



Electric Potential

Concepts and Principles

An Alternative Approach

The electric field surrounding electric charges and the magnetic field surrounding moving electric charges can both be conceptualized as information embedded in space. In both cases, the information is embedded as *vectors*, detailing both the magnitude and direction of each field. Moreover, when this information is “read” by other moving electric charges, the result is a *force* acting on the charge. These forces can be calculated to allow us to determine the subsequent motion of the charge.

Just as in mechanics, there is an alternative to this force-based approach to analyzing the behavior of electric charges. In this chapter I will define a new field, the *electric potential*, which surrounds every electric charge. This field differs from the electric field because it is a *scalar* field, meaning that this field has only a magnitude at every point in space and no associated direction. Moreover, the information in this field, when read by other electric charges, does not result in a force on the charge but rather determines the *electric potential energy* the charge possesses at that point in space.

Understanding how to calculate this new field, how this field relates to energy, and how this field is related to the electric field will be the focus of this chapter.

A Gravitational Analogy

Rather than thinking in terms of the gravitational force and Newton’s Second Law, an alternative way to examine mechanics scenarios is by using the concept of gravitational potential energy and the conservation of energy.

In the force approach, we envision a vector field surrounding the earth, regardless of whether a second mass is nearby to interact with this field. If a mass is present, the mass interacts with this field and feels a gravitational force. This idea is captured in the equation:

$$\vec{F}_G = m\vec{g}$$

In the energy approach, we can envision a scalar field, the *gravitational potential*, which is present regardless of whether a second mass is nearby to interact with this field. If a mass is present, the mass interacts with this field and has gravitational potential energy. Near the surface of the earth, the familiar expression for gravitational potential energy:

$$U_G = mgh$$

can be thought of as the product of the mass of the object and this pre-existing gravitational potential, V_G :

$$U_G = mV_G$$

if we define the gravitational potential by:

$$V_G = gh$$

Although we didn't use the concept of gravitational potential while studying mechanics, it will prove to be a very useful concept in our study of electrical phenomenon. The general expression for gravitational potential, valid regardless of distance from a massive object, is:

$$V_G = -\frac{Gm}{r}$$

In summary, just as a mass will interact with the vector gravitational field as a force, a mass will interact with the scalar gravitational potential field as potential energy.

The situation is very similar for electrical phenomenon. We can envision a scalar field, the *electric potential*, which is present regardless of whether a second charge is nearby to interact with this field. If a charge is present, the charge interacts with this field and has *electric potential energy*. The electric potential, V_E , is defined by the relationship:

$$V_E = \frac{kq}{r}$$

where

- q is the source charge, the electric charge that creates the field,
- and r is the distance between the source charge and the point of interest.

This leads to an expression for the electric potential energy of:

$$U_E = qV_E$$

where

- q is the charge on the particle of interest, the charge that is interacting with the field,
- and V_E is the net electric potential at the location of the particle of interest (created by all of the *other* charged particles in the universe).

We will typically leave the subscript off the electric potential and electric potential energy unless the possibility of confusion with the gravitational potential and potential energy are present.

Relating the Electric Field and the Electric Potential

The electric field and the electric potential are not two, independent fields. They are two independent ways of conceptualizing the effect that an electric charge has on the space surrounding it. Just as problems in mechanics can be analyzed using a force-approach or an energy-approach, problems dealing with electrical phenomenon can be analyzed by focusing on the electric field or on the electric potential.

Additionally, just as it is sometimes necessary in mechanics to transfer between force and energy representations, it is sometimes necessary to transfer between the electric field and electric potential representations. The relationship between the two fields can be understood by examining the expression for work, which relates force to transfer of energy.

The work done in moving a particle from an initial point, i , to a final point, f , can be written as:

$$W_{i \rightarrow f} = |F||\Delta r| \cos \phi$$

where Δr is displacement of the particle between the initial and final points. You may also recall that the difference in potential energy between initial and final locations is defined as the opposite of the work needed to move the particle between the two points:

$$\Delta U = U_f - U_i = -W_{i \rightarrow f}$$

Putting these two ideas together yields:

$$\Delta U = -|F||\Delta r| \cos \phi$$

We can now use this result to relate the electric potential and the electric field. Substituting in expressions for potential energy and force in terms of the fields that convey them leads to:

$$\begin{aligned}\Delta(qV) &= -|qE||\Delta r| \cos \phi \\ \Delta V &= -|E||\Delta r| \cos \phi\end{aligned}$$

In English, this result states that the electric potential difference between any two points is defined as the negative of the electric field along a path connecting the two points. (I'm sure that doesn't seem like particularly *clear* English, but this idea will become more tangible once you get to work on some of the fun activities in this chapter.)

It's probably easier to understand if we look at the special case where the electric field and the displacement of the particle are both in the x-direction. Then:

$$\Delta V = -|E||\Delta r|\cos\phi$$

$$\Delta V = -|E_x||\Delta r_x|\cos 0$$

$$\Delta V = -E_x(\Delta r_x)$$

$$E_x = -\frac{\Delta V}{\Delta r_x}$$

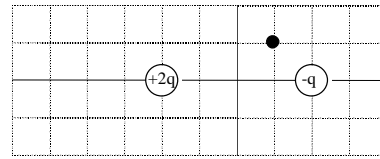
This means that the electric field in the x-direction is the opposite of the *slope* of the electric potential in the x-direction. Thus, if the electric potential is changing rapidly in a region of space the electric field is *large* in that region. Conversely, if the electric field is small in magnitude somewhere in space, the electric potential in that region will be nearly constant.

Electric Potential

Analysis Tools

Point Charges

Find the electric potential at the indicated point.
The charges are separated by a distance $4a$.



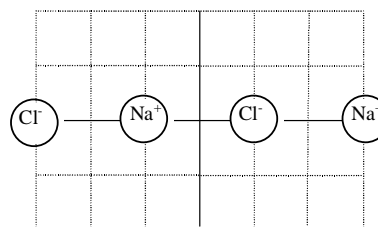
The electric potential at the point specified will be the sum of the electric potential from the left charge (V_L) and the electric potential from the right charge (V_R).

$$\begin{aligned} V &= V_L + V_R \\ V &= \frac{k(+2q)}{\sqrt{(3a)^2 + (a)^2}} + \frac{k(-q)}{\sqrt{(a)^2 + (a)^2}} \\ V &= \frac{2kq}{\sqrt{10}a} - \frac{kq}{\sqrt{2}a} \\ V &= \left(\frac{2}{\sqrt{10}} - \frac{1}{\sqrt{2}} \right) \frac{kq}{a} \\ V &= -0.0746 \frac{kq}{a} \end{aligned}$$

Notice that since the electric potential is a scalar, calculating the electric potential is often much easier than calculating the electric field.

Electric Potential Energy - I

Find the electric potential energy of the left Na^+ ion. The interionic separation is $2.82 \times 10^{-10} \text{ m}$.



The electric potential energy of the left sodium ion is given:

$$U = qV$$

where q is the charge of the sodium ion and V is the electric potential at the location of the sodium ion. Therefore, we must first find the electric potential at this location. The electric potential will be the sum of the potentials from the other three charges (with d equal to the interionic separation):

$$\begin{aligned} V &= V_{\text{leftCl}} + V_{\text{rightCl}} + V_{\text{rightNa}} \\ V &= \frac{k(-e)}{d} + \frac{k(-e)}{d} + \frac{k(+e)}{2d} \\ V &= \frac{-2ke}{d} + \frac{ke}{2d} \\ V &= \frac{-3ke}{2d} \\ V &= \frac{-3(9.0 \times 10^9)(1.6 \times 10^{-19})}{2(2.82 \times 10^{-10})} \\ V &= -7.66 \text{ V} \end{aligned}$$

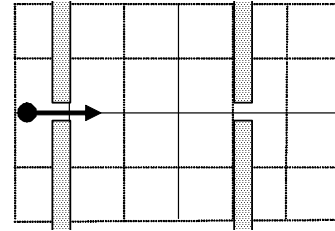
Thus the electric potential energy of the left sodium ion is:

$$\begin{aligned} U &= qV \\ U &= (1.6 \times 10^{-19})(-7.66) \\ U &= -1.23 \times 10^{-18} \text{ J} \end{aligned}$$

In general, if the electric potential energy of a particle is negative the particle is in a *bound state*. The sodium ion at this location is bound by the attractive electric forces due to the two nearby chlorine ions.

Electric Potential Energy - II

In many applications, oppositely charged parallel plates (with small holes cut for the beam to pass through) are used to accelerate beams of charged particles. In this example, a proton is injected at v_i into the space between the plates. The potential difference between the plates is ΔV (the left plate is at higher potential). What is the velocity of the proton as it exits the device?



Since the electric potential changes as the proton moves from the left plate to the right plate, its potential energy changes. This change in potential energy results in a change in kinetic energy of the proton by energy conservation.

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}mv_i^2 + qV_{\text{Left}} = \frac{1}{2}mv_f^2 + qV_{\text{Right}}$$

$$\frac{1}{2}mv_i^2 + e(V_{\text{Left}} - V_{\text{Right}}) = \frac{1}{2}mv_f^2$$

$$v_f = \sqrt{v_i^2 + 2me(\Delta V)}$$

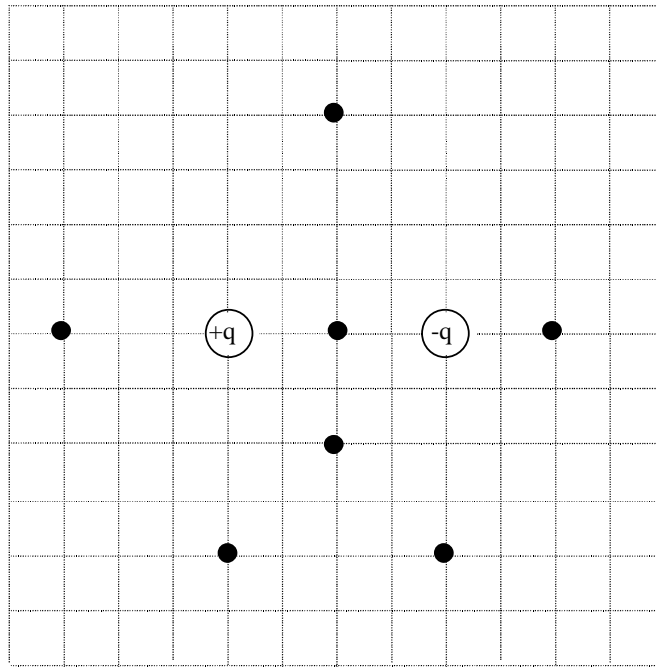


Electric Potential

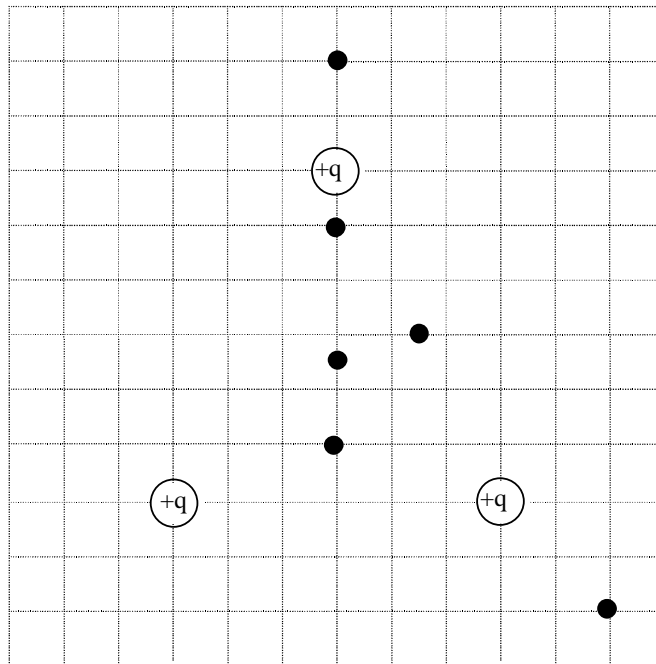
Activities

Determine the direction of the decrease in electric potential at each of the indicated points.

a.

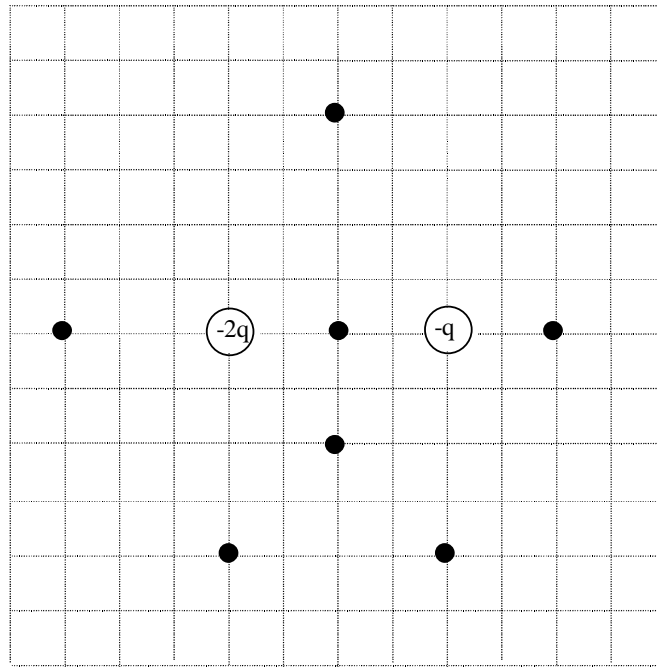


b.

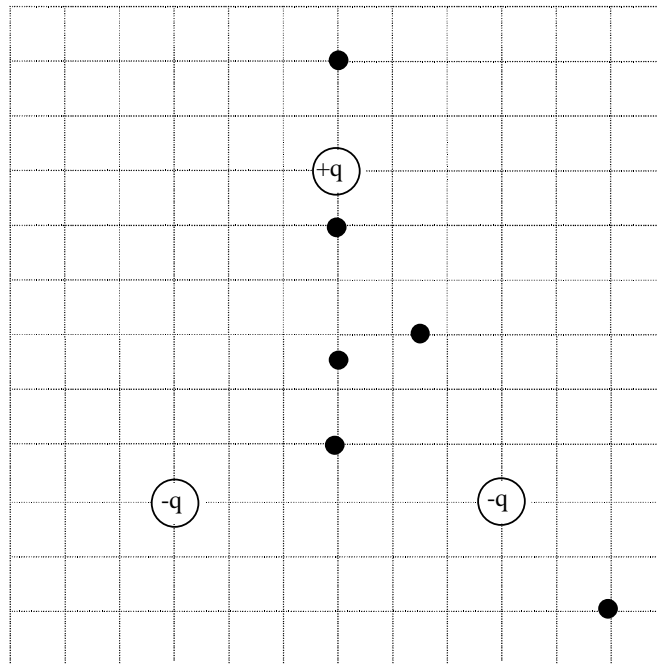


Determine the direction of the decrease in electric potential at each of the indicated points.

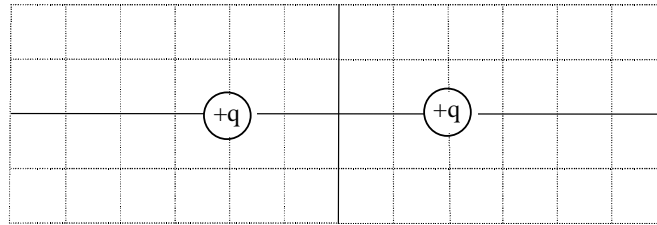
a.



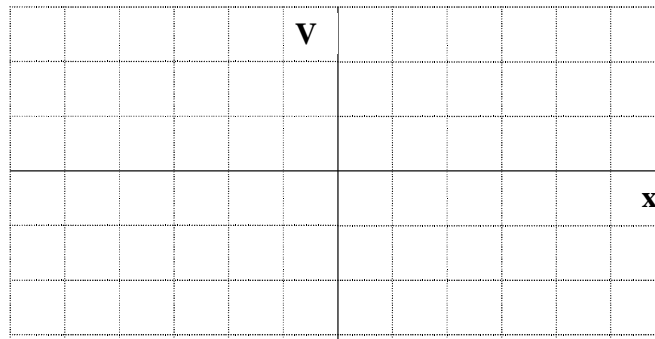
b.



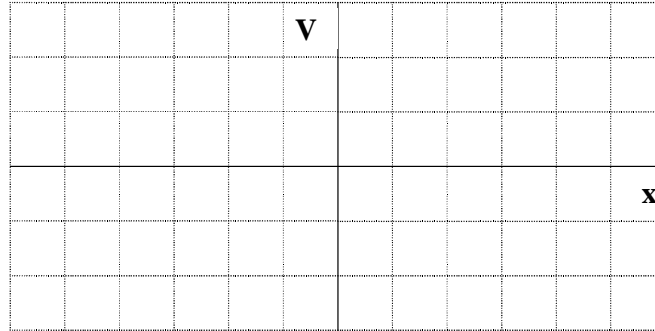
The two charges below are separated by a distance $4a$. Sketch graphs of electric potential for the locations indicated.



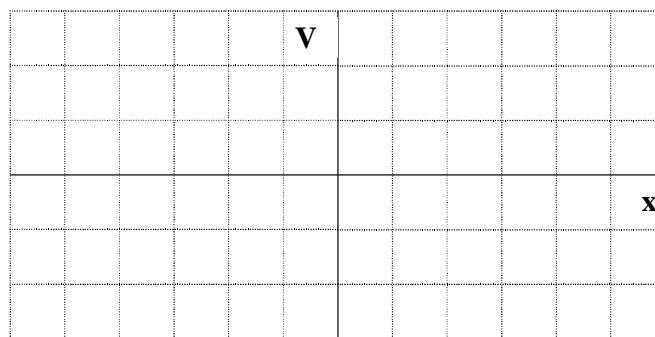
a. At $y = 0$



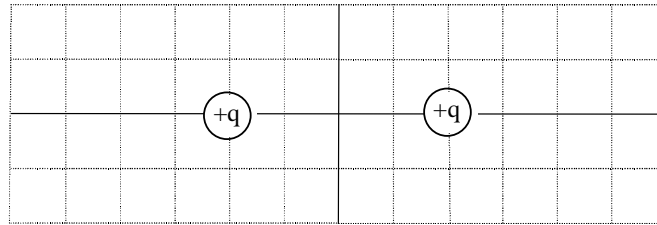
b. At $y = 2a$



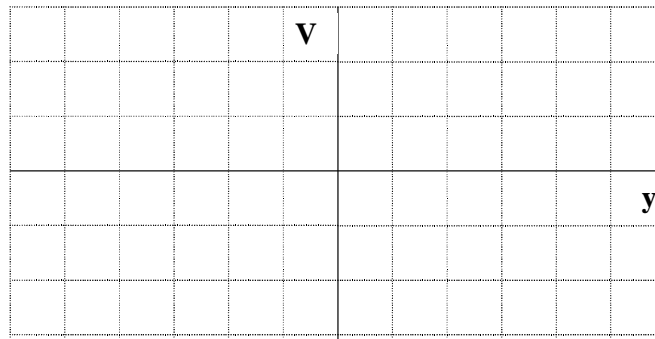
c. At $y = 4a$



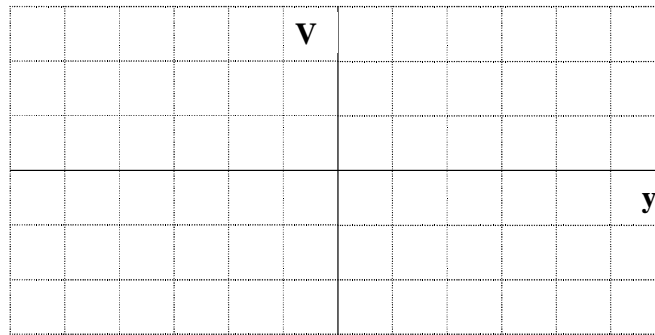
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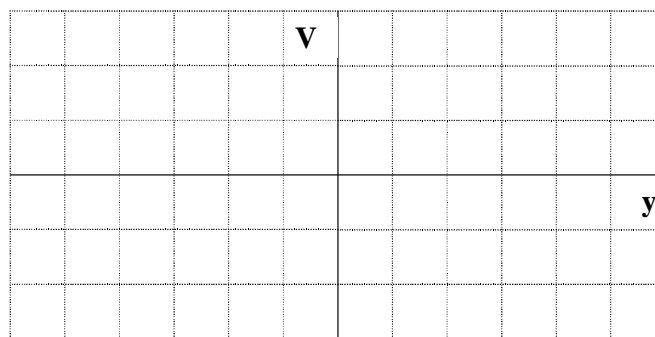
a. At $x = 0$



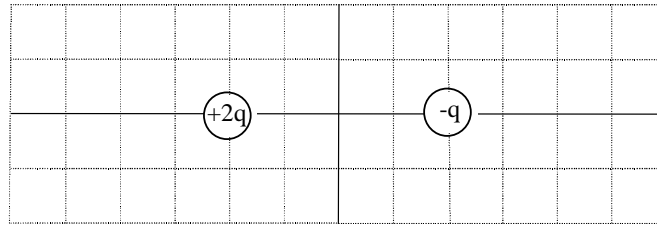
b. At $x = 2a$



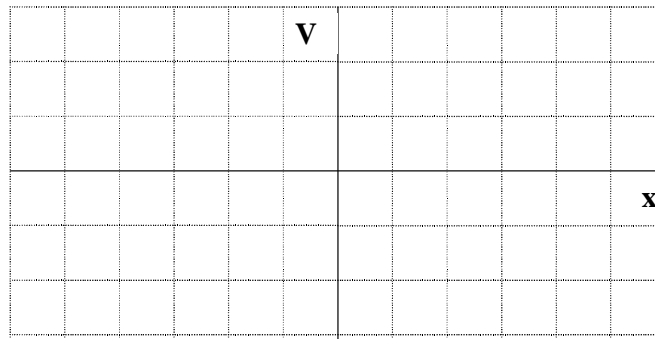
c. At $x = 4a$



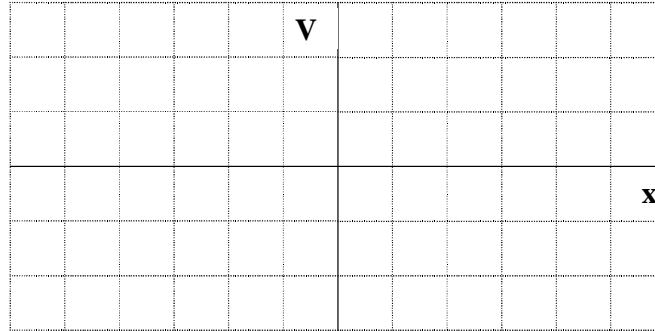
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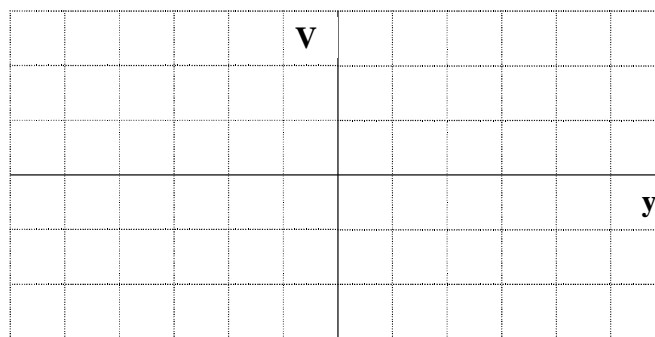
a. At $y = 0$



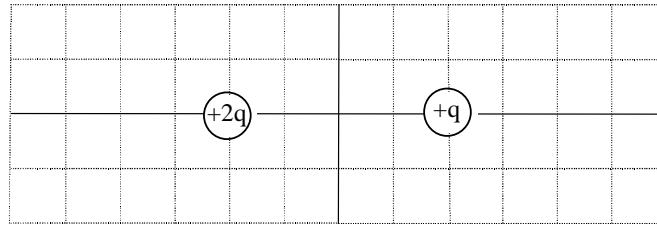
b. At $y = 2a$



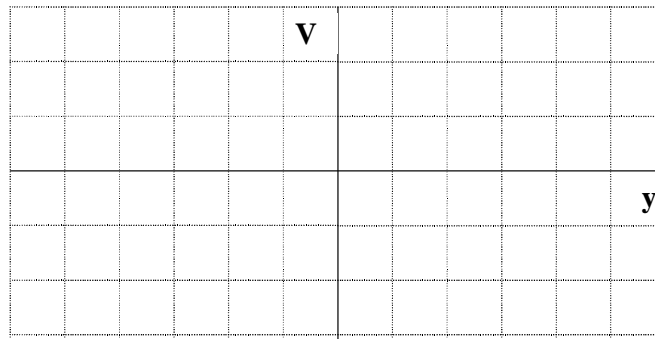
c. At $x = 0$



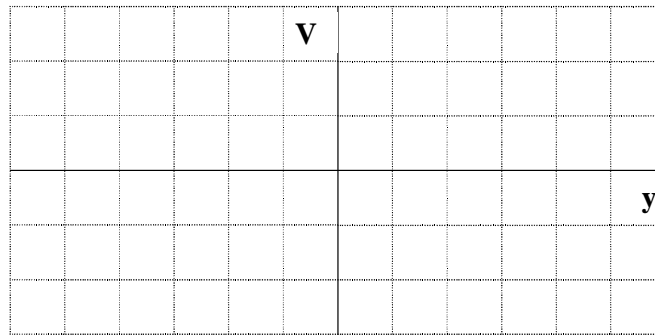
The two charges below are separated by a distance $4a$. Sketch graphs of electric potential for the locations indicated.



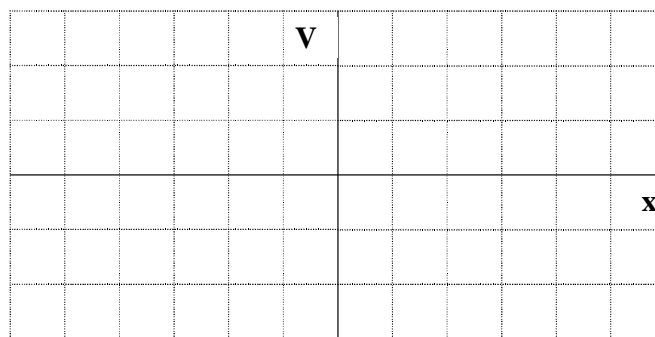
a. At $x = 0$



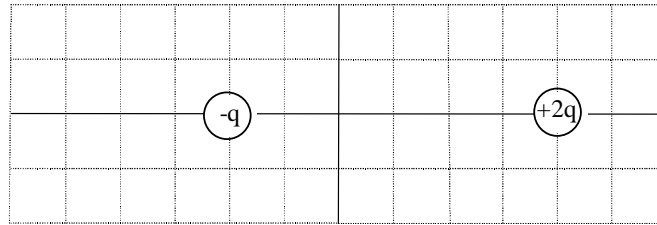
b. At $x = 2a$



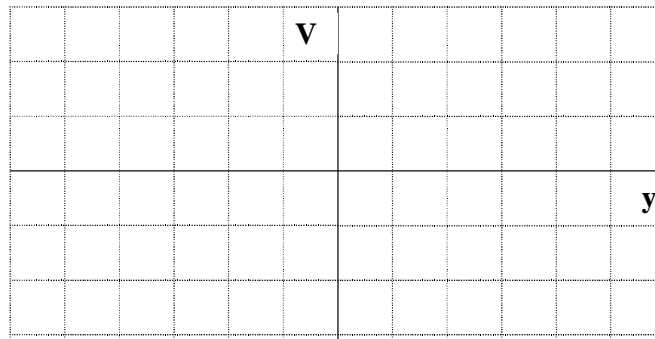
c. At $y = 2a$



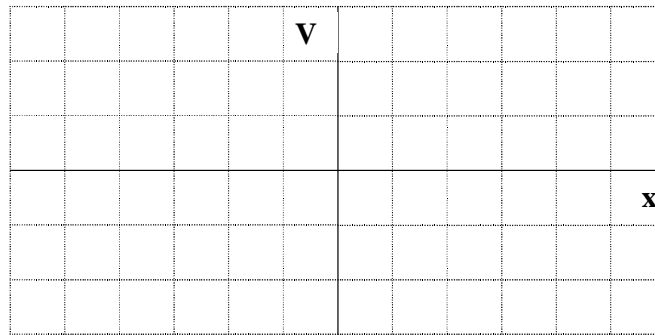
The two charges below are separated by a distance $6a$. Sketch graphs of electric potential for the locations indicated.



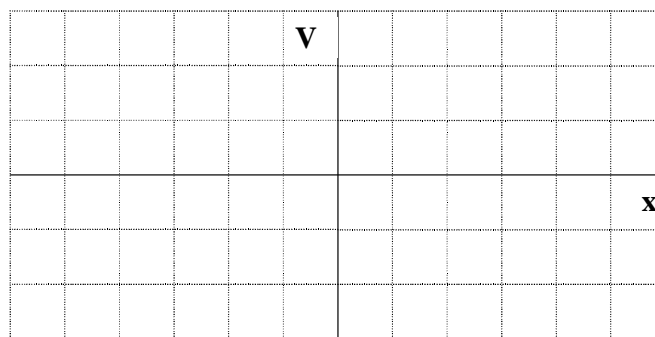
a. At $x = 0$



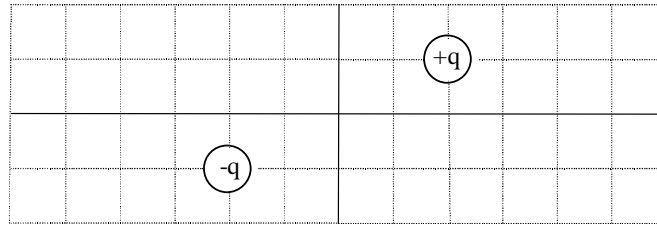
b. At $y = 0$



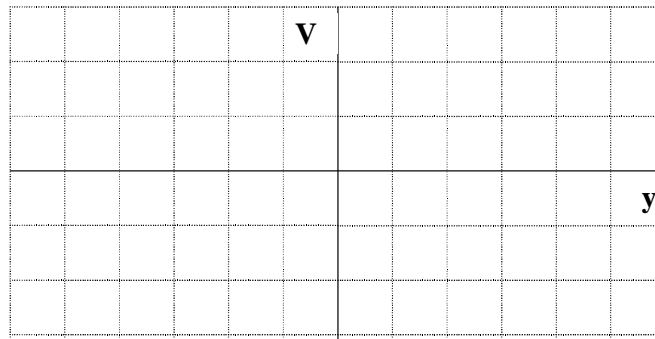
c. At $y = -2a$



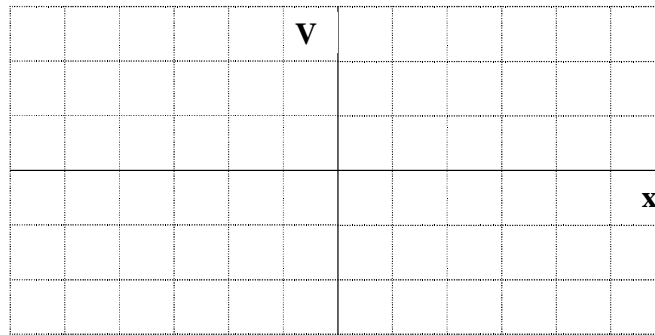
The two charges below are located as shown. Each grid square has width a . Sketch graphs of electric potential for the locations indicated.



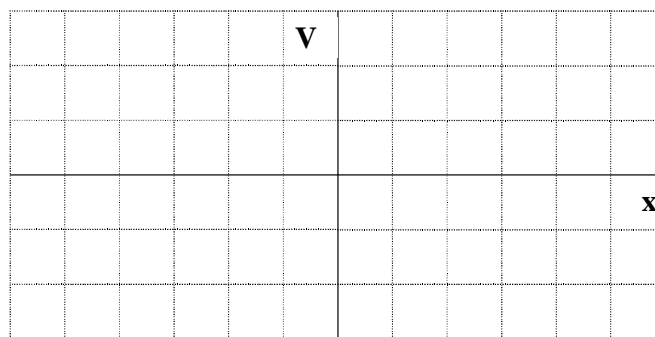
a. At $x = 0$



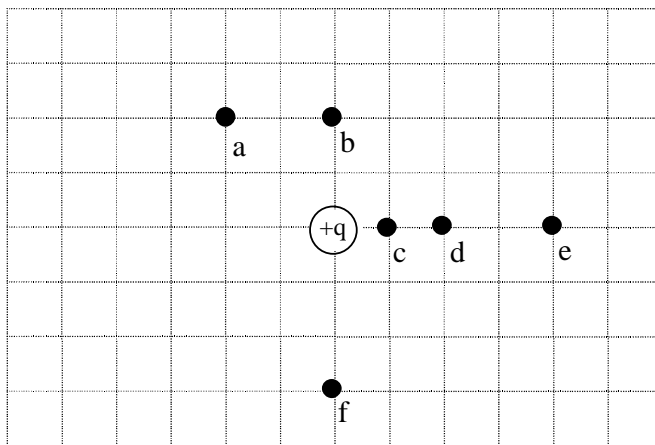
b. At $y = 0$



c. At $y = a$



Rank the locations indicated below on the basis of the electric potential. Rank positive electric potentials as larger than negative electric potentials.



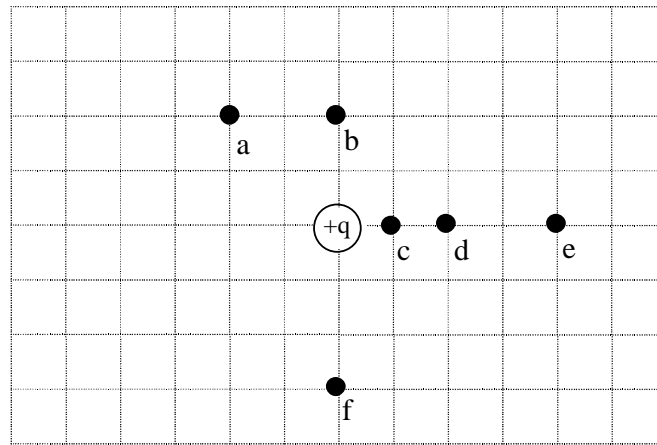
- | | |
|----------|---|
| A | a |
| B | b |
| C | c |
| D | d |
| E | e |
| F | f |

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ 6. ____ Smallest

____ The ranking can not be determined based on the information provided.

Explain the reason for your ranking:

For each of the paths described below, rank the paths on the basis of the change in electric potential.



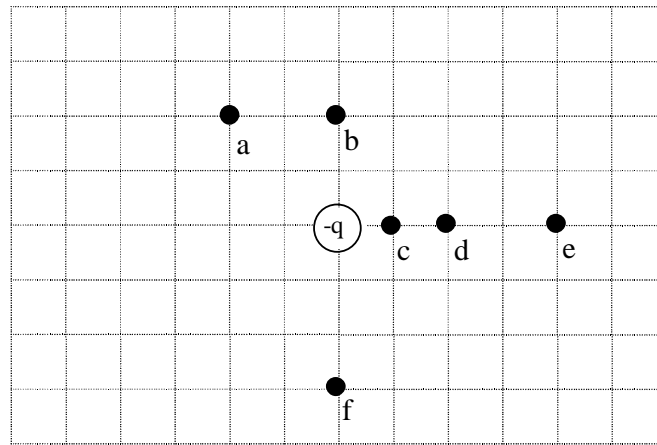
- A** from c to d
- B** from d to c
- C** from d to e
- D** from d to b
- E** from e to f
- F** from b to a

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ 6. ____ Smallest

____ The ranking can not be determined based on the information provided.

Explain the reason for your ranking:

For each of the paths described below, rank the paths on the basis of the change in electric potential.



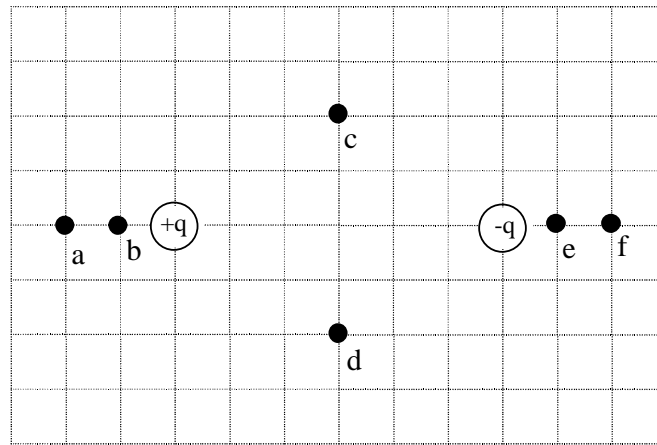
- | | |
|----------|-------------|
| A | from c to d |
| B | from d to c |
| C | from d to e |
| D | from d to b |
| E | from e to f |
| F | from b to a |

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ 6. ____ Smallest

____ The ranking can not be determined based on the information provided.

Explain the reason for your ranking:

For each of the locations indicated below, rank the locations on the basis of the electric potential.



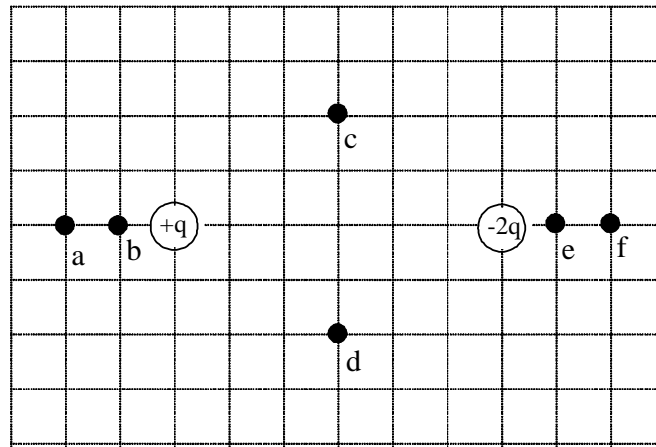
- | | |
|----------|---|
| A | a |
| B | b |
| C | c |
| D | d |
| E | e |
| F | f |

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ 6. ____ Smallest

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Explain the reason for your ranking:

For each of the locations indicated below, rank the locations on the basis of the electric potential.



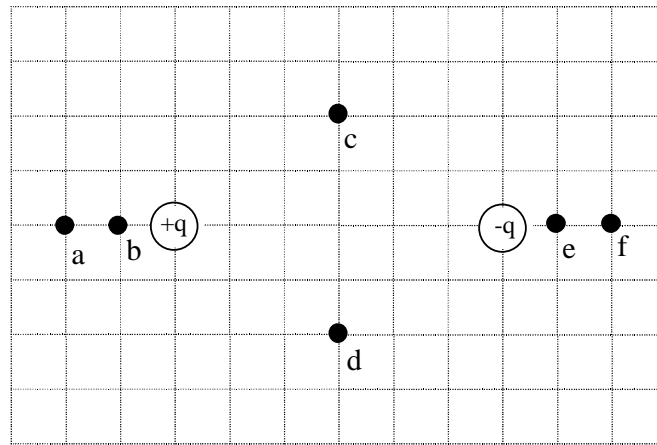
- | | |
|----------|---|
| A | a |
| B | b |
| C | c |
| D | d |
| E | e |
| F | f |

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking can not be determined based on the information provided.

Explain the reason for your ranking:

For each of the paths described below, rank the paths on the basis of the change in electric potential.



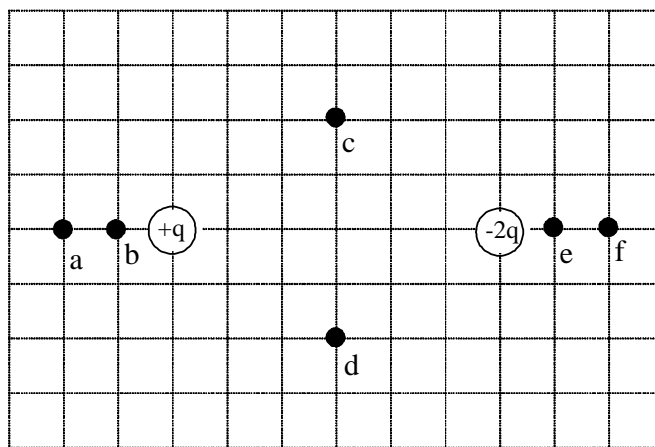
- | | |
|----------|-------------|
| A | from b to a |
| B | from f to e |
| C | from c to d |
| D | from c to e |
| E | from c to b |
| F | from d to a |

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ 6. ____ Smallest

____ The ranking can not be determined based on the information provided.

Explain the reason for your ranking:

For each of the paths described below, rank the paths on the basis of the change in electric potential.



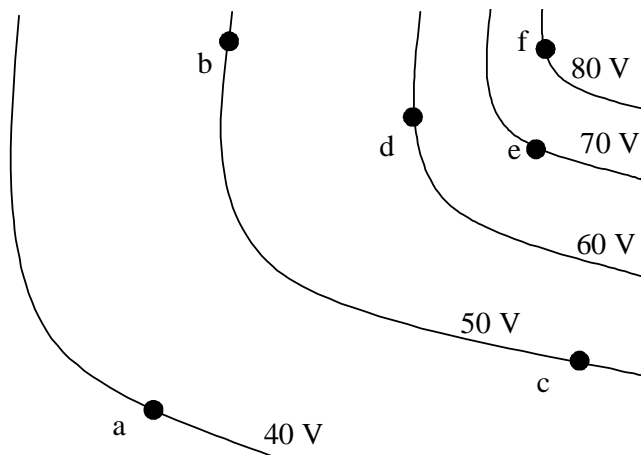
- | | |
|----------|-------------|
| A | from b to a |
| B | from f to e |
| C | from c to d |
| D | from c to e |
| E | from c to b |
| F | from d to a |

Largest 1. ____ 2. ____ 3. ____ 4. ____ 5. ____ 6. ____ Smallest

____ The ranking can not be determined based on the information provided.

Explain the reason for your ranking:

A region of space is filled with the electric potential illustrated below. Various positions within this region are indicated by the points a - f.



- A** an alpha particle (net charge $+2e$) is at rest at a
- B** an electron is at rest at b
- C** a proton is at rest at c
- D** an electron is at rest at d
- E** a neutron is at rest at e
- F** a proton is at rest at f

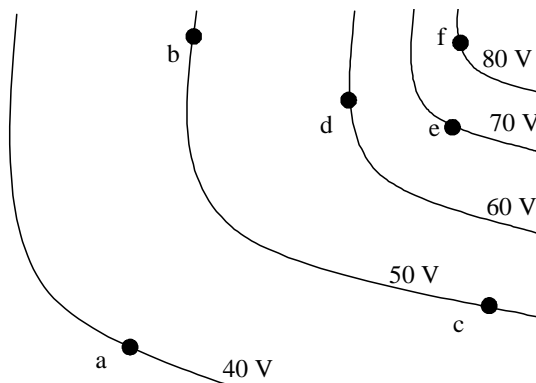
Rank these particles on the basis of their electric potential energy.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking can not be determined based on the information provided.

Explain the reason for your ranking:

A region of space is filled with the electric potential illustrated below. Various positions within this region are indicated by the points a - f. Individual particles are very slowly moved between these positions by an external force.



- A** a proton is moved from a to d
- B** an electron is moved from d to a
- C** a proton is moved from d to f
- D** an electron is moved from b to c
- E** a neutron is moved from a to e
- F** an alpha particle (net charge $+2e$) is moved b to d

a. Rank these motions on the basis of the change in electric potential energy of the particle during the motion.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking can not be determined based on the information provided.

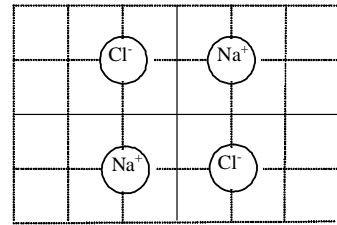
b. Rank these motions on the basis of the work done on the particle by the external force during the motion.

Largest 1. _____ 2. _____ 3. _____ 4. _____ 5. _____ 6. _____ Smallest

_____ The ranking can not be determined based on the information provided.

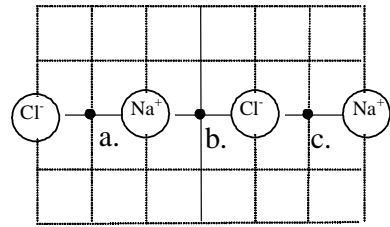
Explain the reason for your ranking:

Find the electric potential at the center of the array of sodium ions (Na^+) and chlorine ions (Cl^-). The interionic separation is $2.82 \times 10^{-10} \text{ m}$.



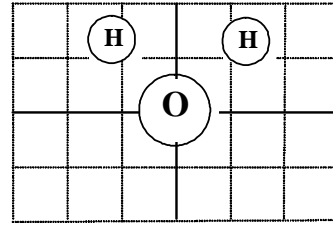
Mathematical Analysis

Find the electric potential at the three indicated points along the chain of Cl^- ions and Na^+ ions. The interionic separation is $2.82 \times 10^{-10} \text{ m}$.



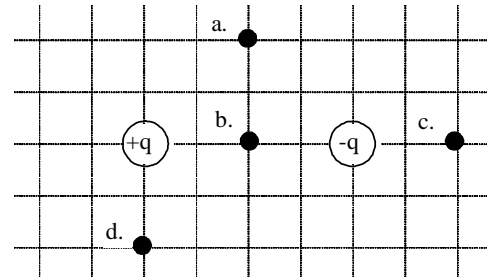
Mathematical Analysis

In the H_2O molecule illustrated at right, the oxygen atom has an effective charge of $-2e$ and each hydrogen atom $+e$. (e is the elementary charge, the charge on a single proton, 1.6×10^{-19} C.) The effective charge separation between the O and each H is 4.9×10^{-12} m. The molecule forms an angle of 105° . Find the electric potential directly between the hydrogen atoms.



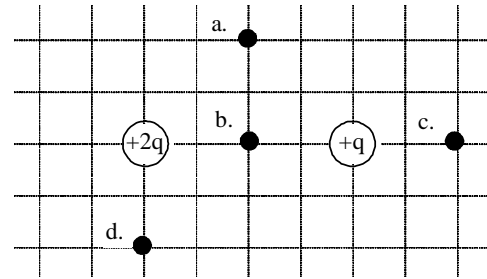
Mathematical Analysis

Find the electric potential at each of the indicated points. The charges are separated by a distance $4a$.



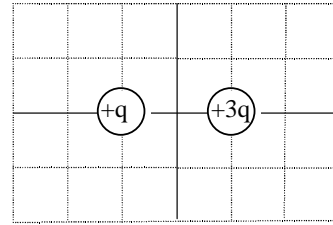
Mathematical Analysis

Find the electric potential at each of the indicated points. The charges are separated by a distance $4a$.



Mathematical Analysis

The two charges at right are separated by a distance $2a$. Find the electric potential at all points on the x -axis and sketch it below.

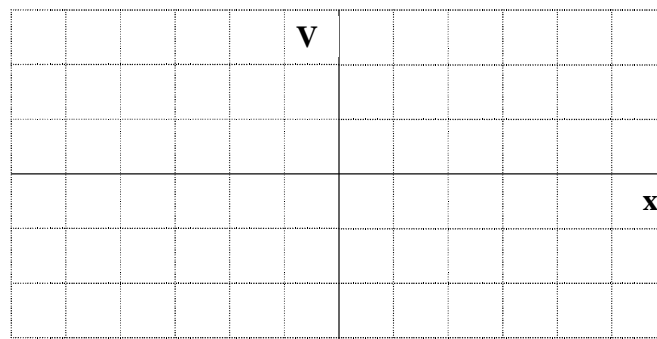


Mathematical Analysis

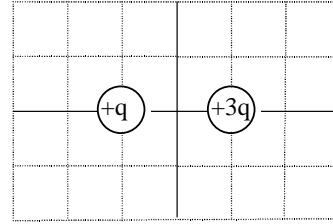
for $x > a$:

for $-a < x < a$:

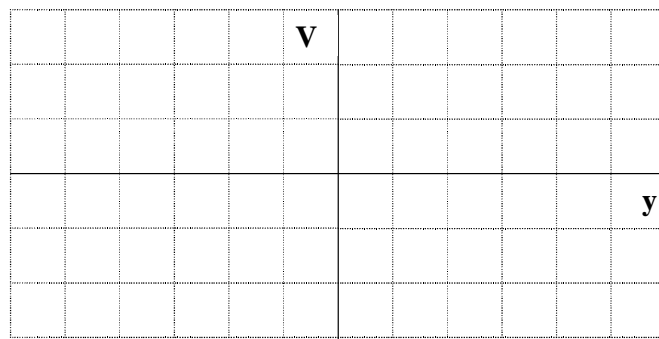
for $x < -a$:



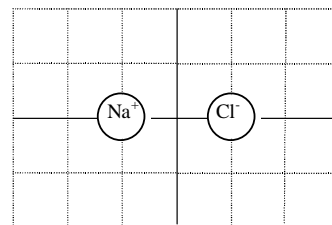
The two charges at right are separated by a distance $2a$. Find the electric potential at all points on the y -axis and sketch it below.



Mathematical Analysis

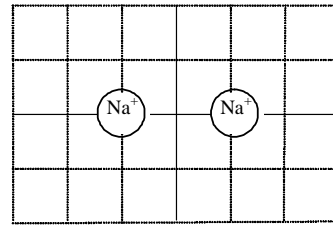


Find the electric potential energy of the Cl^- ion. The sodium ion (Na^+) and chlorine ion (Cl^-) are separated by $2.82 \times 10^{-10} \text{ m}$.



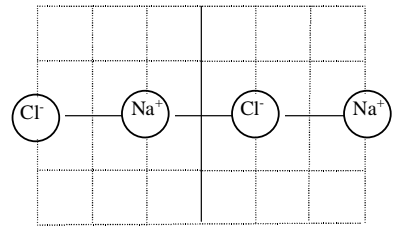
Mathematical Analysis

Find the electric potential energy of the left sodium ion. The two sodium ions are separated by $2.82 \times 10^{-10} \text{ m}$.



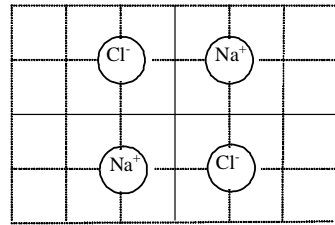
Mathematical Analysis

Find the electric potential energy of the left Cl^- ion. The interionic separation is $2.82 \times 10^{-10} \text{ m}$.



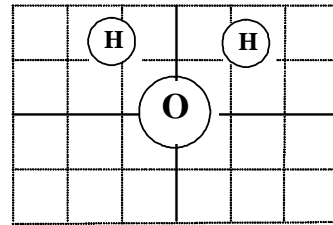
Mathematical Analysis

Find the electric potential energy of the upper left chlorine ion. The interionic separation is $2.82 \times 10^{-10} \text{ m}$.



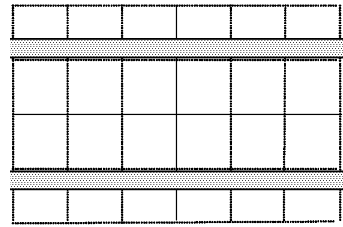
Mathematical Analysis

In the H_2O molecule illustrated at right, the oxygen atom has an effective charge of $-2e$ and each hydrogen atom $+e$. The effective charge separation between the O and each H is $4.9 \times 10^{-12} \text{ m}$. The molecule forms an angle of 105° . Find the electric potential energy of the oxygen atom.



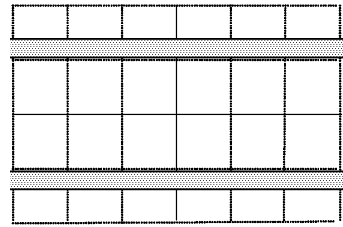
Mathematical Analysis

Two large metal plates, with equal and opposite uniform charge densities of 20 nC/m^2 , are separated by 0.5 cm . Find the potential difference between the plates.



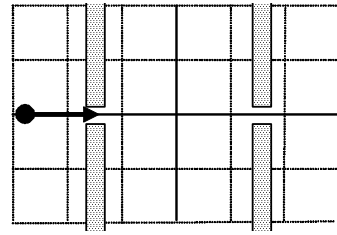
Mathematical Analysis

Two large metal plates are separated by a distance d . A potential difference ΔV is applied between the plates. Find the surface charge density developed on each plate.



Mathematical Analysis

In many applications, oppositely charged parallel plates (with small holes cut for the beam to pass through) are used to accelerate beams of charged particles. In this example, an electron is injected at 2.0×10^7 m/s into the space between the plates.



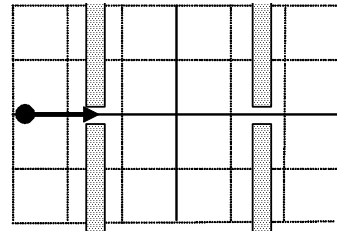
- a. If the electron exits the device at 2.5×10^7 m/s, what is the potential difference between the plates?

Mathematical Analysis

- b. If a proton entered the same device at the same speed, with what speed would it exit the device?

Mathematical Analysis

In many applications, oppositely charged parallel plates (with small holes cut for the beam to pass through) are used to accelerate beams of charged particles. In this example, a proton is injected at 2.0×10^6 m/s into the space between the plates. The potential difference between the plates is 20 kV (the left plate is at higher potential).



a. What is the speed of the proton as it exits the device?

Mathematical Analysis

b. If the charge density on the plates was doubled, with what speed would the proton exit the device?

Mathematical Analysis

In alpha decay, an alpha particle (a bound state of 2 protons and 2 neutrons) escapes from a heavy nucleus and is propelled away due to electric repulsion. For example, a radon nucleus (atomic number 86) will spontaneously transform into a polonium nucleus (atomic number 84) and an alpha particle. Immediately following this transformation, both the polonium nucleus and the alpha particle can be assumed to be at rest, and separated by approximately 50×10^{-15} m. However, the electric repulsion between the two particles will cause the alpha particle to accelerate away from the polonium. Due to its very large mass, the polonium nucleus can be assumed to remain stationary.

- a. What is the initial electric potential energy of the alpha particle?

Mathematical Analysis

- b. What is the velocity of the alpha particle when it is far from the polonium nucleus?

Mathematical Analysis

In an attempt to achieve nuclear fusion, a proton is fired at a fixed deuterium nucleus. (Deuterium is an isotope of hydrogen with one proton and one neutron in the nucleus.) If the distance of closest approach between the proton and the deuterium nucleus is about 1.0×10^{-15} m, the proton and deuterium have a good chance of fusing.

a With what speed must the proton be fired in order to have a good chance of fusing with the deuterium?

Mathematical Analysis

b. If the proton is launched with only half this speed, how close will it get to the deuterium nucleus?

Mathematical Analysis

The Bohr model of the ground state of hydrogen consists of a stationary proton encircled by an electron at radius $5.29 \times 10^{-11} \text{ m}$. In its first excited state, the electron's radius is four times as large.

- a. What is the energy difference between the ground state and the first excited state?

Mathematical Analysis

- b. How much energy is needed to completely ionize hydrogen?

Mathematical Analysis

In a standard x-ray tube, a fine metal filament is heated, "boiling off" electrons. These electrons are then accelerated through a 20 kV potential difference toward a tungsten target. Upon collision with the target, much of the kinetic energy of the electrons is converted into x-rays. How fast are the electrons going when they collide with the tungsten?

Mathematical Analysis