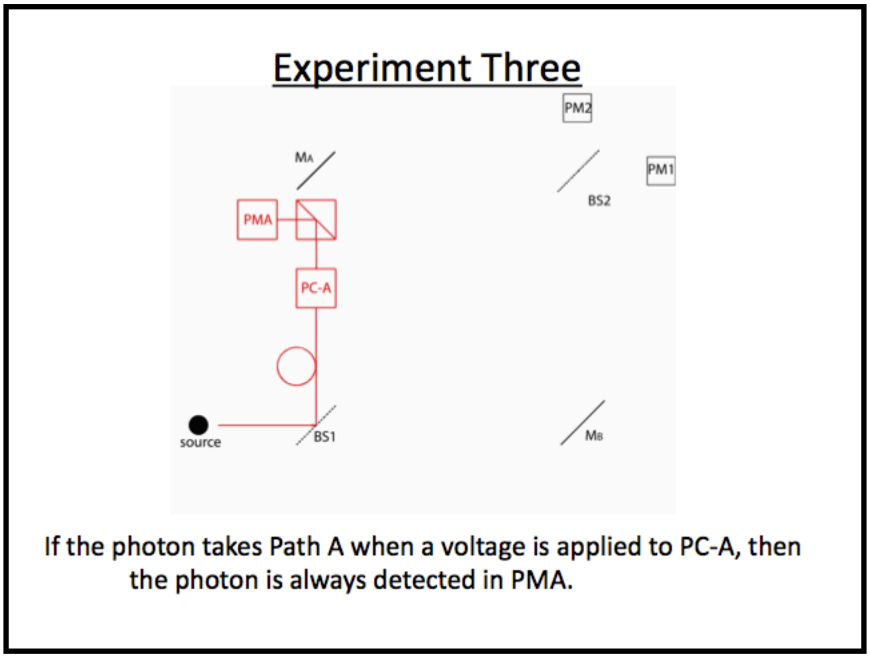
L20.S20. While structurally similar to the first experiment, this one utilizes a laser tuned to such low intensity that there is, on average, only one photon per pulse.



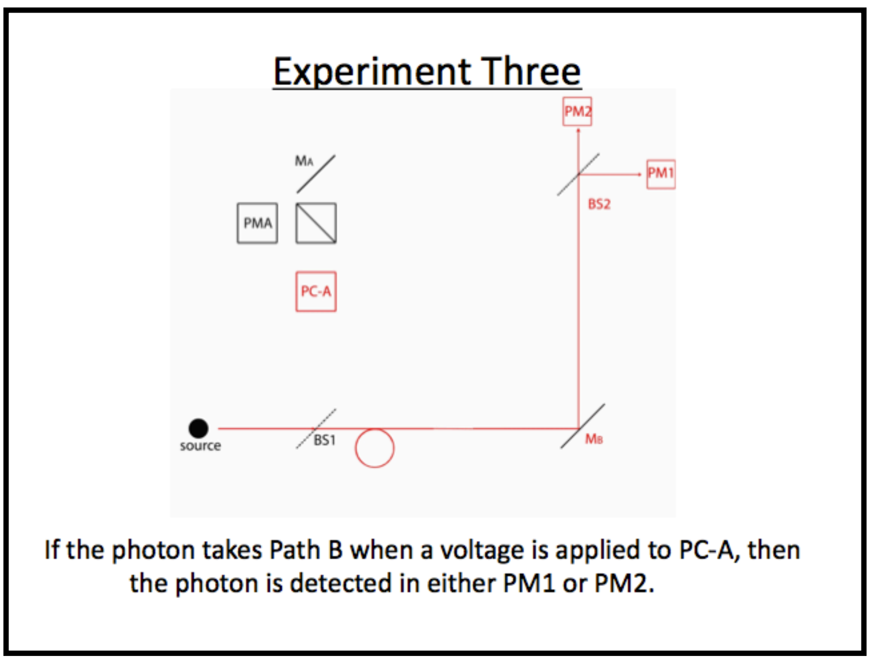
L20.S21. When a voltage is applied to the Pockels cell it rotates the plane of polarization of a photon such that it is always reflected by the Glans prism into PMA. This voltage can be turned on and off with a frequency that is sufficient for the time resolution of this experiment.



L20.S22. Two 10-meter lengths of fiber optic cable introduce a transit delay time of about 30 nanoseconds after the photon has encountered the first beam splitter.

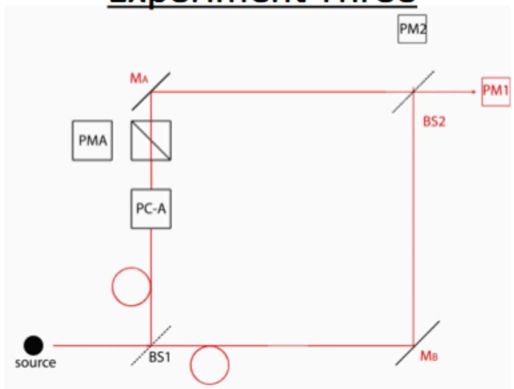


L20.S22. With a voltage applied to the Pockels cell (PC-A), any photon reflected at BS1 will be detected in PMA with 100% probability.



L20.S24. With a voltage applied to the Pockels cell, any photon transmitted at BS1 will have an equal likelihood of being detected in either PM1 or PM2.

Experiment Three



When ***NO*** voltage is applied to PC-A, then both Paths A & B are possible.

Fix the mirrors so that photons are always detected in PM1
(Interference)

L20.S25. With no voltage applied to the Pockels cell, both Path A and Path B are open to the photon. Since self-interference is possible in this case, we may fix the mirrors so that every photon is detected only in PM1 when no voltage is applied.

Experiment Three

No voltage applied to PC-A:

Both paths are possible and photon is detected in PM1 only.

TWO PATHS = INTERFERENCE

Voltage applied to PC-A:

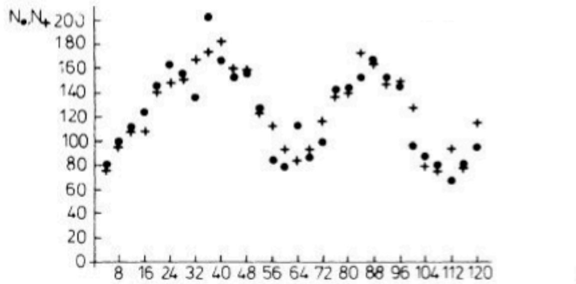
If photon detected in PMA <--> Photon took Path A

If photon detected in PM1 or PM2 <--> Photon took Path B

ONE PATH = NO INTERFERENCE.

L20.S26. This may form the basis of a quantum epistemological tool for students. With only one path possible, no interference effects should be seen (photons behave like particles); two (or more) paths means interference should be visible (photons behave like waves).

Experiment Three



- Dots represent apparatus operating in "normal" mode
  - no voltage applied to either PC.
- Crosses represent apparatus operating in "delayed-choice" mode
  - photon enters apparatus with only one path open.
  - photon should choose one path or the other at BS1
  - paths are unblocked, and interference is still observed.

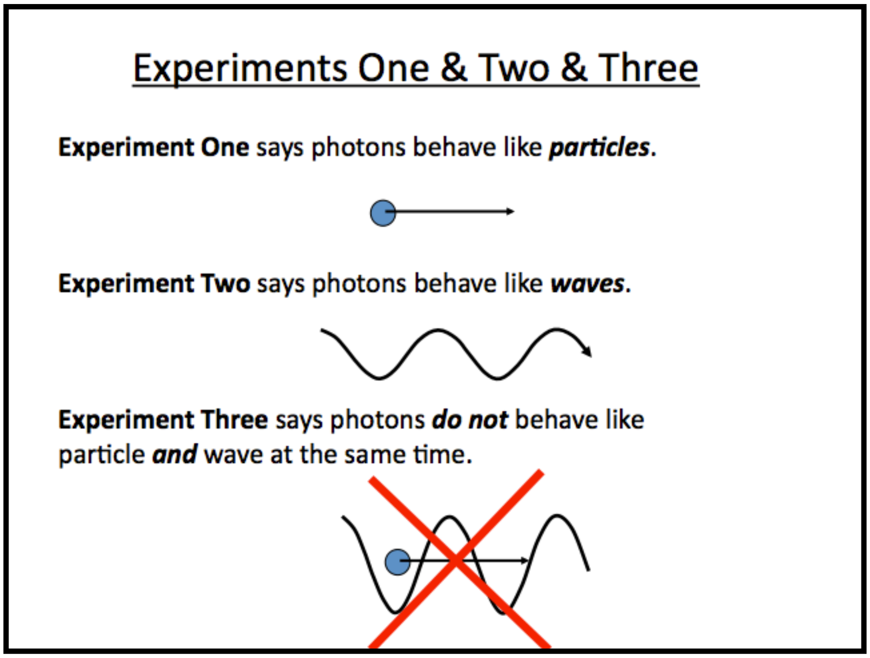
L20.S27. When the experiment is run, interference is seen whenever two paths were open to the photon, and absent when only one path was open, regardless of which was the case at the time the photon encountered the first beam splitter.

What is a Photon?

"The result of [the detection] must be either the whole photon or nothing at all. Thus the photon must change suddenly from being partly in one beam and partly in the other to being entirely in one of the beams."

P. A. M. Dirac, *The Principles of Quantum Mechanics* (1947).

L20.S28. Dirac's interpretation suggests the photon is coherently split into a superposition state at the first beam splitter in all three experiments, and then collapses to a point when (randomly) interacting with a detector.



L20.S29. It is hoped that, by this point, students will not just accept, but conclude for themselves that photons never exhibit both types of behaviors simultaneously.

II.C. Homework

Informal interviews with modern physics instructors have revealed a common concern that a proper treatment of the interpretive aspects of quantum theory requires an understanding and knowledge base that is beyond the reach of most introductory students, and may only open a Pandora's Box of unanswerable questions that could ultimately lead to more confusion. We believe, however, that this end result is more likely in a course where students are not given the requisite tools, including language, to fully appreciate the arguments against classical thinking in quantum contexts; and that it is precisely these kinds of open questions in physics that inspire the excitement and imagination of our students. We also believe that realist preferences are common, and so intuitive to students that many are simply lacking a name for beliefs they had already articulated in their pre-instruction survey responses. The full implications of nonlocality in quantum phenomena might not be appreciated by every student, but most will readily agree that a measurement performed on one of two physically separated systems should have no influence on the outcome of a measurement performed on the second. We wish to address here just how accessible some of the formal definitions of concepts associated with *Local Realism* are to students, following their discussion in class and in the assigned reading. [16]

One of the homework essay questions from Week 7 asks students to articulate their own understanding of the terms *realism*, *locality*, and *completeness*, and to provide some examples of *hidden variables*:

Student A: To me, realism can be described as the idea that things happen whether someone is there to witness it. For example, if a tree falls in the middle of the woods and there is nothing around to hear it, does it still make a sound? Locality represents an intuition that objects around us can only be directly influenced by other objects in its immediate surrounding. Completeness is a description of the world that is represented by the smallest physical attributes such as particles, electrons, waves, atoms, etc. Completeness describes the complete world as one. A great example of hidden variables is the example referred to in class about 2 socks being put into different boxes, mixed up and sent to opposite sides of the universe. Once you discover the color of one sock, you know the color of the other one... entanglement. These socks are hidden variables until one sock's color is discovered.

Student B: Realism is a property in which every measurable quantity exists. In other words, everything is definite, and there is no superposition. The only thing that keeps us from knowing what all the quantities are is our ignorance. Completeness refers to a theory that can describe everything without leaving anything unknown. By this definition, quantum physics is not complete because when we measure a certain quantity such as the projection of the atom in the Z direction, then we can't know its projection in the X direction.

Locality is the concept of being able to relate all actions to actions that occurred before them. For example, locality can describe a car accident – all the events that lead up to the car accident are clear and relate to one another. Bohr's interpretation of entanglement is not local, because we have no way of explaining how the observation of one atom collapses the wave such that the other atom (which would be miles apart) instantaneously is affected.

Student C: Locality: Locality of the two particles that are being separated and measured means that in some way the particles are linked to each other. These two linked particles are then able to influence each other with out traveling faster than the speed of light.

Realism: Realism suggests that no quantum superposition exists. If I see a red sock in the classic two socks in box experiment, the sock was red all along and the other sock was blue all along.

Completeness: If the sum total parts of any experiment is known, the outcome can be predicted. There is completeness to an experiment that can always be predicted. Quantum mechanics suggests otherwise.

Hidden Variables: A hidden variable could influence the outcome of an experiment and explain the non-locality of entangled particles. A tachyon is an example of a hidden variable, it is something that can travel faster than the speed of light.

Student D: Realism states that a quantity in a measured system has an objectively real value, even if it isn't known. For example, under a realist interpretation, an atom always has a particular spin, we are simply unable to know that spin before we measure it (it is "hidden"). Locality is the concept that there must always be a causative chain in the real world linking two events, in other words, that one object may only effect another by causing a change in its local surroundings that may eventually propagate to cause a change in the second object through its local surroundings. Entanglement appears to violate this principle by allowing two particles to influence the state of each other regardless of their physical separation or the material in-between them. For a physical theory to be "Complete" according to the guidelines set by EPR, it must be able to explain the nature and behavior of everything in physical reality. In this sense, quantum mechanics is not complete; if locality is not to be violated quantum mechanics cannot explain all of the physical properties of a system at the most basic level.

Not surprisingly, the coherence of Student D's overall response indicates a solid understanding of each of these terms, not only individually, but also in how they relate to each other in making up EPR's argument for the incompleteness of quantum mechanics. Student B's responses are also satisfactory, and a careful reading reveals his continued preference for realist notions: his specific choice of language implies that an atom can indeed have a definite spin projection along multiple axes, and that our quantum mechanical knowledge of the system is therefore incomplete. Student A's definition of *completeness* seems not far off the mark, though his last statement on the matter is somewhat vague – does he mean that a complete theory consists of a complete description of everything in the universe, or that a complete theory describes everything as a complete and undivided whole? Student C's ideas about *completeness* are linked with determinism: knowing all of the relevant variables would make the outcomes of measurements predictable. In defining *locality*, Student C actually describes a state of *entanglement*, though he later correctly refers to entanglement as being *non-local* in his description of hidden variables. He is also correct in asserting that, should tachyons exist, their unknown presence may have some hidden influence on the outcome of measurements, but we consider it preferable that students focus their attention on more concrete examples of hidden variables (such as position or momentum), as opposed to exotic, hypothetical phenomena.

Fortunately, this was not the last opportunity for students to wrestle with the meaning of these terms, and all that they imply. During Weeks 6-8, students responded each week to an online reading quiz, which merely asked them to pose (at least) one question about something (anything) from the reading assignments for that week. These questions were then compiled and used as seeds for an online class discussion forum. For each of the subsequent five weeks, students were asked to make a contribution to the discussion board as part of their weekly homework assignments, but no efforts were made to verify their participation, and students were free to put as little or as much effort as they liked into their postings. Student postings were anonymous (even to the instructors), though we could verify at the end of the semester how many postings a student had made. Figure 5.1 shows how a large majority (> 75%) of students made at least four contributions to the discussion board during the course of the semester (the few students who made zero contributions are not shown).

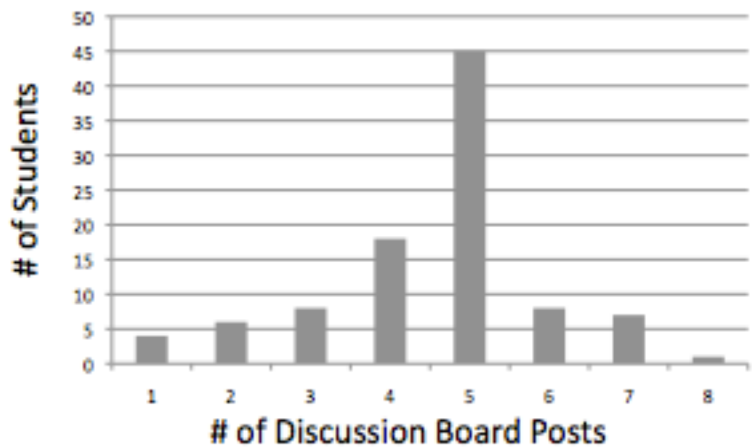


FIG. 5.1. Total number of postings made by students by the end of the Fall 2010 semester. Well over 3/4 of the enrolled students made at least four contributions to the discussion board over the course of the semester.

Our overall assessment would be that students engaged each other in a thoughtful and creative exchange of ideas, sometimes within topics that were fairly removed from our immediate focus (tachyons, time travel, warped space, and the like…). Many of the discussion threads centered on students clarifying their understanding of specific concepts (with the *occasional* intervention of an instructor, in order to stem the propagation of misconceptions), but a good deal more showed how many of the students didn't struggle so much with understanding what the interpretations were about; they struggled more with what they implied about the nature of science and reality. In just one excerpt from a discussion thread, [see Appendix F for a larger selection] we see how students are troubled by the idea of collapsing wave functions – is it some ad hoc rule invented to make the theory conform with observation? We see opposing views on questions of ontology: a literal switch between categories, or a switch between descriptions, or do photons belong to a category all their own? What are our everyday experiences with quantum phenomena, and where do we draw the line between the classical and the quantum world?

Subject: Delayed-Choice Experiments
Date: October 12, 2010 10:53 PM

[…] It seems that what's important for the argument is what's going on at the first beamsplitter. I think Dirac is saying that we can think of each photon always taking both paths and then the collapse of the wavefunction forces the photon to suddenly go from being in both paths to being in just one?

> Date: October 17, 2010 7:46 PM
>
> I got the same message from Dirac's statement that "each photon interferes only with itself" and that the photon is wavelike until observed as a particle. Or innocent until proven guilty if you will ;)
>
> Still, riddle me this, how can a propagating wave suddenly switch to particle like behavior?
>
> And the weirdness of quantum mechanics persists.
>
>> Date: October 19, 2010 3:34 AM
>>
>> That has been tough to grasp for me as well, how do we understand that there is some mechanism for the wave to switch to particle behavior?
>>
>> We have only the wave equation collapse and probability which seem like the algorithms we discarded earlier in the semester for the "farmer and the seed". I know there isn't an answer yet of the process its what me have to accept for now since the math coincides with experimentation so perfectly. (My observations thus far)
>
>> Date: October 19, 2010 9:56 AM
>>
>> I've been thinking about the nature of photons and the like, and I've decided that "behaving like a particle/wave" doesn't say anything about what the photon actually is. These comparisons just give us something to relate them to, at certain times. Photons are in a category all their own, and behave like nothing we know classically.
>>
>>> Date: November 3, 2010 12:39 AM
>>>
>>> Like so much in our world: words can never suffice.
>>>
>>> It's just so very perturbing to me: the idea a wave acts like a wave when we want it to and vice versa with the particle. Why is the measurement so important? Have particles such as photons always acted this way even when we were ignorant of things not just at the quantum level, but at simply the cellular level? I sometimes wonder if the world behaves in a quantum manner just because we are observing it behave in a quantum manner, like the whole of existence is just a hypothetical wave in someone's photon experiment and there's a whole other particle-side out there which we don't know about. Is it just a question of making an effort to find it?
>>
>>> Date: November 9, 2010 7:14 PM
>>>
>>> I wholeheartedly agree. Light quanta is a concept used to explain certain phenomena we perceive in certain experiments, not the absolute truth. What the photon actually is can only be described in partially complete terms "wave or particle" that end up confusing the people.
>>>
>>> But light behaves in a so called "classical" manner, does it not? You perceive light all the time. As you are reading this light is stimulating nerves in your eyes. You know the effects of light well. So, do photons truly behave like nothing we know classically?
>>>
>>>> Date: November 15, 2010 9:49 PM
>>>>
>>>> We've discussed plenty of times that objects that were previously believed to have only "classical" properties behave in a quantum manner. Bucky balls for instance are quite "large" especially compared to an electron or photon and in general I would say that we would think of the Bucky ball behaving "classically." That said, we've seen interference patterns from them which is strictly a quantum behavior. What is your justification for light behaving "classically"? Remember that your retina is a measurement device and will destructively alter the quantum state of a photon.

II.D. Exam Materials

One learning goal for this section of the course was for students to be able to identify a perspective as being realist, and to have some facility with the arguments in favor or against any particular interpretation. Since our usual post-instruction essay question on the double-slit experiment had proven useful in our interviews (in terms of eliciting students' attitudes toward some interpretive themes), we thought it appropriate to adapt this question for the second midterm exam. The problem statement for the exam question was identical to its presentation in the post-instruction online survey, but here students were asked first to identify and characterize the assumptions of Student One in terms of the interpretations of quantum mechanics we had discussed in class:

Student A: Student One interprets this sequence of screen shots classically, he obviously is thinking of this problem not quantum mechanically because if he did he would think the electron is going through both slits at the same time although he is thinking of this in terms of the Bohr model a bit. I think this is because he knows that we don't know the true position of the electron which means he is also thinking of it in terms of the uncertainty principle too. He thinks classically because he thinks it can't go through 2 slits at the same time.

Student B: Student One believes that the electron is indeed just a particle the whole time, but is moving around so fast in a random way that we can't detect it. He does not believe in wave-particle duality of electrons. He does believe that there are hidden variables (i.e., position). He also does not believe that there is a superposition. Overall, he has a realist point of view that the electron has a specific path but we just don't know it.

Student C: Student 1 is taking a somewhat realist perspective. They are assuming the electron traveled through one slit or the other. They claim the reality of the situation is the particle-like electron existed in a cloud of probability, and passes through one slit or the other as the cloud moved through the double slits. This explanation does not mention the probability density predicted by the wave equation.

Student D: Student 1's statement is consistent with that of someone who holds realism to be true. He/she assumes that: 1) The electron was always a particle with a fixed position in space and time; and 2) The only reason that the probability field is so large is because we are unable to determine its position (a "hidden variable") prior to it striking the screen. Thus, he believes that the properties of the electron are always the same, but we (the observer) are only able to observe those properties under a given set of circumstances (when the particle hits the screen).

Like Student A, there were some students who didn't utilize the specific terminology we had developed in class (e.g., distinguishing only between *classical* and *quantum* thinking, or *particle* and *wave* perspectives, without employing terms like *realism*); virtually every single student was regardless able to recognize that Student One's belief in localized electrons was an assumption. The second part of the essay question asks students to list any rationale or evidence that favors or refutes the first two statements; and to explain whether the third statement is claiming the first two are wrong, and why such a stance might or might not be favored by practicing physicists:

Student A: For Student 1, I agree that the prob. density is large because we don't know position of the electron – we never do. I disagree that this can't be represented quantum mechanically. From experiments in the past it is proven that we get fringes (pattern).

For Student 2, I disagree that the electron is the blob because in the brighter part of the blob there is a higher probability that an electron will be detected than in the dimmer part. Although I agree the electron acts as a wave, I disagree that a single electron can be described as a wave packet.

The third student isn't saying the first 2 are wrong. All he is saying is that the interference patterns are a result of probability not classical physics and that both are right. We don't know how we get the results we do so we work with probabilities.

Student B: Since Student One believes that the electron was traveling within the blob and went through only one slit, he believes that electrons act as particles. This would mean that he would never observe interference. This is not true though because the experiment shows that over a long time, interference is observed. (Even the nickel atoms in a crystal lattice experiment shows this too.) Since Student 2 believes that the electron acts as a wave packet, he suggests that we have a small uncertainty in its position (and large uncertainty in its momentum). However, if we had a small uncertainty in its position, then we could later predict where it would show up on the screen. The double-slit experiment shows this. In other words, the blob doesn't represent the electron, but rather the probability density of the electron to be detected. Experiments show that we don't really know what the electron is doing before we detect it. Student 3 is indeed disagreeing with Students 1 & 2 by saying that Students 1 & 2 can't make some of their claims, as we really just can't tell what the electron is doing between being emitted from the gun and being detected on the screen. He might not be stating that Students 1 & 2 are necessarily wrong, but he says that quantum mechanics can't conclude their conclusions. A practicing physicist would most likely agree with Student 3 because it is consistent with the Aspect experiment for photons.

Student C: Student 2 describes the electron as a wave packet. When a double slit experiment is performed, the interference pattern that is observed corresponds to a probability density that can be described by a wave-packet equation. A packet of waves would interfere with itself, creating a probability of the electron to pass through both slits. Also, which slit the electron went through cannot be measured without altering the uncertainty in the momentum.

Student D: **Rationale/Evidence for Student 1 (aka EPR):**
Realism argument: all objects must have definite properties within the system regardless of observation. Location is real but hidden variable. Makes intuitive sense.

Against Student 1:
Idea of definite quantities for all states (Local Realism) does not hold to experiment. Probabilistic provides correct explanation, deterministic does not. Single-photon interference experiments.

Rationale/Evidence for Student 2 (aka Bohr):
Electron is a wave function that collapses to a determinate state at plate. Consistent with matter waves argument put forward by deBroglie. Allows for interference with only one electron.

Against Student 2:
Fails when applied quantitatively; no mechanism for wave collapse yet developed.

No, Student Three is simply stating the theory behind the interpretations put forth by the first two students. In other words, he is limiting his assessment of the experiment to what can be predicted and explained through existing QM theory. A practicing physicist would tend to agree with Student 3 because his description requires the least assumptions and adheres to what we know as opposed to what we postulate.

Once again, Student D offers a near textbook response. Student B employs standard arguments against a strictly particle view of electrons, and in favor of a wave representation, but is explicit in saying that the wave corresponds to the probability for where an electron might be found, and not the electron itself. He is also cognizant of the incompatibility of the two statements – it is not possible for both of the fictional students to be correct. Not every student saw these two views as contradictory, in the sense that they reduced the two statements down to simply representing either a particle view or a wave view, without considering how each statement makes an explicit assertion regarding the behavior of the electron at the slits – it either goes through one slit or it goes through both. In other words, not every student took a definitive stance on the question of whether an electron always passes through one slit or both, focusing more on the legitimacy of particle or wave views in this context.

Interestingly, Student A's response is an almost exact recapitulation of Student R3's reasoning in Chapter 4: they both agree the electron is somehow behaving like a wave in this experiment, but object to the idea that a wave packet can describe an individual particle. Student A also indicates a belief that we can never know the true position of an electron, hence the large probability density. At this stage, it seems that Student A is not yet *split* in his beliefs – he hasn't conceded that an authoritative stance trumps his intuitive views, and indeed implies that scientists might believe that Students One & Two are both right, and that we can't really know why we observe what we do. Student C is not explicit in arguing against

Student One, but instead explains why Student Two's description conforms to observation. As we shall see in the final portion of this exam question, Student C still believes in a continuously localized existence for electrons in this experiment:

(Part III) Which student(s) (if any) do you ***personally*** agree with? If you have a different interpretation of what is happening in this experiment, then say what that is. Would it be reasonable or not to agree with ***both*** Student 1 & Student 2? This question is about your personal beliefs, and so there is no "correct" or "incorrect" answer, but you will be graded on making a reasonable effort in explaining why you believe what you do.

Student A: I think from what I have learned in this class that Student 3 is correct. Probability can show us patterns but we really don't know what's going on before. It is reasonable to agree with both Student One who thinks classically and Student 2 who thinks quantum mechanically because that allows you to form your own ideas about what is going on but the truth is that we don't know what's going on between emission and the screen.

Student B: I personally believe that the electron acts like a wave until we observe it. This is Dirac's interpretation. Student 1 & Student 2 can't both be right because that would suggest that the electron acts like a wave and particle at the same time, and there is experimental evidence that refutes this.

Student C: Since electrons show both wave and particle like behavior, it would be reasonable to side with either Student 1 or 2. Student 2 used a more wave-like interpretation, Student 1 used a more particle like interpretation.

I personally visualize the situation as a flow of some fluid that travels through the two slits in waves. It appears through all space as soon as the electron is fired. The electron then rides this chaotic fluid toward the screen and strikes in a location that is somewhat determined by the interference patterns of the fluid. Trying to measure this fluid flow collapses the waves created.

Student D: I personally agree with Student 3. I see no reason to jump to a conclusion regarding the electron's behavior without a quantitative mechanism to explain its behavior between source and the plate. We know from this experiment that an electron exhibits behavior consistent with that of a wave, but we do not know exactly why or how that is so. That being said, I find Student 2's statement a more convenient way to think about the electron's behavior.

Student A merely restates his earlier stance: we require probabilistic descriptions because we can't really know what is going on between source and detection, and so either point of view might be equally legitimate. In the end, it seems this student is asserting his right to believe as he chooses when science has no definitive answer. At this point, we would characterize Student A as *Agnostic* – he recognizes the implications of competing perspectives, but is unwilling to take a stance on which might best describe reality.

Student B does not explicitly say which student he agrees with, but reports his belief in Dirac's matter-wave interpretation. Notice, however, that he says the electron *acts* like a wave, and not that an electron *is* a wave. Without further information from Student B, his views at this point might be consistent with either a *Quantum* or a *Copenhagen* perspective, since his stance on the reality of the wave function, and the nature of its collapse, is unclear.

We may easily place Student C within the *Pilot-Wave* category; indeed, his response sounds eerily similar to Student P3 (from Chapter 4) – the interference of nonlocal quantum waves determines the trajectories of localized particles. These two students arrived at the same conclusions independently; we made only cursory mention of Bohm's interpretation in our class, and it was not discussed at all in Student P3's class. This suggests that such ideas may be more prevalent among students than it seemed at first glance.

Student D's sentiments are not so different from Student A – it isn't known why quanta behave as they do, and so being agnostic requires the fewest assumptions (though he does mention that he finds it useful to employ a wave description in this situation). It seems reasonable to characterize Student D as subscribing to a *Copenhagen/Agnostic* perspective at this stage of the course.

The class as a whole performed well on this exam question: ~75% of students received full credit for their responses; the remaining students primarily lost one or more points (usually not more than three, from a total of ten points) for providing incomplete responses (very few students made any assertions that were unequivocally false). Overall, we would say that several of our learning goals surrounding this material were met by the majority of our students: they were able to identify the realist assumptions of the first fictional student, and to contrast them with an alternative perspective; they could provide evidence that favors or refutes competing points of view; and they were able to articulate their own beliefs regarding the interpretation of this quantum experiment. All of this regardless of whether they actually employed the exact terminology that had been developed in class (though most students did indeed use terms like *realism* and *hidden variables* in their argumentation). 18% of students chose to explicitly agree with Student One, though only one of them agreed with this statement exclusively; the remaining students were split between agreeing with both of the first two statements, or agreeing with all three. 46% of students said they agree with Student Two, or with both of the last two statements, while 36% preferred Student Three's statement exclusively.

II.E. Assessing Outgoing Perspectives

As part of their final homework assignment, students were asked to respond to the same post-instruction attitudes survey that had been administered in other courses. We report here the final class wide responses to each survey item, juxtaposed with how they responded at the beginning of the semester. We similarly offer complete responses from Students A, B & C. Student D did not respond to this final survey, but we shall hear from him again in our discussion of the final essay assignment below. [Section II.F]

1. It is possible for physicists to carefully perform the same measurement and get two very different results that are both correct.

	Agree	Neutral	Disagree
POST (N=90)	**0.78**	**0.06**	**0.17**
PRE (N=94)	**0.65**	**0.13**	**0.22**

Student A: **(Disagree)** Take the example of hidden variables. If you put one red sock and one blue sock into identical boxes and both socks are identical beside their color, and you send them across the universe, then your technically performing the same measurement. When you open one box you find out what color the sock is in that box and it can be either red or blue, two different results. At the same time you also know what is in the other box every time you perform the experiment, in that respect, you are kinda getting the same result.
(PRE: Agree)

Student B: **(Strongly Agree)** This is possible especially when it comes to measuring the position of an electron. This is because there is no definite position to begin with. All we can know is the probability of finding the electron in a particular position, but probability does not determine where the electron will be when we measure it.
(PRE: Agree)

Student C: **(Strongly Agree)** Two very different results could confirm the same fact. Being correct is nothing more than confirming a fact.
(PRE: Strongly Agree)

Students shifted towards more agreement with this question (and less neutrality), but drawing conclusions from overall agreement or disagreement should be done with caution, for there are quantum mechanical reasons for disagreeing with this statement. For example, it has been argued by students that, in practice, scientists perform a number of measurements in any given experiment, and it is the statistical distribution of data that is the final result, which should be always be the same for similar experiments:

> "…if we are measuring the position of an electron, we will measure a different position each time. But if we compile all our results we will find positions that correspond to the wave function. I strongly disagree with the above statement because if an experiment is performed correctly it should produce the same results!"

The distribution in Table 5.II of the kinds of reasoning invoked by students at pre- and post-instruction (by the same categorization scheme employed in Chapter 2) shows that students shifted dramatically in their preferences for deterministic and hidden-variable style thinking (Categories D & E). Students shifted from 47% to 17% in providing Category D & E responses (whether in agreement or disagreement). And while only 17% of students invoked quantum phenomena (Category A) at the outset of the course, 65% of post-instruction responses made reference to quantum systems. Most students agreed with this statement before and after instruction, but learning about quantum mechanics caused most of them to consider it in a new light. For example, Student B has confirmed his pre-instruction suspicion that quantum mechanics might allow for this statement to be true. Student A originally agreed because of wave-particle duality, but now disagrees through an example of hidden variables and classical ignorance. Student C strongly agreed in both cases, first providing a Category D response, and then one more consistent with Category C.

TABLE 5.II. Categorization (as in Chapter 2) and distribution of reasoning provided at pre- and post-instruction, in agreement or disagreement with the statement: *It is possible for physicists to carefully perform the same measurement and get two very different results that are both correct*; standard error on the proportion ≤ 5% in each case.

CATEGORY DESCRIPTION				
A	Quantum theory/phenomena			
B	Relativity/different frames of reference			
C	There can be more than one correct answer to a physics problem. Experimental results are open to interpretation.			
D	Experimental/random/human error Hidden variables, chaotic systems			
E	There can be only one correct answer to a physics problem. Experimental results should be repeatable.			
CATEGORY	**PRE-INSTRUCTION (N=94)**		**POST-INSTRUCTION (N=90)**	
	AGREE	**DISAGREE**	**AGREE**	**DISAGREE**
A	15%	2%	58%	7%
B	4%	0	0	0
C	13%	0	10%	0
D	29%	3%	9%	4%
E	1%	14%	0	4%
TOTAL	62%	19%	77%	15%

2. The probabilistic nature of quantum mechanics is mostly due to physical limitations of our measurement instruments.

	Agree	Neutral	Disagree
POST (N=90)	**0.18**	**0.21**	**0.61**
PRE (N=94)	**0.46**	**0.32**	**0.22**

Student A: **(Strongly Agree)** The probabilistic nature of quantum mechanics comes from the fact that there are aspects of quantum mechanics that can't be measured due to physical limitations of our measurement instruments. For instance how the uncertainty principle interacts with electrons orbiting a nucleus. Electrons are too small and move too fast for humans to know exactly where an electron is at a certain moment, so we can only perform one measurement at a time. Position and momentum of a particle can't be known at the same time, we can only calculate the probability of finding them there.
(PRE: Neutral)

Student B: **(Strongly Disagree)** It seems that the probabilistic nature of quantum mechanics is mostly due to the nature of sub-atomic particles rather than the limitations of our measurement instruments. If the particles were in definite states and definite positions to begin with, or even if there were a wave function that could define the exact state of the particles at any time, then one could argue that the problem is our measurement instruments. Perhaps such a formula will exist in the future, but that would mean that the limitation is our knowledge, not our instruments.
(PRE: Strongly Agree)

Student C: **(Neutral)** I have no idea.
(PRE: Neutral)

There was a strong shift away from agreement and in favor of disagreement by the end of the class; without passing judgment on students who feel neutrally towards this statement (after all, we do not consider agnosticism to be unsophisticated), we would at least like for our student to *not agree* with the notion that technology might one day reduce the need for probabilistic descriptions of quantum phenomena. Student B's response is desirable, in that he identifies uncertainty in quantum mechanics as fundamental, and not a consequence of experimental uncertainty. Student A's response is consistent with his reasoning on atomic electrons at the beginning of the course: their chaotic, rapid motion precludes knowledge of their true positions. We placed Student A in the *Agnostic* category at the time of the second exam, but we shall now see his explicit preference for realism:

3. When not being observed, an electron in an atom still exists at a definite (but unknown) position at each moment in time.

	Agree	Neutral	Disagree
POST (N=90)	**0.26**	**0.18**	**0.57**
PRE (N=94)	**0.72**	**0.09**	**0.19**

Student A: **(Strongly Agree)** Every physical thing exists whether it is being observed or not. This is the idea of realism, and I completely agree with it. An electron is a particle therefore I believe that it has a physical manifestation. An electron will definitely still exist at a definite position at every moment in time. This correlates with my answer above.
(PRE: Strongly Agree)

Student B: **(Disagree)** This thought process only makes sense if one were to view electrons as particles (like billiard balls). However, we know from experimentation that the electron has wave-like properties and can be described in the form of an electron cloud (Schrodinger's model). Thus, we can have an idea of where we are likely to find the electron if we make a measurement, but when we don't make a measurement, the electron should not be acting like a particle. But then again, we can't be 100% sure of what's happening when we aren't measuring...
(PRE: Strongly Agree)

Student C: **(Neutral)** If an electron orbits a nucleus in a forest and no physicist is there to observe it, does it obey the uncertainty principle?
(PRE: Agree)

As with the second survey item, we would have liked for our students to *not* choose to agree with this statement, and only 26% of them did by the end of the semester. We may not infer too much from Student C's tongue-in-cheek response, except to suggest his neutral attitude implies this question may now have as little (or as much) meaning to him as considering the sound of one hand clapping – at a minimum, his response has shifted away from agreement. In his disagreement, Student B explicitly addresses the wave-like properties of atomic electrons, though he also expresses a modicum of tentativeness in his beliefs.

Even though Student A has come through this course with explicitly realist notions intact (perhaps even reinforced), we would still consider his response to be in keeping with at least some of our learning goals: he has given conscious consideration to his intuitive beliefs and confirmed them to himself, and he can now articulate those beliefs in terms of language that been previously unavailable to him. At the very least, he did not use such language in his pre-instruction responses, which focused more on the tentativeness of scientific knowledge. Let us consider these students' last thoughts on the double-slit experiment before drawing any final conclusions on their overall outgoing perspectives:

Student A: I agree with Student 1 mostly except for the fact that the electron could be going through both slits at the same time for all we know. I also agree with student 2 because I think that the electron is acting as a wave and again possibly go through both slits at the same time. Therefore I agree more with student 3 because we really don't know what is happening between the moment the electron is shot from the gun and it hits the detection screen.

Student B: I agree with student three because it seems that the electron can act as a wave until we observe it. Even if this isn't the reality, there's nothing we can know about it from when the electron is emitted to when it is detected. However, student one and student two cannot be both correct because the electron cannot act like a wave (student 2) and a particle (student 1) at the same time, because there is experimental evidence that refutes this.

Student C: Student One is assuming the electron is always a particle. Student Two is assuming that the electron is pretty much a wave until it gets smooshed by the screen. Student three is sticking to the fact that the electron has a probability of going in certain places on the screen. I think there will always be a more accurate description of observations and quantum mechanics is, for now, an accurate description of reality.

And so it would have been premature to consider Student A to be a confirmed *Realist*, seeing how he maintains an explicit tentativeness regarding what can actually be known in this experiment, and so we might best characterize his overall final responses as *Realist/Agnostic.* Student B's earlier exam responses placed him somewhere between the *Quantum* and *Copenhagen* categories, but his overall language has consistently referred to the *behavior* of quanta, and he has explicitly refused to equate the wave with the particle it describes. Considering his final agreement with Student Three, and his concession that a wave description of quanta may ultimately not conform to reality, Student B's outgoing perspective on quantum mechanics is most consistent with the *Copenhagen* category. Student C's final response requires some thought: we believe he is suggesting there will one day be a *more accurate* description of reality, but that quantum mechanics is currently a *sufficiently accurate* description of that reality, and so we don't interpret his response as implying that quantum mechanics is necessarily incomplete. Student C expressed beliefs in non-local realism at mid-semester, and we did not ask him for his own interpretation of the double-slit experiment in the post-instruction survey, but his overall final response indicate he would be best described as being in the *Agnostic* category.

A final look at the overall class responses to this post-instruction essay question, in conjunction with their responses on atomic electrons, provides some insight into the consistency of student perspectives, which was part of our original motivations for our investigations. [Chapter 2] Only five of the 87 students who provided clear responses to this survey item explicitly agreed with Student One, and three of them did so in their expression of agreement with all three statements. Of these five students, three of them agreed with the statement on atomic electrons, one was neutral, and the other replied in disagreement. This means that 23% of students who chose to *not agree* with Student One in the double-slit experiment essay question offered a response to the statement on atomic electrons that would be consistent with realist expectations. Even though we are only considering five students here (meaning there is significant statistical error), we note that this distribution of responses on atomic electrons for students who had expressed realist preferences in the double-slit experiment matches our findings in Chapter 2 exactly. We also note that this 23% (±4%) of students evidencing inconsistent thinking across these two contexts is significantly less than the 33% (±6%) found in our initial studies ($p<0.001$, by a one-tailed t-test). We believe these results allow us to conclude that another of our learning goals had been achieved for a majority of our students – the consistency of student perspectives between these two contexts has been significantly increased over prior incarnations of modern physics courses.

We conclude this section by considering the level of personal interest in quantum mechanics expressed by students at the end of the semester:

4. I think quantum mechanics is an interesting subject.

	Agree	Neutral	Disagree
POST (N=90)	0.98	0.02	0.0
PRE (N=94)	0.85	0.13	0.02

Student A: **(Strongly Agree)** I found quantum mechanics to be an interesting subject because the concepts around it are not proven. A lot of what is behind quantum mechanics is qualitative which is very different than most physics classes which are quantitative. It is nice to look at a complex subject such as physics from a qualitative manner because for the past two years I've been taking all engineering classes which are all involving math significantly.
(PRE: Strongly Agree)

Student B: **(Strongly Agree)** The fact that there are truths associated with quantum mechanics that still can't be explained is a very interesting concept. I have never been taught something in school that is proven in experiments but still lacks a proper reasoning (such as entanglement). I also think it's very interesting to learn how sub-atomic particles behave so differently than macroscopic particles.
(PRE: Strongly Agree)

Student C: **(Strongly Agree)** Quantum mechanics is strange and interesting and mind stretching. This has been a great course.
(PRE: Neutral)

We find it remarkable that virtually every student expressed an interest in quantum mechanics by the end of the course, and that only two students responded neutrally – these final numbers are contrary to the usual decrease in interest among engineering students, and are on par with what is typically seen in a course populated with physics majors, where it is fairly safe to assume that nearly every student is already interested in learning about quantum mechanics coming into the course. [Chapter 6.] Still, considering the relatively high rate of incoming interest in quantum mechanics for students from our course, it is not entirely clear how effective we were in influencing student attitudes without considering a more detailed breakdown of their responses. In all other cases, *agreement* and *strong agreement* had been collapsed into a single category, and similarly for *disagreement* and *strong disagreement*; we therefore consider the number of students who became *more emphatic* in their agreement. Initially, 32% of students merely agreed that quantum mechanics is an interesting subject, and 53% were in strong agreement – these numbers shifted by the end of the course to 20% and 78%, respectively. We may therefore conclude that this curriculum, as implemented, was successful in not only maintaining student interest in physics, but in promoting it as well. As a final comment, we note that Students A, B & C all express a strong interest in the subject, and their responses suggest that it is precisely the still-open questions in quantum mechanics that inspire their fascination – Pandora's Box has been opened, and we don't have to be afraid!

II.E. Final Essay

In lieu of a long answer section on the final exam, students were asked to write a 2-3 page (minimum) final essay on a topic from quantum mechanics of their choosing, or to write a personal reflection on their experience of learning about quantum mechanics in our class (an option chosen by ~40% of students). As opposed to a formal term paper, this assignment was meant to give students the opportunity to explore an aspect of quantum mechanics that was of personal interest to them. Topics selected by students for their final essays (ones that were not personal reflections) included: quantum cryptography; quantum computing; enzymatic quantum tunneling; bosons and fermions; the Quantum Zeno Effect; string theory; atomic transistors; quantum mechanics in science fiction; and more… The nearly universally positive nature of the feedback provided by students in their personal reflections is evidence for the popularity and effectiveness of our transformed curriculum, and its practical implementation. [Excerpts from *each* of the submitted personal reflections from the Fall 2010 semester are collected in Appendix E.]

We recall from earlier in this chapter that Student D had entered this course with a relatively sophisticated view on quantum mechanics, but one that was explicitly realist/statistical. We are interested, of course, in whether this curriculum has something new to offer students with a high degree of background knowledge coming into the semester. Though he did not complete the end-of-term attitudes survey, we may still draw some conclusions regarding the effectiveness of this curriculum at influencing Student D's interpretive stances:

> "Upon entering the class, I was most excited to learn about the various interpretations put forth to explain quantum mechanical phenomena. I already had a fairly strong footing in the actual mathematics of the material, both from my own independent studies and from an exceptional AP Physics course I had taken in my senior year in high school. However, neither of those pursuits had given me a strong grounding in the overarching theoretical principles behind the material, especially when it came to interpreting the experimental data in the more recent work such as Aspect's single photon experiments and electron diffraction. I came in understanding the results of those experiments, but not their implications for the nature of light and matter. This class did a fantastic job of patching those holes in my understanding. […] Although this class has not significantly changed my ideas about physics and the practice of science, it has been one of the few courses I have taken that accurately portrays the scientific method of careful observation. The course was exceptional in how it handled conclusions drawn from experimental results, the most memorable example being the refutation of the "hidden variable" interpretation. The class was at its best when discussing the interpretations of experiments and the implications of their results; Aspect's single photon experiments were explained with particular clarity and care."

We may not know precisely how Student D would have responded to the post-instruction survey, but we may infer from his statements that he no longer personally subscribes to the notion of *hidden variables*. We assert that Student D successfully transitioned from a *Realist*/*Statistical* perspective on quantum mechanics, to one that is more aligned with the beliefs of practicing physicists (*Copenhagen*).

References (Chapter 5):

1. S. B. McKagan, K. K. Perkins and C. E. Wieman, Reforming a large lecture modern physics course for engineering majors using a PER-based design, *PERC Proceedings 2006* (AIP Press, Melville, NY, 2006).

2. W. K. Adams, K. K. Perkins, N. Podolefsky, M. Dubson, N. D. Finkelstein and C. E. Wieman, A new instrument for measuring student beliefs about physics and learning physics: the Colorado Learning Attitudes about Science Survey, *Phys. Rev. ST: Physics Education Research* **2**, 1, 010101 (2006).

3. E. Redish, J. Saul and R. Steinberg, Student expectations in introductory physics, *Am. J. Phys.* **66**, 212 (1998).

4. S. B. McKagan, K. K. Perkins and C. E. Wieman, Why we should teach the Bohr model and how to teach it effectively, *Phys. Rev. ST: Physics Education Research* **4**, 010103 (2008).

5. R. Hake, Interactive-Engagement Versus Traditional Methods: A Six-Thousand-Student Survey of Mechanics Test Data for Introductory Physics Courses, *Am. J. Phys.* **66** (1), 64 (1998).

6. L. C. McDermott, Oersted Medal Lecture 2001: "Physics Education Research – The Key to Student Learning," *Am. J. Phys.* **69**, 1127 (2001).

7. S. B. McKagan, K. K. Perkins, M. Dubson, C. Malley, S. Reid, R. LeMaster and C. E. Wieman, Developing and Researching PhET simulations for Teaching Quantum Mechanics, *Am. J. Phys.* **76**, 406 (2008).

8. http://phet.colorado.edu/simulations/sims.php?sim=QWI

9. C. Davisson and L. H. Germer, Diffraction of electrons by a crystal of nickel, *Phys. Rev.* **30**, 705 (1927).

10. S. Frabboni, G. C. Gazzadi, and G. Pozzi, Nanofabrication and the realization of Feynman's two-slit experiment, *App. Phys. Letters* **93**, 073108 (2008).

11. A. Tonomura, J. Endo, T. Matsuda, T. Kawasaki and H. Exawa, Demonstration of single-electron buildup of an interference pattern, *Am. J. Phys.* **57**, 117 (1989).

12. http://rdg.ext.hitachi.co.jp/rd/moviee/doubleslite.wmv

13. New lecture materials and summaries are available online at:
http://www.colorado.edu/physics/EducationIssues/baily/dissertation/

14. V. Otero, S. Pollock and N. Finkelstein, A Physics Department's Role in Preparing Physics Teachers: The Colorado Learning Assistant Model, *Amer. J. Phys.* **78**, 1218 (2010).

15. R. D. Knight, *Physics for Scientists and Engineers: A strategic approach*, 2nd Ed. (Addison Wesley, San Francisco, CA, 2004).

16. A. Shimony, The Reality of the Quantum World, *Scientific American* (January 1988, pp. 46-53).

17. N. D. Mermin, Is the moon there when nobody looks? Reality and the quantum theory, *Phys. Today* **38** (4), 38 (1985).

18. D. F. Styer, *The Strange World of Quantum Mechanics* (Cambridge University Press, Cambridge, 2000).

19. E. Cataloglu and R. Robinett, Testing the development of student conceptual and visualization understanding in quantum mechanics through the undergraduate career, *Am. J. Phys.* **70**, 238 (2002).

20. S. B. McKagan and C. E. Wieman, Exploring Student Understanding of Energy Through the Quantum Mechanics Conceptual Survey, *PERC Proceedings 2005* (AIP Press, Melville, NY, 2006).

21. S. Wuttiprom, M. D. Sharma, I. D. Johnston, R. Chitaree and C. Chernchok, Development and Use of a Conceptual Survey in Introductory Quantum Physics, *Int. J. Sci. Educ.* **31 (5)**, 631 (2009).

CHAPTER 6

Teaching Quantum Interpretations – Comparative Outcomes and Curriculum Refinement

I. Introduction

In the previous chapter, we considered the design and implementation of a transformed modern physics curriculum for engineers, taught at the University of Colorado in the Fall 2010 semester. The accessibility and effectiveness of this new curriculum was discussed in terms of some measures that were entirely new and specific to that course – student responses to homework and exam questions relevant to the physical interpretation of quantum mechanics. But we have also gauged learning outcomes according to measures that had been employed in prior studies, (Chapter 3) and so we shall address in this chapter how some of the outcomes for this transformed course compare with three previous modern physics offerings.

Naturally, the outcomes from this course would be less significant if our learning goals had not represented a challenge for our students, or for ourselves as instructors and curriculum designers. Any newly implemented curriculum will certainly have need for refinement, requiring first the identification of specific student difficulties with the new material, which may then inform our suggestions for improvement. In light of our focus throughout this dissertation, it seems most appropriate to discuss problems students had in understanding the single-photon experiments, as revealed through their responses to another long-answer exam question from the second midterm. At the same time, we may also assess their use of some of the epistemological tools we had worked to establish in lecture. We will also consider aggregate and individual student responses to several of the multiple choice questions from our exams and the post-instruction content survey, which may indicate other student difficulties requiring future study.

II. Comparative Outcomes

We have already seen how certain instructional approaches with respect to interpretation can be associated with specific student outcomes (e.g., there is a greater prevalence of realist beliefs in contexts where instruction has been less explicit in promoting an alternative perspective, or in topic areas where realist/statistical interpretations were deliberately promoted). There are many similarities between our course from Fall 2010 and the four courses discussed in detail in Chapter 3, Section 3.II - they were all large-lecture courses (N > 60) where interactive engagement was employed during class, and covered roughly the same progression of topics from quantum mechanics and its applications. And all but the course taught from a realist/statistical perspective utilized many of the same lecture materials that had been developed during the first round of course transformations in 2005-2007. Yet they all differed in their instructional approaches to interpretation, though we would say that Course B2 (as denoted in Section 3.II) was most similar to our own, in that the instructor was explicit in promoting a matter-wave interpretation of the double-slit experiment, and significant lecture time was given toward the very end of the semester to discussions of measurement and interpretation in quantum mechanics (but without specific reference to atomic systems). There were no significant differences in the wording or presentation of the online attitudes survey administered in each course. Before making direct comparisons of student outcomes, we first (briefly) remind ourselves of our characterizations of the courses with which we'll be making our comparisons, and establish how they will be denoted in this chapter. [Table 6.I]

TABLE 6.I Summary of the four courses to be compared in this section, including a characterization of each instructor's approach to interpretive themes. For reference, how each course was denoted in Chapter 3 is also included [n/a = not applicable].

STUDENT POPULATION	COURSE	INSTRUCTIONAL APPROACH	CH. 3 DENOTION
Engineering	ENG-FA10	*Matter-Wave*	n/a
	ENG-R/S	*Realist/Statistical*	A
	ENG-MW	*Matter-Wave*	B2
Physics	PHYS-C/A	*Copenhagen/Agnostic*	C

ENG-R/S is the only engineering class considered in our studies that was taught from a realist/statistical perspective. ENG-MW is the engineering course most similar to ours (ENG-FA10), in that similar lecture materials were used, a matter-wave perspective was promoted, and interpretive themes were discussed near the end. PHYS-C/A is a class for physics majors that also used many of the same lecture materials, but with less emphasis on interpretation.

II.A. Student Interest in Quantum Mechanics

It is now well known in physics education research that student attitudes toward physics have a tendency to become less positive after instruction in introductory courses of all kinds, including ones where specific attention had been paid to student attitudes and beliefs. [3, 4] Similar effects have been seen in modern physics courses; one study showed that traditional modern physics instruction typically led to significant negative shifts in student attitudes (as measured by the CLASS [4]), while a curriculum transformed using principles from PER saw no significant pre/post-instruction shifts, meaning overall student attitudes at least did not get worse. [1] By combining pre-instruction survey responses on their reported interest in quantum mechanics from six semesters of engineering courses (including the Fall 2010 semester), we see that incoming interest for engineers is on average between 75-80% favorable. [Fig. 6.1] The average post-instruction interest among engineering students from five of these course offerings dropped to below 70%, while negative responses increased significantly ($p<0.001$) – approximately 1/3 of engineering students would not agree that quantum mechanics is an interesting subject after having learned about it in our modern physics courses! Students from the Fall 2010 semester were nearly unanimous (98%) in their reported interest in quantum physics, and not one student responded with a negative opinion. [Relative to the number of students who completed the final exam, the response rate for our post-survey was ~90%.]

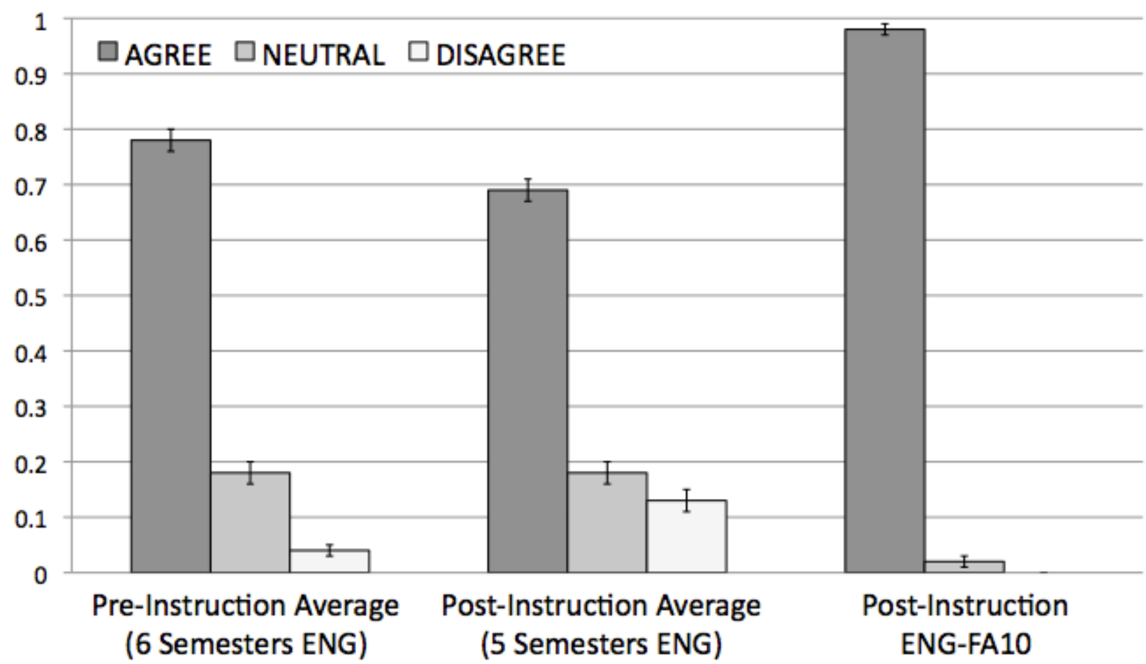


FIG. 6.1. Average pre- and post-instruction student responses to the statement: *I think quantum mechanics is an interesting subject*, from five modern physics courses for engineers, plus the FA10 semester. [Error bars represent the standard error on the proportion; N ~ 50-100 for each course].

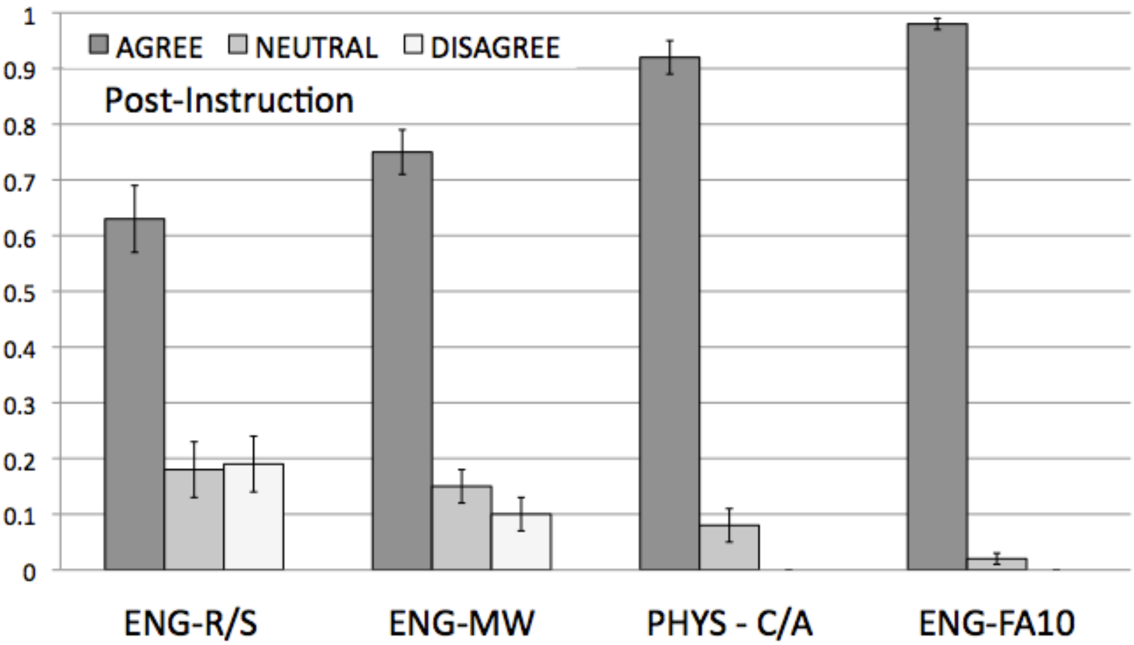


FIG. 6.2. Post-instruction student responses to the statement: *I think quantum mechanics is an interesting subject*, from four modern physics offerings, as denoted in Table 6.I. [Error bars represent the standard error on the proportion; N ~ 50-100 for each course]

It would be too great an assumption to conclude that shifts in student interest are necessarily directly correlated with the interpretive approach of the instructor, or with the student population. There are surely myriad other affective considerations, such as instructor popularity or choice of textbook, and we have seen courses for physics majors where overall interest in quantum mechanics declined. Nonetheless, we note that the Fall 2010 course had the greatest proportion of students reporting positive post-instruction attitudes towards quantum mechanics, including the course for physics majors; [Fig. 3.2] and that end-of-term student evaluations from ENG-MW, PHYS-C/A and ENG-FA10 ranked all of those instructors in the top 25%, relative to departmental averages (the instructor for ENG-R/S was ranked lower, at 32%). Different results were achieved by instructors of comparable popularity, and the responses from students to the newly introduced topics were overwhelmingly positive, which leads us to conclude that the new curriculum was at least partly responsible for the increased popularity of the course.

II.B. Interpretive Attitudes

We may assess the relative impact of our transformed curriculum on student perspectives by further considering their post-instruction survey responses in relation to outcomes from previous modern physics offerings. The overall distribution of student responses from our course to the double-slit essay question is consistent with prior results, which had shown them to be generally reflective of each instructor's specific approach to that particular topic, whether *Realist*, *Quantum* or *Agnostic*. [Fig. 6.3] Considering this question had been adapted for use on the second exam, and that exam solutions detailing "acceptable" responses were later available online, it might be reasonably argued that the near absence of student preference for a realist interpretation of this experiment is mere confirmation of the effect of explicit instruction in that context.

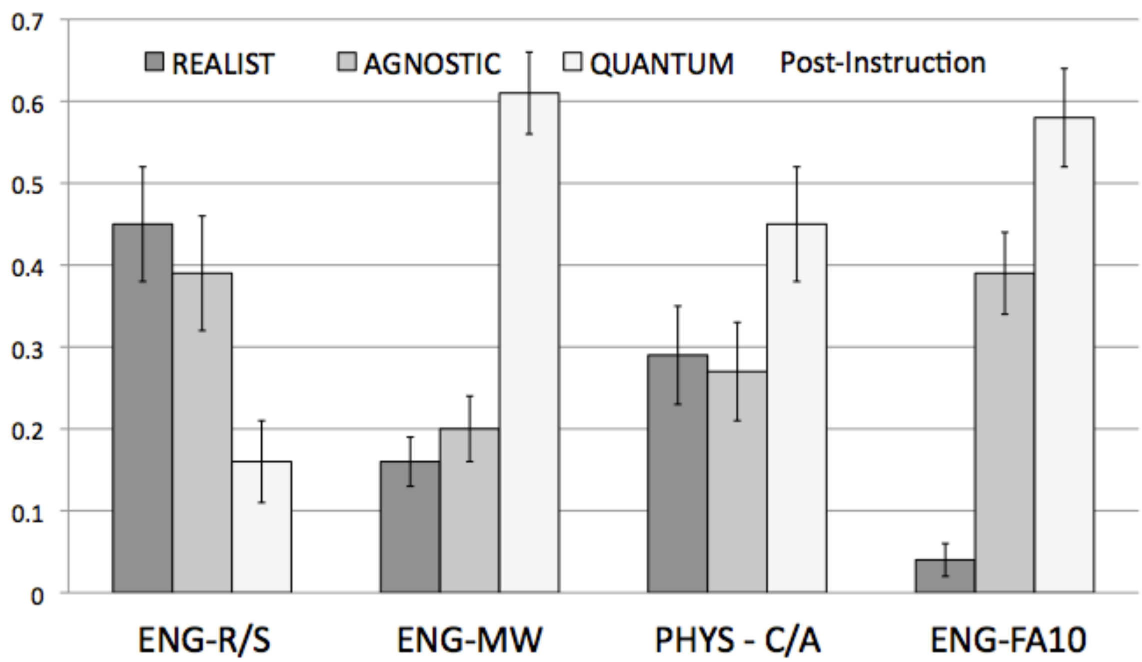


FIG. 6.3. Post-instruction student responses to the double-slit essay question, from four different modern physics courses, as denoted in Table 3.I. [Error bars represent the standard error on the proportion; N ~ 50-100 for each course.]

However, we made no mention during the entire semester of student responses to the pre-instruction attitudes survey, and did not give students any indication they would be revisiting these questions at the end of the course. We offered no explicit instruction as to what kinds of responses would be considered "acceptable", and repeatedly emphasized in the survey and in the homework assignments that we were most interested in what students actually believed. The lecture materials used during our treatment of the Schrödinger model of hydrogen were essentially the same as those used in ENG-MW and PHYS-C/A, with a few notable exceptions. Like the instructors for those two courses, we showed students how the Schrödinger model predicts zero orbital angular momentum for an electron in the ground state, and contrasted this result with the predictions of Bohr and de Broglie. But we continued by explicitly arguing how this result has implications for the physical interpretation of the wave function – for how could conservation of angular momentum allow for a localized particle to exist in a state of zero angular momentum in its orbit about the nucleus? This difficulty is removed when we choose to view atomic electrons as delocalized standing waves in quantized modes of vibration. More importantly, having already established language and concepts specific to interpretive themes, we were able to explicitly identify the position of an atomic electron as yet another example of a hidden variable, which we had argued couldn't exist as a matter of principle. Ours is the only course among these four where a significant majority of students chose to disagree with the idea of localized atomic electrons at the end of the semester. [Fig. 6.4]

The instructor for ENG-R/S told students during lecture that they *should* think of atomic electrons as localized, and overall responses from his course reflect this instruction. More specifically, he explained that quantized energy levels represent the average behavior of electrons over a time scale that is long relative to their orbital frequency, and that atomic electrons may be found to have a continuous range of energies when the time scale of the energy measurement is short (as enforced by the time-energy uncertainty relation); hence the broadening of spectral lines. This kind of reasoning is not unique among physicists, [5] and has therefore likely been utilized by modern physics instructors elsewhere.

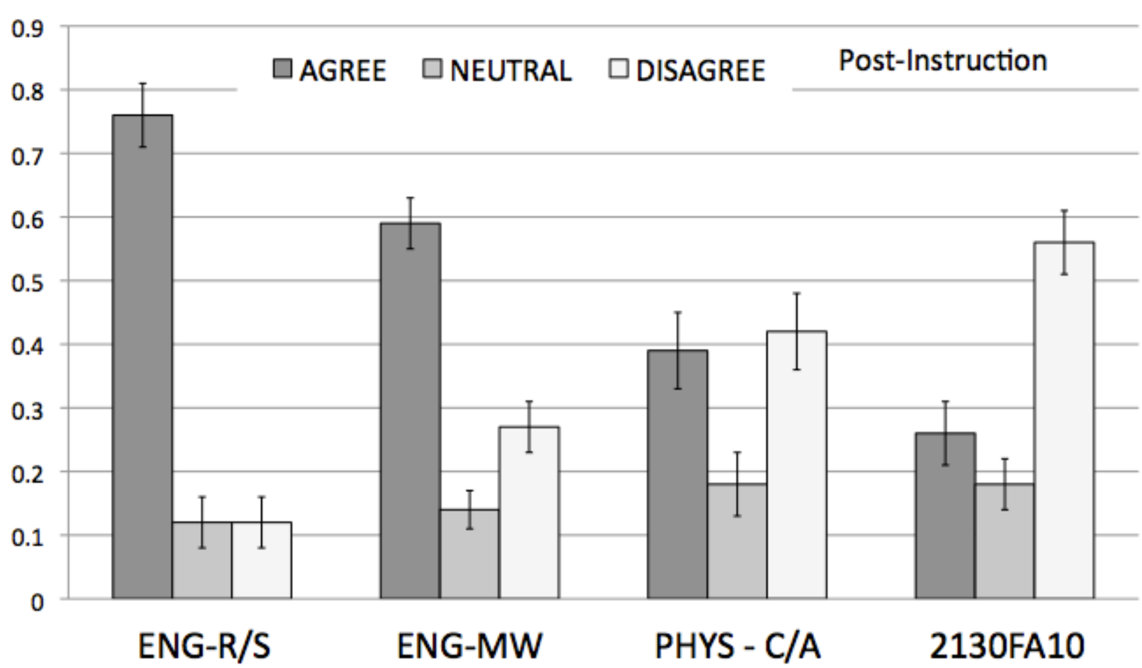


FIG. 6.4. Post-instruction student responses to the statement: *When not being observed, an electron in an atom still exists at a definite (but unknown) position at each moment in time*, from four different modern physics courses, as denoted in Table 3.I. [Error bars represent the standard error on the proportion; N ~ 50-100 for each course.]

We have previously characterized the other two courses as having de-emphasized matters of interpretation in the latter parts of the course, [Chapter 3] and heard from one instructor [PHYS-C/A; Instructor C in Chapter 3] about what had influenced his instructional choices – he felt that giving students a facility with the mathematical tools of quantum mechanics should take precedence over a detailed exploration of its physical interpretation, which might anyways be beyond the sophistication of introductory students. Though different in his overall interpretive approach, it turns out the other instructor [ENG-MW; Instructor B2 in Chapter 3] offered similar reasoning for having made a similar choice, and so it is worthwhile to consider one last time in detail what we consider to be a common motivation for the de-emphasis of interpretive themes in introductory modern physics courses, according to the instructor for ENG-MW:

> "This [probabilistic] aspect of quantum mechanics I feel is very important, but I don't expect undergraduate students to grasp it after two months. So that's why I can understand why [the statement on atomic electrons] was not answered to my satisfaction, but that was not my primary goal of this course – not at this level. We don't spend much time on this introduction to quantum mechanics, and there are many aspects of it that are significant enough at this level – it is really great for students to understand how solids work, how does conductivity work, how does a semiconductor work, and these things you can understand after this class. If all of the students would understand how a semiconductor works, that would be a great outcome. I feel that probably at this level – especially with many non-physics majors – I think that's more important at this point. But still, they have to understand the probabilistic nature of quantum mechanics, and I hope, for instance, that this is done with the hydrogen atom orbitals, not that everyone would understand that, but if the majority gets it that would be nice. These are very hard concepts. At this level, I feel it should still have enough connections to what they already understand, and what they want to know. They want to know how a semiconductor works probably much more than where is an electron in a hydrogen atom. […] I don't think the [engineering] students will be more successful in their scientific endeavors, whether it's a personal interest or career, by giving them lots and lots of information about how to think of the wave function. The really important concept I feel is to see that there is some sort of uncertainty involved, which is new, which is different from classical mechanics. […] At the undergraduate level, I feel it is important to make the students curious to learn more about it – and so even if they don't understand everything from this course, if they are curious about it, that's more important than to know where the electron really is, I think."

We see the instructor for ENG-MW *would have liked* for his students to disagree with this statement, and yet 75% of them chose to *not disagree*. Recall that this instructor made his own modifications to the first modern physics curriculum, to include an entire lecture on quantum measurement and interpretation towards the end of the course (but without specific reference to atomic systems).

At the end of the introduction to matter waves, our transformed course and ENG-MW both utilized a lecture slide similar to the one shown in Fig. 6.5 – note that both courses offered similar explicit guidance, albeit decontextualized, on how to think of electrons when not being observed: as delocalized waves. We believe this kind of general guidance is not by itself sufficient to cause most students to reconsider their conceptions of atomic electrons, as evidenced by the distribution of responses from a course that did not apply more specific guidance in the context of atoms. [Fig. 6.4] But specifically telling students to think of atomic electrons as delocalized would also not by itself be sufficient for significantly influencing students' overall perceptions of uncertainty in quantum mechanics.

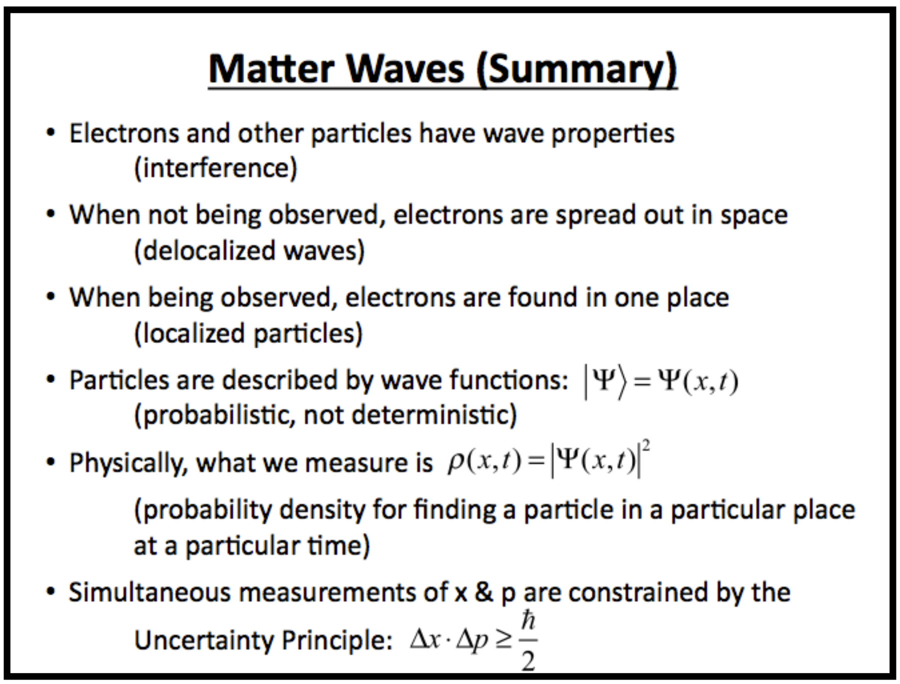


FIG. 6.5. A lecture slide equivalent to one used in each of two modern physics courses for engineers, ENG-MW & ENG-FA10. This slide offers explicit, but decontextualized, guidance on how to think of matter when not being observed.

We may conclude this from our observation that explicit instruction in one context does not necessarily influence student perspectives in other contexts, but also by other considerations. Even if the physical interpretation of atomic wave functions is not a primary learning goal for every instructor, we may safely say that our course shared with ENG-MW and PHYS-C/A a common learning goal that *was* primary: recognizing a difference between the experimental uncertainty of classical mechanics and the fundamental uncertainty of quantum physics. How do these four courses compare with respect to student responses to our last attitudes statement on the probabilistic nature of quantum mechanics? Realist expectations might lead incoming students to favor agreement with the statement: *The probabilistic nature of quantum mechanics is mostly due to the limitations of our measurement instruments.* We find that the incoming percentage of students from all three of the engineering courses agreeing with this statement is nearly identical, [ENG-R/S: 45%; ENG-MW: 48%; ENG-FA10: 46%] but that incoming attitudes for physics majors were significantly more favorable (with only a quarter of them agreeing, and over half disagreeing before instruction).

It would seem from his explanation of atomic energy quantization that the *Realist/Statistical* instructor would consider the uncertainty in quantum mechanics as being introduced by the *measurement process*, which is not the same as asserting that quantum uncertainty is experimental in origin, or that technology might one day find a way around these fundamental limits on observation. Regardless, there was a mild uptick in students from his course agreeing with this survey statement at the end of the semester. [Fig. 6.6] The instructor for PHYS-C/A had the greatest proportion of favorable responses at post-instruction – despite a de-emphasis on interpretive themes, he was successful in positively influencing student perspectives on uncertainty in quantum mechanics, though we must keep in mind the student population of his course, and the already relatively favorable incoming attitudes of his students.

In fact, the differential impact on student responses from these four modern physics courses is most dramatically illustrated by normalizing (post – pre) shifts in student agreement with this survey statement, according to their rate of agreement at the start of the course.[1] [Fig. 6.7] By this measure, our course had the greatest positive impact on student attitudes regarding the relationship between fundamental uncertainty in quantum mechanics and classical experimental uncertainty.

We conclude this section with some comments on the statements of the instructor for ENG-MW, regarding what we might like for students to take away from our introductory courses. First, if the aim of instruction is not necessarily a universal understanding of concepts, but for students to come away with a continued interest in modern physics, then we would claim that our course was the more successful of the two: student interest in quantum mechanics increased from 70% to 75% for his course (with 10% responding negatively at post-instruction), but the reported interest among students from our course increased from 85% to 98%, which we have argued must be in part attributable to the transformed curriculum itself. Second, we shouldn't presume to know exactly where the interests of our engineering students lie. The results from our curriculum implementation would suggest that students are in fact *just as interested*, if not more so, in questions about the nature of reality, as they are in learning about the theory of semiconductors.

[1] We define *favorable gain* as the negative of this, since we consider a *decrease* in agreement with this statement as favorable. This definition is equivalent to the definition of *normalized gain* = (post – pre)/(1 – pre), except that the target response rate is zero, and not 100%.

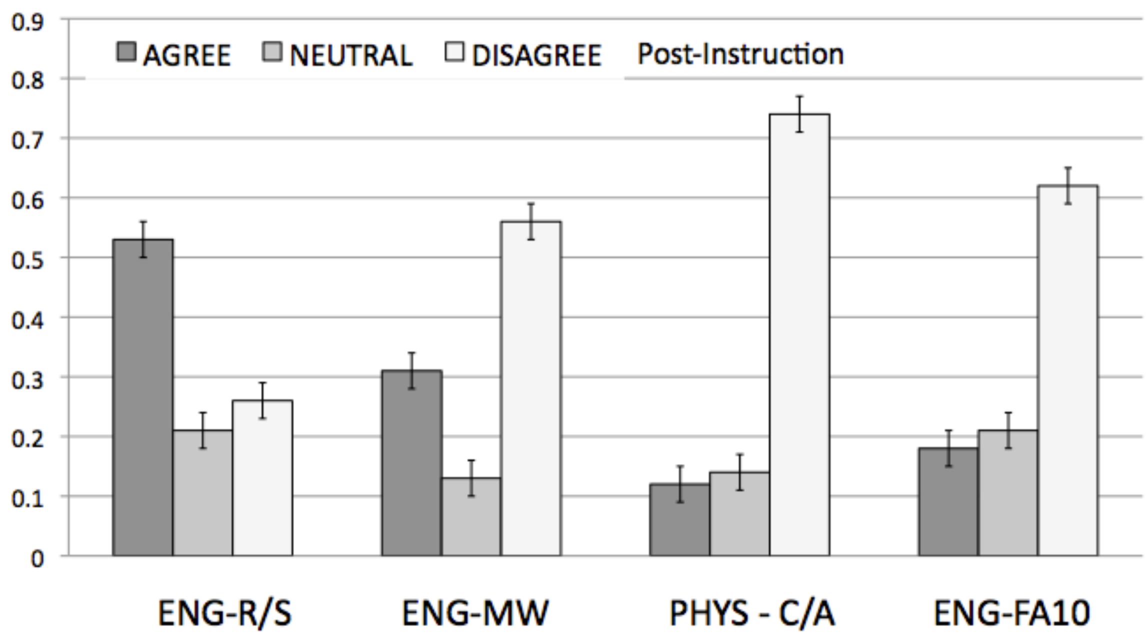


FIG. 6.6. Post-instruction student responses to the statement: *The probabilistic nature of quantum mechanics is mostly due to the limitations of our measurement instruments,* from four different modern physics courses, as denoted in Table 3.I. [Error bars represent the standard error on the proportion; N ~ 50-100 for each course.]

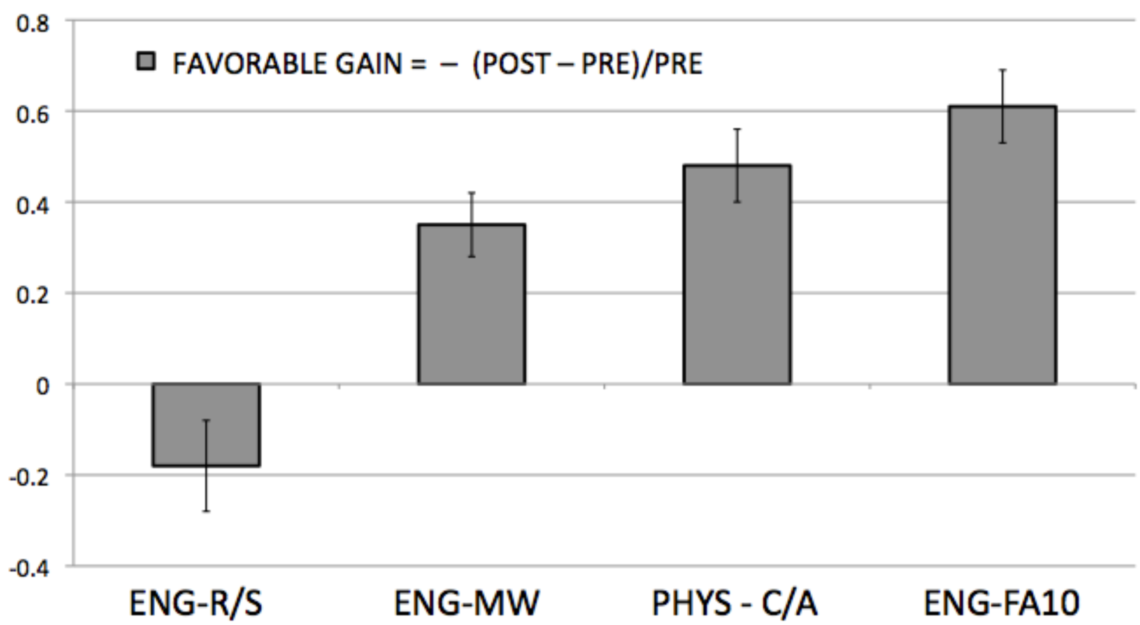


FIG. 6.7. Normalized favorable gain in the (post – pre) rate of student agreement with the statement: *The probabilistic nature of quantum mechanics is mostly due to the limitations of our measurement instruments,* from four different modern physics courses, as denoted in Table 3.I. A positive favorable gain is defined as a *decrease* in agreement with this statement. [N ~ 50-100 for each course.]

And finally, we didn't just give our students lots and lots of information about how to think of the wave function – we also gave them lots and lots of information about molecular bonding, conduction banding, semiconductors, transistors and diodes; as well as lasers, scanning tunneling microscopes, and nuclear energy; not to mention applications of nonlocality to quantum cryptography and computing. We had time for this because our course omitted topics from special relativity, which have generally cost other modern physics courses a minimum of three weeks out of a 15-week semester. We wouldn't claim that special relativity is not a relevant and worthy topic for engineering students, but the original decision to omit special relativity was in part a response to an overall consensus among engineering faculty at the University of Colorado, that their students would be better served by a curriculum that emphasized the quantum origins of material structures, and other real-world applications. [1] Every modern physics instructor at CU has had the option of removing special relativity from the engineering curriculum, and its re-emergence following the first round of course transformations is symbolic of a deep sense of tradition surrounding the topic, and stands in recognition of the profound influence its development has had on modern scientific thinking.

Our students had ample opportunity to contemplate the myriad contributions of Einstein's genius to the twentieth-century, but many of them were even more fascinated by the idea that Einstein could have been wrong about *anything*! And his glory was in no way diminished by telling our students this story of his confusion; for as we wove this tale of classical and quantum reality, he became a champion for those who expressed a deep commitment to their intuitions, which had become all the more apparent to them when we made their own beliefs (and not just our own) a topic of discussion. In the end, it is a question for each instructor of the pedagogical costs and benefits when deciding which story from the history of physics to tell our students, but we have made our best argument that the benefits may far outweigh any costs when we make the physical interpretation of quantum mechanics a central theme of our modern physics courses.

III. Curriculum Refinement and Other Future Directions

For the sake of future implementations of this curriculum, efforts should be made to assess where students had the most difficulty, so that suggestions for improvement can be made. Given the volumes of data collected in this dissertation project, we must confine our discussion here to specific examples of potentially fruitful changes, and suggestions for future studies.

III.A. Single-Photon Experiments

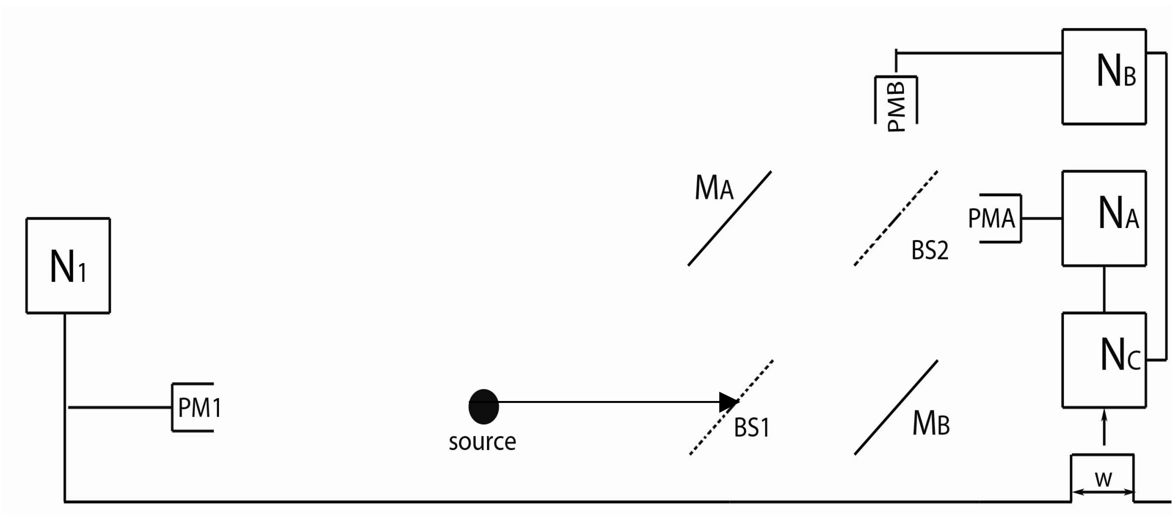
We begin by examining student responses to another essay question from the second midterm exam, designed to test student understanding of the single-photon experiments; we focus on this specific topic area for several reasons. First, single-quanta experiments with electrons and photons were the topics most commonly cited by students in their personal reflections as having influenced their perspectives on quantum physics, indicating this to be a key component of this curriculum's successful implementation. Second, the content of this lecture is fairly self-contained, and might easily be adapted by instructors who wish to augment their own courses without adopting the entire curriculum, and is therefore worthy of extra attention. Third, we are unaware of any instructional materials having yet been developed for introductory modern physics students concerning such experiments, and so have had no basis for judging ahead of time whether their implications for the meaning of wave-particle duality would be fully appreciated by our students.

For this midterm, students were required to answer the first essay question on interpretations of the double-slit experiment, but were given the option of answering just one of the remaining two essay questions; if students chose to answer both of the remaining two problems, they received credit for the higher of the two scores. Naturally, we will have no insight into the difficulties faced by students who opted out of answering this question, but 75% of the 103 students who took the exam did respond, which should represent a fair sampling of overall student understanding of this topic. Generally speaking, students performed well on this question: the average total score was 6.75 out of 8 points, and 85% of responses received a total score of 6 or better. We shall first give the problem statement below, and then consider individual responses of our four students (A–D) from Chapter 5. Their individual answers will help to illustrate the coding scheme that emerged in our analysis of aggregate student responses, but also the quality of responses from students with whom we are already somewhat familiar.

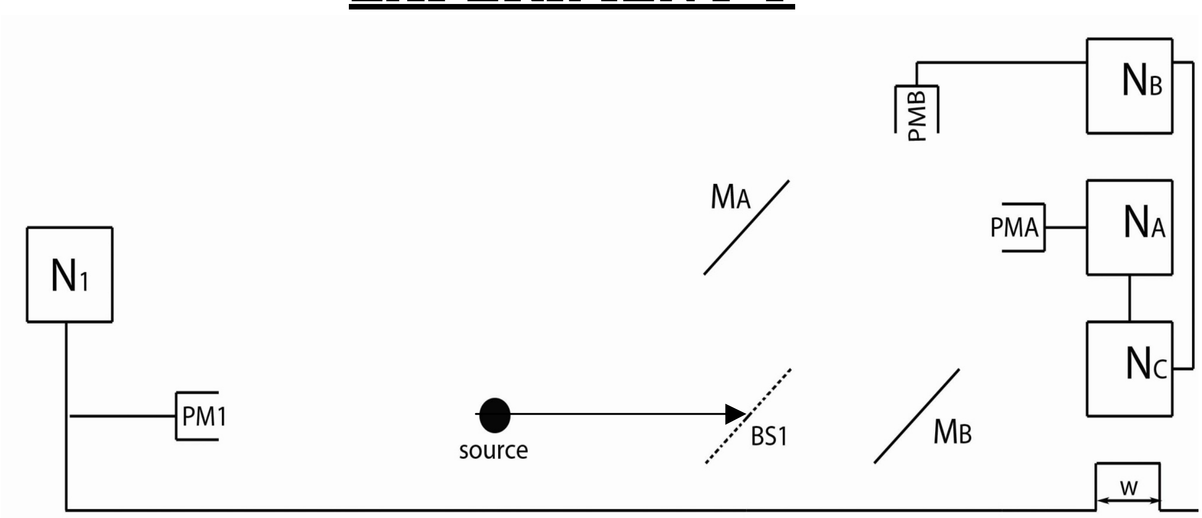
The beginning of each of the first two parts asks students to identify for which experimental setup, X or Y, (see below) they would expect photons to exhibit particle-like behavior, and which for wave-like behavior. Calling these two experiments X and Y (instead of 1 and 2, as in the lecture slides; see Chapter 5), and reversing their order of presentation seemed to have no impact on student responses, since all students but one were correct in their identification for each case. We felt a key step in assessing student understanding of the implications of these experiments would be to determine whether they could describe in what sense the photon is behaving like a particle or wave in each setup. We were also interested in finding out which kinds of epistemological tools would be favored by students in justifying why each type of behavior could be expected in a given situation. The final part of the this essay question concerns a delayed-choice experiment that is the reverse of the situation described during lecture: here, the second beam splitter is in place at the time a photon encounters the first beam splitter. If the second beam splitter were to be quickly removed before the photon had passed through the apparatus, there would be no opportunity for the photon to interfere with itself, meaning there is an equal likelihood for it to be detected in either photomultiplier.

E3. (OPTION TWO – 3 PARTS, 8 POINTS TOTAL) For the diagrams below depicting Experiments X & Y, M = Mirror, BS=Beam Splitter, PM = Photomultiplier, N = Counter. In each experiment a single-photon source sends photons to the right through the apparatus one at a time.

EXPERIMENT X



EXPERIMENT Y



E3.A (3 Points) For which experimental setup (X or Y) would you expect photons to exhibit particle-like behavior? Describe in what sense the photon is behaving like a particle during this experiment. What features of the experimental setup allow you to draw this conclusion without actually conducting the experiment?

Student A: In setup Y, the photons exhibit particle like behavior because the photon can only have one path to get to a particular photomultiplier. I know this because beamsplitter one will either allow the photon through or reflect it. If it reflects it it will go to PMA, if it is let through it will go to PMB. It can't take Path A to get to PMB thus there is one path to take, it acts as a particle.

Student B: Particle-like behavior expected in setup Y. Photon's path is predictable depending on the detector in which it was detected. It either gets reflected or transmitted at BS1, thus if detected at PMA, it must have been reflected and if detected at PMB, it must have been transmitted. We also know that $\alpha = P_C/(P_A P_B) = 0$ if there is only one photon in the apparatus during the time constant. This implies that $P_C = 0$ and no wave like behavior, acts like a particle. There is only one BS, so it will act like a particle (we know this even before conducting exp.)

Student C: Experiment Y should show photons acting like a particle. This is due to the fact that which path the photon takes can be determined by which photomultiplier is triggered. If the photon struck mirror B, PMB will fire, if the photon struck mirror A, PMA will fire. If there was truly only a single photon in the source only one of the photomultipliers will fire, and each would fire with a 50/50 chance.

Student D: Experiment Y (Aspect's 1st Experiment)
The photon may take one of 2 paths, but not both, and thus travels along a defined path consistent with the behavior of a particle. The way the experiment is set up, a photon may only take one of:
  source – beamsplitter – mirror A – photomultiplier A
  source – beamsplitter – mirror B – photomultiplier B
If a photon is to be detected in PM1, its pair must have exited the source in exactly the opposite direction, and by geometry can only take one of the two paths listed above.

Of the three parts to this essay question, this one presented the least problems for students, and 95% of them received full credit for their responses. Students were fairly uniform in the types of argumentation and reasoning they employed, and a simple coding scheme was almost immediately apparent. Many students offered multiple justifications for their answers, and so we ranked each type of argument according to its prominence in the student's response, or by which appeared first if they seemed to carry equal weight; we report here statistics only on students' primary responses.

In describing the behavior of a photon in Experiment Y, 58% of students said that, as a particle, it is only taking one path or other on its way from source to detector (Students A & D); and 40% said its particle nature is demonstrated by being detected in either one PMT or the other, but not both (Students B & C). It seems significant that the majority of students associated particle behavior with definite trajectories (taking a single path), while fewer students associated particles with localized detections. This focus is also reflected in their identification of which features of the setup would allow them to predict particle-like behavior: 66% cited the fact that only a single path existed between source and each detector; 14% claimed the ability to determine which path a given photon had taken was sufficient for predicting this specific behavior. [16% focused on the literal difference between the two experiments – the absence of a second beam splitter.] So, a relatively small number of students relied on the new and more abstract epistemological tool developed in lecture, the availability which-path *information* as a determiner of behavior (as opposed to the existence of a single path). Not only did fewer students associate particles with localized detections, only 10% of all students made mention of measuring the anticorrelation parameter, or referred to counting rates and coincidence detections, even though these had been significant aspects of our presentation. This suggests that students are not entirely comfortable with the statistical nature of the argument for interpreting particle-like behavior in this experiment, which likely has implications for why students had greater difficulties with the flip side to this question:

E3.B (3 Points) For which experimental setup (X or Y) would you expect photons to exhibit wave-like behavior? Describe in what sense the photon is behaving like a wave during this experiment. What features of the experimental setup allow you to draw this conclusion without actually conducting the experiment?

Student A: In setup X, the photons exhibit wave like behavior because the photon can take either Path A or Path B and still get to PMA or PMB, we don't know which path it took, thus since it is unpredictable, it acts like a wave. Since it can take either path and still get to either photomultiplier, I know it can be represented as a wave.

Student B: Wave-like behavior expected in Setup X. Photon behaves like a wave because there is interference if we change the path length (move BS2). Thus it seems to interfere with itself. In this experiment, we can't know which path the photon takes due to the existence of BS2 (it could be detected by either PM, and have taken either path). We can also change BS2's location such that all the photons are detected in PMA or PMB. Throughout the experiment, it seems that the photon somehow "knows" that there are both paths. The BS2 lets us conclude this before starting the experiment (that it can behave like a wave).

Student C: Experiment X should show photons acting like waves. The path the photon took is undeterminable. Mirror B could have been hit with a photon and either PMB or PMA could fire. This implies a wave is being propagated through both possible paths. The wave then describes an equal probability of triggering each photomultiplier provided each path is the same length. Interference can happen if the paths are different length and cause only one photomultiplier to trigger.

Student D: Experiment X (Aspect's 2nd Experiment)
The exact path taken by the photon is rendered indeterminate by the second beamsplitter; we can't know which path the photon actually took to PMA or PMB. If we vary the path length of A or B, and observe interference as a result in the detectors, a logical explanation is that the wave that represents the photon split at beamsplitter 1, and then (due to the difference in phase created by the changed path length) interfered with itself to produce the observed results. The presence of the 2nd beamsplitter essentially randomizes whether a photon travelling along path A or B ends up in PMA or PMB (50% chance of either for fixed path length), thus rendering the path of the photon indeterminate, which allows for the above conclusions to be drawn.

Only 51% of students received full credit for their responses to this part of the question, but a total of 90% were given a score of 2/3 or better. 43% said that photons manifest their wave behavior in the form of interference (Students B, C & D), and 35% claimed that wave-like photons take both paths in this experiment. This is not precisely what Student A said – he mentions that photons are *capable* of taking both paths, but not that photons *are* taking both paths. In fact, his responses to this part of the question and the last suggest that he associates wave-like behavior with indeterminacy – photons are still presumed to take only one of two paths – it is our knowledge of which that is indefinite.

Most significant was the finding that 21% of students mistakenly believed, in Experiment X, that photons would be detected in both PMT's simultaneously; 5% explicitly stated that measuring the anticorrelation parameter as greater than one (coincidental detection) would be evidence of the photon's wave behavior in this case. In fact, for the data run presented in class demonstrating interference through path length modulation, the anticorrelation parameter was calculated to be 0.18 (less than unity, as it should be). We believe this confusion may be likely attributed to two factors. First, we only implied individual photon detections in our comparison of counting rates, but did not explicitly point out that the anticorrelation parameter had been found here to also be less than one. The specific wording of Slide 12 from this lecture [Fig. 6.8] could also be confusing for students. We want them to associate wave-like behavior in this experiment with what each photon does at the beam splitter, yet this slide could lead them to believe that wave behavior should be universally associated with coincidental detections. This misunderstanding could be directly addressed by placing greater emphasis on the connection between wave behavior and self-interference, or indefinite trajectories; and by placing greater emphasis on the continued particle-like *detection* of photons, focusing student attention instead on the behavior of the photons at each beam splitter.

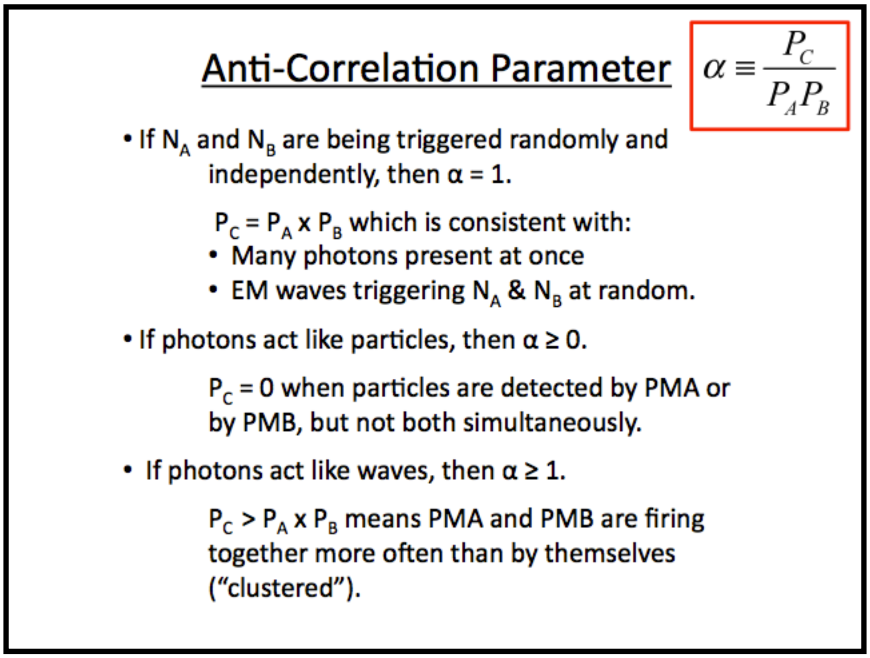


FIG. 6.8. Slide 12 from Lecture 20 (Single-Photon Experiments, see Chapter 5). In the first experiment, wave behavior is associated with coincidental detection; it is associated with indefinite trajectories and self-interference in the second.

We have further indication that students are uncomfortable with how wave interference is manifested in this experiment, which is different from directly observing a fringe pattern. Of all the students who mentioned interference as evidence of wave behavior, only half specifically said that it would be observed by making changes to the relative path lengths; the other half only commented that interference would be observed. Moreover, only 26% correctly spoke of interference in terms of modulated detection rates in the two photomultipliers, and 5% incorrectly believed that fixing the mirrors would cause every photon to take just one of the paths (as opposed to being detected in just one of the PMT's). Whereas only 16% of students had cited the absence of the second beam splitter as being the key feature of Experiment Y, a full 38% of students focused on its presence in Experiment X as being key to determining what kind of behavior would be observed. 36% employed an epistemological tool developed in class: no *which-path information* would be available; and 23% said the availability of two paths for the photon was key to predicting wave-like behavior in this experiment.

These results, and those from the first part of the question, suggest that students attach greater significance to the question of which path a photon takes (strong associations with particle behavior), and focus less on its behavior at the beam splitter (weak associations with wave behavior). The argument for wave behavior presented in class centered on the behavior of the photon at the beam splitter, and so perhaps this emphasis was not properly communicated to students; but we may also consider exploiting the strength of student preference for which-path arguments by giving them greater prominence in our argumentation. After all, we had been trying to develop the concept of *which-path information* as an epistemological tool, which might be aided by placing less emphasis on the response of photons to beam splitters, where students are less likely to have had any exposure to in previous classes. We had discussed them earlier in the context of the Michelson-Morley experiment, but perhaps there was insufficient connection made between the coherent 50/50 splitting of a classical EM wave, and a 50/50 probability for transmission or reflection of a photon.

Responses to the third part of this question show that the subtlety of the delayed-choice experiments was not entirely lost on students, but also provide additional evidence of student difficulties with probabilistic descriptions of measurement outcomes:

E3.C (2 Points) Suppose we are conducting Experiment X (the second beam splitter (BS2) is present) when a photon enters the apparatus and encounters the first beam splitter (BS1). Afterwards, while the photon is still travelling through the apparatus (but before it encounters a detector), we suddenly remove the second beam splitter (switch to Experiment Y). Can we determine the probability for the photon to be detected in PMA? If not, why not? If so, what would be that probability? Explain your reasoning.

Student A: No, we could not because we don't know which path the photon took, it could have taken path A in which it which it would be detected by photomultiplier A or it could have taken path B and not been detected by PMA. Since it has not been detected yet we can't determine the probability it's already on a definite path.

Student B: This is the delayed-choice experiment. We can indeed predict the path that the photon took if BS2 is not present depending on the detector in which it was detected. Thus, the probability of being detected in PMA would be 50/50 (0.5). It would act just as if we ran experiment Y and behave like a particle. Put the beam splitter back and it acts like a wave again. There is no "tricking" the photon!

Student C: First assume that experiment X is set up so that interference occurs and only PMA is firing. If the photon is still traveling through the apparatus, and BS2 is then suddenly removed, the photon will switch to acting like a particle. The photon will no longer only fire in PMA due to interference, but will instead show particle-like behavior and trigger either PMB or PMA with a 50/50 probability. BS1 results in either path from BS1 being 50/50 probable. Because when BS2 is removed, the path the photon took is now better known and particle like behavior is observed. In other words, once BS2 is removed PMB firing means MB was hit by a photon, and PMA firing means MA was hit by a photon.

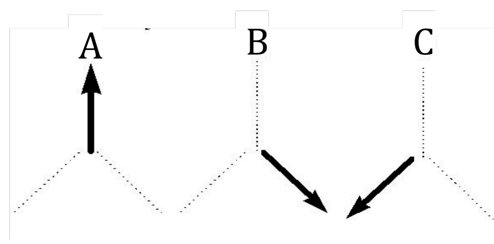
Student D: Yes, the probability will be 0.5 – same result as Experiment X with equal path lengths, but with a definite path for any given photon. A photon may exhibit either wave-like or particle-like properties, but not both in the same instant. Removing the 2nd beamsplitter "forces" the photon to exhibit particle-like behavior by making its path definite retroactively – example of a "delayed choice" experiment.

We note that each of the four students suggested that removing the second beam splitter forces a photon into taking a definite path (not that self-interference would no longer be possible), and only Student C's response makes explicit mention of the lack of interference. Again, student associations seem to be strongest between particles and definite paths. 81% of students said that the probability for detection in PMA could be known, but only 3/4 of those students explicitly stated that probability as being 50%. Student A seems to be close to drawing this conclusion, but there appears to be a disconnect between a completely indeterminate outcome and a 50/50 likelihood for either occurrence. Regardless of whether they felt the probability could be known, almost 40% of students did not state that the probability for detection in PMA is 0.5; this suggests that students require more practice with the use of probabilities, beyond the single lecture we devoted to classical probability and probability distributions.

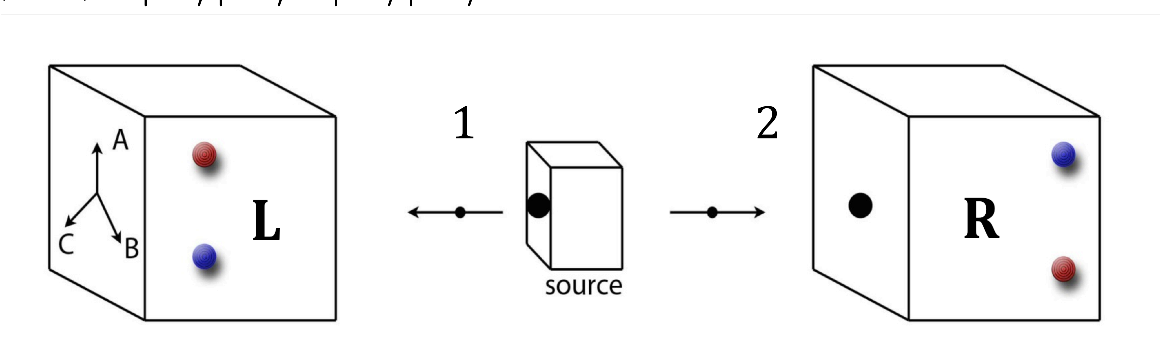
II.B. Entanglement and Correlated Measurements

The need for future studies into student difficulties with our transformed curriculum is illustrated by responses to a multiple-choice exam question concerning distant, anticorrelated measurements performed on entangled atom pairs:

6. Suppose we have two "Local Reality Machines" (Stern-Gerlach analyzers capable of being oriented along three different axes: A, B, & C, each oriented at 120^0 to each other, as shown) set up to detect atom-pairs emitted in an entangled state:



$$\left|\Psi_{12}\right\rangle = \left|\uparrow_1\right\rangle\left|\downarrow_2\right\rangle + \left|\downarrow_1\right\rangle\left|\uparrow_2\right\rangle$$



The leftward travelling atom (1) reaches the left analyzer (L) before the rightward traveling atom (2) reaches the right analyzer (R). The left analyzer is set on A and measures atom 1 to be "up" along the vertically oriented A-axis (it exited from the plus-channel). A short time later, atom 2 enters the right-side analyzer. If the right analyzer is set on B (120^0 from the vertical axis), what is the probability for atom 2 to exit from the plus-channel of the right analyzer?

TABLE 6.II Distribution of student responses to multiple-choice question #6 from the second midterm exam – correct response highlighted in bold.

A) 0	B) 1/4	C) 1/2	**D) 3/4**	E) 1	RIGHT	WRONG
14%	34%	12%	**42%**	0	42%	58%

The majority of the class got this question wrong (58%, see Table 6.II), but we have little insight into the reasons for this, since each option is a significant distractor, with many potential sources of confusion. We note that no student selected option E (1), and so we may at least conclude that students did not believe that a measurement of "up" in the first detection requires an "up" measurement for the second. Option C (1/2) was chosen by 12% of students, which may indicate they did not recognize how the entangled state of the atom pair, and therefore the outcome of the first measurement, establishes a definite state of "down" for the second particle along Axis-A; this response would be correct if there were no influence of the first measurement on the second. Option B (1/4) was the most popular incorrect response, which comes from using ($120^0/2$) as the relevant angle in calculating the probability for an "up" measurement along Axis-B, when it is actually ($60^0/2$) – it varies according to the cosine squared of the half-angle between incoming state and axis of analyzer orientation. Students who correctly identified this angle may have forgotten to divide by two; or they may have correctly applied the formula, but thought the second atom would also be measured as "up" along Axis-A; or they may have simply been distracted by the prominence of the 120^0 angle in the problem statement. Option A (0) was also a popular response (14%), which may imply these students felt that an "up" measurement for the first particle precluded an "up" measurement for the second particle along *any* axis. Adapting this specific question into a short-answer problem, where students would be required to provide their reasoning, would be a first step toward understanding some of the difficulties students have with entanglement and distant correlated measurements.

II.C. Atomic Models and Probability

One of the questions adopted from the QMCS [1] for our post-instruction content survey was designed to elicit common student misconceptions regarding the outcome of a position measurement for an atomic electron in the ground state of hydrogen:

30. The electron in a hydrogen atom is in its ground state. You measure the distance of the electron from the nucleus. What will be the result of this measurement?

A. You will measure the distance to be the Bohr radius.
B. You could measure any distance between zero and infinity with equal probability.
C. You are most likely to measure the distance to be the Bohr radius, but there is a range of other distances that you could possibly measure.
D. There is a mostly equal probability of finding the electron at any distance within a range from a little bit less than the Bohr radius to a little bit more than the Bohr radius.

TABLE 6.III Distribution of student responses to multiple-choice question #30 from the post-instruction content survey – correct response highlighted in bold.

A	B	**C**	D	E	%Correct	%Incorrect
30%	2%	**49%**	18%	0	49%	51%

An analysis of student responses to a midterm exam question on atomic models showed that only ~10% of students exclusively employed a planetary model in their descriptions of hydrogen, yet 30% of students incorrectly answered on the post-instruction survey that the electron would definitely be found at the Bohr radius, and 18% thought it was equally likely be found somewhere in that vicinity. This apparent disconnect may be explained by further difficulties students have in using probabilities to describe the outcome of quantum measurements, but it may also indicate realist commitments that were not revealed by the attitudes survey statement on atomic electrons. [See above, Section II.B.] Option D may have been popular among students that favor the de Broglie atomic model over a planetary description, but we must only speculate without the opportunity to further question students on the reasons for their responses, which is impossible in an end-of-term, multiple-choice format.

IV. Concluding Remarks

Perhaps the most important take-home message from these studies is that students will develop their own attitudes (right or wrong, sophisticated or not) regarding the physical interpretation of quantum mechanics when we, as instructors, do not explicitly attend to the realist beliefs that are so common among our introductory modern physics students. We have frequently heard that a primary goal when introducing students to quantum mechanics is for them to recognize a fundamental difference between classical and quantum uncertainty. The notorious difficulty of this has lead many instructors to view this learning goal as superficially possible, but largely unachievable in a meaningful way for most introductory students. We believe our studies demonstrate otherwise. By addressing the physical interpretation of quantum phenomena across a variety of contexts, but also by making questions of classical and quantum reality a central theme of our course, we were able to positively influence student thinking across a variety of measures, both attitudinal and in content-specific topic areas.

We have developed a framework for understanding student interpretations of quantum mechanics, which show how their overall perspectives may be influenced by their specific attitudes toward several individual themes central to the question of probabilistic measurement outcomes. Is the wave function physically real, or a mathematical tool? Is the reduction of quantum superpositions to definite states an ad hoc rule established to make theory agree with observation, or does it represent some kind of physical transition not described by any equation? Is an electron, being a form of matter, strictly localized at all times? We have identified student attitudes regarding these questions as playing a key role when formulating their thoughts on quantum phenomena, and have seen how the myriad ways in which these attitudes may combine can lead to a variety of overall interpretive stances. If we wish to have significant influence on student perspectives, and if we are to take seriously the lessons learned from education research on the impact of *hidden curricula*, then we must choose to explicitly address these beliefs in our introductory courses.

We also believe that a *static* view of student and expert ontologies, however useful in addressing student difficulties in classical physics, is too limited to account for the contextually sensitive and highly dynamic thought processes of our students when it comes to ontological attributions. We have seen students blend attributes from the classically distinct categories of particles and waves; they may switch between views according to their cognitive needs of the moment; and they often distinguish between their intuitive perspectives, and what they have learned from authority. At the very least, we may conclude that ontological flexibility does not come easily to most students, and that the contextual sensitivity of their responses is most consistent with students engaging in a piecewise altering of their perspectives, rather than some wholesale shift (or replacement) in ontologies. Most importantly, many of our students demonstrated exactly the kind of ontological flexibility that is required for a proper understanding of quantum mechanics. We believe this learning goal is more easily achieved by placing greater emphasis on the meaning of wave-particle duality, and by providing experimental evidence that favors dualistic descriptions, but also by explicitly addressing in class the commonly held beliefs of students revealed by our studies. Among the many learning goals for our transformed curriculum was for students to be consciously aware of their own (often intuitive and tacit) beliefs, but also for them to acquire the necessary language and conceptual inventory to identify and articulate those beliefs. This was accomplished in part by presenting them with specific terminology relevant to perceptions of reality and locality, but also by making the beliefs of students (and not just the beliefs of scientists) a topic of discussion in our course.

It would be too simplistic to say that our aim was for students to consistently *not agree* with realist interpretations of quantum phenomena. After all, there are a variety of situations in quantum mechanics where the physical interpretation of the wave function has no relevance or bearing on the outcome of a calculation. It is not that a particle view of matter is entirely illegitimate in quantum mechanics; it is simply that its consistent application in all contexts is not adequate in accounting for all of what we observe in nature. We suggest that a significant amount of the confusion introductory students feel when learning about quantum mechanics results from the paradoxical conclusions that come as a consequence of realist expectations and ontological inflexibility.

Nor would we wish to connote too much negativity with the fact that students are relying on their intuition as a form of sense making. It is true we are telling them that their everyday thinking can be misleading in quantum physics, but that is not a sufficient argument for the wholesale abandonment of productive epistemological tools. Indeed, our approach to teaching quantum interpretations frequently required an appeal to student intuitions about the classical behavior of particles (they are transmitted or reflected; they are localized upon detection), and similarly with waves. A more important goal is for students to achieve more internal consistency in their thinking, which may be cultivated by developing epistemological tools that aid in deciding which type of behavior should be expected in which type of situation. Considering the observed strong associations students make between particles and definite paths, it seems that framing such tools in terms of *which-path information* [two paths = interference; one path = no interference] may be particularly useful for students.

Of the many potential studies that might be conducted as an improvement on those presented here, we believe that focusing on the thinking associated with *Agnostic* students would be particularly beneficial. We have never considered an agnostic perspective to be unsophisticated; in fact, our *Agnostic* category [as defined in Chapter 4] was meant to include both students *and* experts who acknowledge the potential legitimacy of competing perspectives, without taking a definitive stance. Agnosticism, by this definition, would therefore involve an acknowledgement of evidence that favors more than just a single interpretation, which is clearly different from students who exclusively assert the legitimacy of their realist intuitions *in spite of* evidence to the contrary. At the same time, an agnostic stance may be indicative of the perception that nothing can truly be known or understood in science, since many of the assumptions we make about the world turn out to be demonstrably false, and so much in quantum physics cannot be directly observed. Either way, an agnostic stance among students may be interpreted as an intermediary stage in the transition away from realism, but might also signal unfavorable perceptions on the *nature of science*. Negative perceptions about what can and can't be known in science might be an unintended consequence of our curriculum transformations, and require further detailed consideration.

References (Chapter 6)

1. S. B. McKagan, K. K. Perkins and C. E. Wieman, Reforming a large lecture modern physics course for engineering majors using a PER-based design, *PERC Proceedings 2006* (AIP Press, Melville, NY, 2006).

2. S. B. McKagan, K. K. Perkins and C. E. Wieman, Why we should teach the Bohr model and how to teach it effectively, *Phys. Rev. ST: Physics Education Research* **4**, 010103 (2008).

3. E. Redish, J. Saul and R. Steinberg, Student expectations in introductory physics, *Am. J. Phys.* **66**, 212 (1998).

4. W. K. Adams, K. K. Perkins, N. Podolefsky, M. Dubson, N. D. Finkelstein and C. E. Wieman, A new instrument for measuring student beliefs about physics and learning physics: the Colorado Learning Attitudes about Science Survey, *Phys. Rev. ST: Physics Education Research* **2**, 1, 010101 (2006).

5. G. Greenstein and A. J. Zajonc, The Quantum Challenge: Modern Research on the Foundations of Quantum Mechanics, 2nd ed. (Jones & Bartlett, Sudbury, MA, 2006).

BIBLIOGRAPHY

W. K. Adams, K. K. Perkins, N. Podolefsky, M. Dubson, N. D. Finkelstein and C. E. Wieman, A new instrument for measuring student beliefs about physics and learning physics: the Colorado Learning Attitudes about Science Survey, *Phys. Rev. ST: Physics Education Research* **2**, 1, 010101 (2006).

D. Z. Albert and R. Galchen, A Quantum Threat to Special Relativity, *Scientific American* (March 2009, pp. 32-39).

A. Aspect, P. Grangier and G. Roger, Experimental Tests of Realistic Local Theories via Bell's Theorem, *Phys. Rev. Letters* **47**, 460 (1981).

C. Baily and N. D. Finkelstein, Development of quantum perspectives in modern physics, *Phys. Rev. ST: Physics Education Research* **5**, 010106 (2009).

C. Baily and N. D. Finkelstein, Teaching and understanding of quantum interpretations in modern physics courses, *Phys. Rev. ST: Physics Education Research* **6**, 010101 (2010).

C. Baily and N. D. Finkelstein, Refined characterization of student perspectives on quantum physics, *Phys. Rev. ST: Physics Education Research* **6**, 020113 (2010).

L. E. Ballentine, The Statistical Interpretation of Quantum Mechanics, *Rev. Mod. Phys.* **42**, 358 (1970).

L. E. Ballentine, Resource letter IQM-2: Foundations of quantum mechanics since the Bell inequalities, *Am. J. Phys.* **55**, 785 (1987).

L. E. Ballentine, Quantum Mechanics: A Modern Development (World Scientific Publishing, Singapore, 1998).

J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 1987).

D. Bohm and Y. Aharonov, Discussion of Experimental Proof for the Paradox of Einstein, Rosen, and Podolsky, *Phys. Rev.* **108**, 1070 (1957).

D. Bohm and B. J. Hiley, *The Undivided Universe* (Routledge, New York, NY, 1995).

N. Bohr, Can quantum mechanical description of physical reality be considered complete? *Phys. Rev.* **48**, 696 (1935).

A. Buffler, S. Allie, F. Lubben and B. Campbell, The development of first year physics students' ideas about measurement in terms of point and set paradigms, *Int. J. Sci. Educ.* **23,** 11 (2001).

O. Carnal and J. Mlynek, Young's Double-Slit Experiment with Atoms: A Simple Atom Interferometer, *Phys. Rev. Lett.* **66** (21), 2689 (1991).

E. Cataloglu and R. Robinett, Testing the development of student conceptual and visualization understanding in quantum mechanics through the undergraduate career, *Am. J. Phys.* **70**, 238 (2002).

M. T. H. Chi, Common sense misconceptions of emergent processes: Why some misconceptions are robust, *Journal of the Learning Sciences* **14**, 161 (2005).

J. F. Clauser, M. A. Horne, A. Shimony and R. A. Holt, Proposed Experiment to Test Local Hidden-Variable Theories, *Phys. Rev. Lett.* **23**, 880 (1969).

J. G. Cramer, The Transactional Interpretation of Quantum Mechanics, *Rev. Mod. Phys.* **58**, 647 (1986).

J. W. Creswell, *Education Research*, 2nd Ed. (Prentice Hall, Englewood Cliffs, NJ, 2005), pp. 397-398.

C. Davisson and L. H. Germer, Diffraction of electrons by a crystal of nickel, *Phys. Rev.* **30**, 705 (1927).

P. A. M. Dirac, *The Principles of Quantum Mechanics, 3rd ed.*, (Clarendon Press, Oxford, 1947).

A. A. diSessa, "A history of conceptual change research: Threads and fault lines," in *Cambridge Handbook of the Learning Sciences*, K. Sawyer (Ed.) (Cambridge University Press, Cambridge, 2006), pp. 265-281.

M. Dubson, S. Goldhaber, S. Pollock and K. Perkins, Faculty Disagreement about the Teaching of Quantum Mechanics, *PERC Proceedings 2009* (AIP, Melville, NY, 2009).

A. Einstein, "Autobiographical Notes" in *Albert Einstein: Philosopher-Scientist*, P. A. Schillp (Ed.) (Open Court Publishing, Peru, Illinois 1949).

A. Einstein, *Letters on Wave Mechanics*, K. Przibram (Ed.) (Philosophical Library, New York, NY, 1986).

A. Einstein, B. Podolsky and N. Rosen, Can quantum mechanical description of physical reality be considered complete? *Phys. Rev.* **47**, 777 (1935).

B.-G. Englert, M. O. Scully and H. Walther, The Duality in Matter and Light, *Scientific American* (December 1994, pp. 86-92).

S. J. Freedman and J. F. Clauser, Experimental test of local hidden-variable theories, *Phys. Rev. Lett.* **28**, 938 (1972).

J. Falk, Developing a quantum mechanics concept inventory, unpublished master thesis, Uppsala University, Uppsala, Sweden (2005). Available at: http://johanfalk.net/node/87 (Retrieved January, 2011).

S. Frabboni, G. C. Gazzadi, and G. Pozzi, Nanofabrication and the realization of Feynman's two-slit experiment, *App. Phys. Letters* **93**, 073108 (2008).

R. Gaehler and A. Zeilinger, Wave-optical experiments with very cold neutrons, *Am. J. Phys.* **59** (4), 316 (1991).

S. Goldhaber, S. Pollock, M. Dubson, P. Beale and K. Perkins, Transforming Upper-Division Quantum Mechanics: Learning Goals and Assessment, *PERC Proceedings 2009* (AIP, Melville, NY, 2009).

P. Grangier, G. Roger, and A. Aspect, Experimental Evidence for a Photon Anticorrelation Effect on a Beam Splitter: A New Light on Single-Photon Interferences, *Europhysics Letters* **1**, 173 (1986).

K. E. Gray, W. K. Adams, C. E. Wieman and K. K. Perkins, Students know what physicists believe, but they don't agree: A study using the CLASS survey, *Phys. Rev. ST: Physics Education Research* **4**, 020106 (2008).

B. R. Greene, *The Elegant Universe* (Norton, New York, NY, 2003).

G. Greenstein and A. J. Zajonc, The Quantum Challenge: Modern Research on the Foundations of Quantum Mechanics, 2nd ed. (Jones & Bartlett, Sudbury, MA, 2006).

D. J. Griffiths, *Introduction to Quantum Mechanics, 2nd Ed.* (Prentice Hall, Upper Saddle River, New Jersey, 2004).

A. Gupta, D. Hammer and E. F. Redish, The case for dynamic models of learners' ontologies in physics, *J. Learning Sciences* **19**, 285 (2010).

R. Hake, Interactive-Engagement Versus Traditional Methods: A Six-Thousand-Student Survey of Mechanics Test Data for Introductory Physics Courses, *Am. J. Phys.* **66** (1), 64 (1998).

D. Hammer, Student resources for learning introductory physics, *Am. J. Phys.* **68**, S52 (2000).

D. Hammer, A. Elby, R. E. Scherr and E. F. Redish, "Resources, Framing and Transfer" in *Transfer of Learning*, J. Mestre (Ed.) (Information Age Publishing, 2005) pp. 89-119.

D. Hammer, A. Gupta and E. F. Redish, On Static and Dynamic Intuitive Ontologies, *J. Learning Sciences* **20**, 163 (2011).

S. W. Hawking, *A Brief History of Time* (Bantam, New York, NY, 1988).

W. Heisenberg, The Physical Content of Quantum Kinematics and Mechanics, in *Quantum Theory and Measurement*, J. A. Wheeler and W. H. Zurek (Eds.) (Princeton, NJ, 1983) p. 62. Originally published under the title, "Uber den anschaulichen Inhalt der quantentheoretischen Kinematik und Mechanik," *Zeitschrift fur Physik*, **43**, 172 (1927); translation into English by J. A. Wheeler and W. H. Zurek, 1981.

T. Hellmuth, H. Walther, A. Zajonc and W. Schleich, Delayed-choice experiments in quantum interference, *Phys. Rev. A* **35**, 2532 (1987).

D. Hestenes, M. Wills and G. Swackhamer, *Physics Teacher* **30**, 141 (1992).

R. D. Knight, *Physics for Scientists and Engineers: A strategic approach*, 2nd Ed. (Addison Wesley, San Francisco, CA, 2004)

L. Lising and A. Elby, The impact of epistemology on learning: A case study from introductory physics, *Am. J. Phys.* **73**, 372 (2005).

K. Mannila, I. T. Koponen and J. A. Niskanen, Building a picture of students' conceptions of wave- and particle-like properties of quantum entities, *Euro. J. Phys.* **23**, 45-53 (2002).

A. R. Marlow (Ed.), *Mathematical Foundations of Quantum Mechanics* (Academic Press, New York, NY, 1978).

E. Mazur, *Peer instruction: A user's manual* (Prentice Hall, New York, NY, 1997).

T. L. McCaskey, M. H. Dancy and A. Elby, Effects on assessment caused by splits between belief and understanding, *PERC Proceedings 2003* **720**, 37 (AIP, Melville, NY, 2004).

T. L. McCaskey and A. Elby, Probing Students' Epistemologies Using Split Tasks, *PERC Proceedings 2004* **790**, 57 (AIP, Melville, NY, 2005).

L. C. McDermott, Oersted Medal Lecture 2001: 'Physics Education Research – The Key to Student Learning', *Am. J. Phys.* **69**, 1127 (2001).

S. B. McKagan and C. E. Wieman, Exploring Student Understanding of Energy Through the Quantum Mechanics Conceptual Survey, *PERC Proceedings 2005* (AIP Press, Melville, NY, 2006).

S. B. McKagan, K. K. Perkins and C. E. Wieman, Reforming a large lecture modern physics course for engineering majors using a PER-based design, *PERC Proceedings 2006* (AIP Press, Melville, NY, 2006).

S. B. McKagan, K. K. Perkins, M. Dubson, C. Malley, S. Reid, R. LeMaster and C. E. Wieman, Developing and Researching PhET simulations for Teaching Quantum Mechanics, *Am. J. Phys.* **76**, 406 (2008).

S. B. McKagan, K. K. Perkins and C. E. Wieman, Why we should teach the Bohr model and how to teach it effectively, *Phys. Rev. ST: Physics Education Research* **4**, 010103 (2008).

N. D. Mermin, Is the moon there when nobody looks? Reality and the quantum theory, *Phys. Today* **38** (4), 38 (1985).

N. D. Mermin, What's Wrong with this Pillow? *Phys. Today* **42**, 9 (1989).

R. G. Newton, *How Physics Confronts Reality: Einstein was correct, but Bohr won the game* (World Scientific, Singapore, 2009).

V. K. Otero and K. E. Gray, Attitudinal gains across multiple universities using the Physics and Everyday Thinking curriculum, *Phys. Rev. ST: Physics Education Research* **4** (1), 020104 (2008).

V. Otero, S. Pollock and N. Finkelstein, A Physics Department's Role in Preparing Physics Teachers: The Colorado Learning Assistant Model, *Amer. J. Phys.* **78**, 1218 (2010).

A. Pais, *Niels Bohr's Times, in Physics, Philosophy and Polity* (Clarendon Press, Oxford, 1991).

G. J. Posner, K. A. Strike, P. W. Hewson and W. A. Gertzog, Accommodation of a scientific conception: Toward a theory of conceptual change, *Sci. Educ.* **66**, 211 (1982).

S. Pollock, No Single Cause: Learning Gains, Student Attitudes, and the Impacts of Multiple Effective Reforms, *PERC Proceedings 2004* (AIP Press, Melville, NY, 2005).

S. Pollock and N. D. Finkelstein, Sustaining Change: Instructor Effects in Transformed Large Lecture Courses, *PERC Proceedings 2006* (AIP Press, Melville, NY, 2006).

E. Redish, J. Saul and R. Steinberg, Student expectations in introductory physics, *Am. J. Phys.* **66**, 212 (1998).

S. Redner, Citation Statistics from 110 Years of Physical Review, *Phys. Today* **58**, 52 (2005).

M. Reiner, J. D. Slotta, M. T. H. Chi and L. B. Resnick, Naïve physics reasoning: A commitment to substance-based conceptions, *Cognition and Instruction* **18**, 1 (2000).

A. L. Robinson, Quantum Mechanics Passes Another Test, *Science* **217**, 435 (1982).

A. L. Robinson, Loophole Closed in Quantum Mechanics Test, *Science* **219**, 40 (1983).

E. Schrödinger, "The Present Situation in Quantum Mechanics," in *Quantum Theory and Measurement*, J. A. Wheeler and W. H. Zurek (Eds.) (Princeton, NJ, 1983) p. 152. Originally published under the title, "Die gegenwärtige Situation in der Quantenmechanik," *Die Naturwissenschaften*, **23**, 807 (1935); translation into English by J. D. Trimmer, 1980.

A. Shimony, The Reality of the Quantum World, *Scientific American* (January 1988, pp. 46-53).

C. Singh, Student understanding of quantum mechanics, *Am. J. Phys.* **69**, 8 (2001).

C. Singh, Assessing and improving student understanding of quantum mechanics, *PERC Proceedings 2006* (AIP Press, Melville, NY, 2006).

C. Singh, Student understanding of quantum mechanics at the beginning of graduate instruction, *Am. J. Phys.* **76**, 3 (2008).

J. D. Slotta, In defense of Chi's Ontological Incompatibility Hypothesis, *Journal of the Learning Sciences* **20**, 151 (2011).

J. D. Slotta and M. T. H. Chi, The impact of ontology training on conceptual change: Helping students understand the challenging topics in science, *Cognition and Instruction* **24,** 261 (2006).

W. H. Stapp, The Copenhagen Interpretation, *Am. J. Phys.* **40**, 1098 (1972).

D. F. Styer, *The Strange World of Quantum Mechanics* (Cambridge University Press, Cambridge, 2000).

M. Tegmark and J. A. Wheeler, 100 Years of Quantum Mysteries, *Scientific American* (February 2001, pp. 68-75).

W. Tittel, J. Brendel, B. Gisin, T. Herzog, H Zbinden and N. Gisin, Experimental demonstration of quantum correlations over more than 10 km, *Phys. Rev. A* **57** (5), 3229 (1998).

A. Tonomura, J. Endo, T. Matsuda, T. Kawasaki and H. Exawa, Demonstration of single-electron buildup of an interference pattern, *Am. J. Phys.* **57**, 117 (1989).

N. G. van Kampen, The Scandal of Quantum Mechanics, Am. J. Phys **76**, 989 (2008).

J. von Neumann, "Measurement and Reversibility" and "The Measuring Process" in *Mathematical Foundations of Quantum Mechanics* (Princeton University Press, Princeton, New Jersey, 1955), pp. 347-445.

A. Watson, Quantum Spookiness Wins, Einstein Loses in Photon Test, *Science* **277**, 481 (1997).

A. Whitaker, *Einstein, Bohr and the Quantum Dilemma: From Quantum Theory to Quantum Information* (Cambridge University Press, Cambridge, 1996).

S. Wuttiprom, M. D. Sharma, I. D. Johnston, R. Chitaree and C. Chernchok, Development and Use of a Conceptual Survey in Introductory Quantum Physics, *Int. J. Sci. Educ.* **31 (5)**, 631 (2009).

Appendices available online at:

http://per.colorado.edu/baily/dissertation

Modern Physics course materials available online at:

http://per.colorado.edu/ModernPhysics