Student Epistemology About Mathematical Integration In A Physics Context: A Case Study

Joshua Von Korff[1], Andrew Elby[2], Dehui Hu[3], N. Sanjay Rebello[3]
[1]Georgia State University, [2]University of Maryland, [3]Kansas State University

1. Research Questions

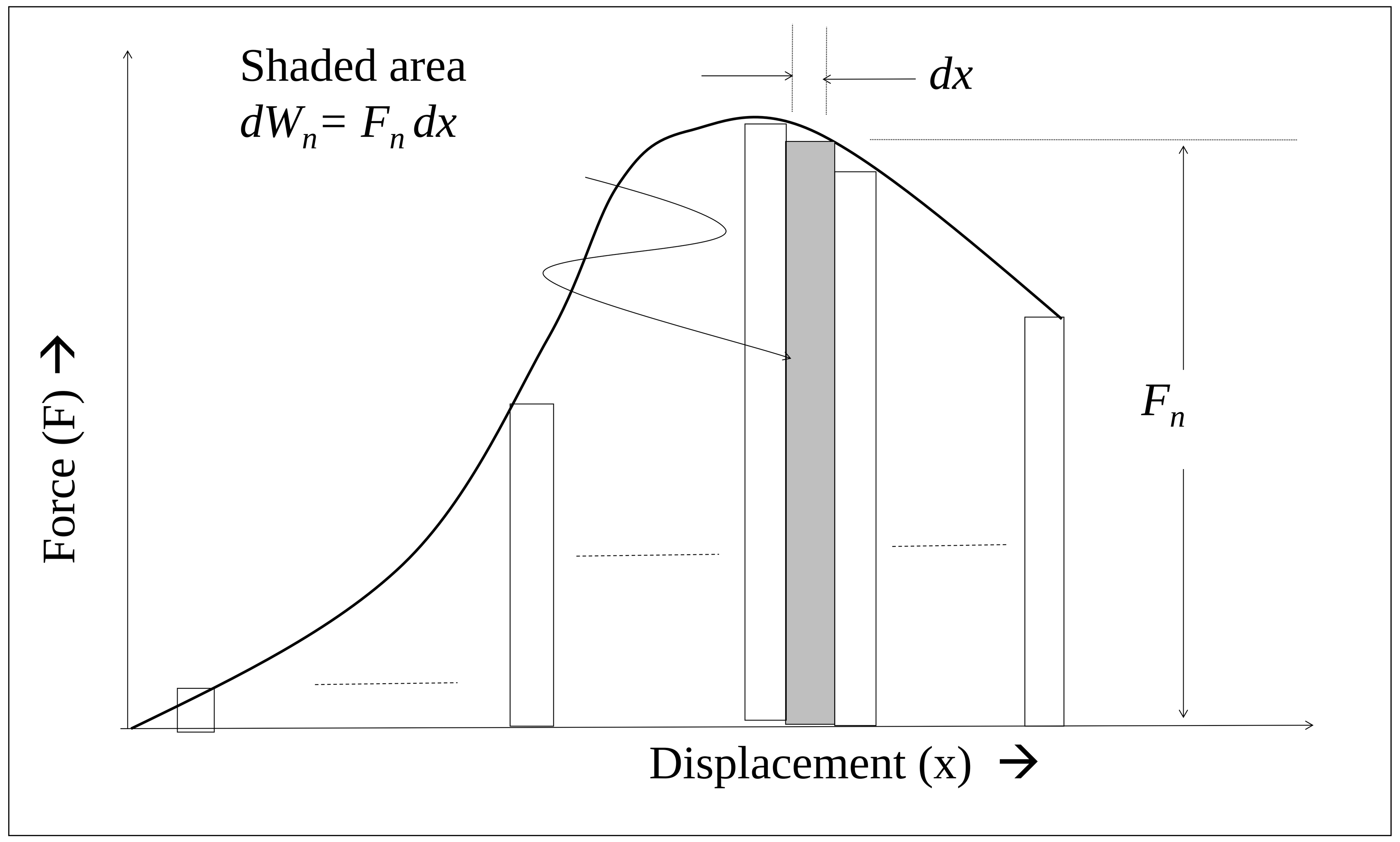
- RQ1: How do students reason about the relationship between integrals and sums in physics?
- RQ2: How do students think about proofs and explanations about setting up integrals?

2. Methods

Initial study: 52 introductory calculus-based physics students.
Students worked on a problem in which a net displacement was to be computed using a table of times, velocities, and accelerations. (E.g. by summing the small displacements $\Delta x = v\,\Delta t$.) They also solved a similar problem in groups using $W = F\,\Delta x$.

One month later: Five students chose to participate in individual interviews. Students attempted the same problems they had worked on one month ago. If they had difficulties, they were given hints. As the interview progressed, students were also permitted to review and comment on their old worksheets.

3. A "layered" understanding of integration



Products and sums: $W = dW_1 + dW_2 + \cdots = F_1\,dx + F_2\,dx + \ \ldots$

4. Layers vs. average height explanation

Question: "Let's say you know this equation for constant force (W = F x). And you wanted to … explain to someone why for a varying force the work is the area under the curve … How would you explain why this is true?"

Our preferred "layers" explanation: The equation $\Delta W = F\,\Delta x$ shows that the area of a small rectangle on the F(x) graph is a small amount of work. These small amounts of work are added up to produce a large amount of work, which is the area under the curve.

Bryce's preferred "average height" explanation: Bryce stated that if you multiply the "average height" of the curve F(x) by the width of the curve Δx, then according to the equation $\Delta W = F\,\Delta x$, you obtain the total work.

Unfortunately Bryce's explanation is flawed, since the equation $\Delta W = F\,\Delta x$ had not been shown to be applicable to Bryce's idea of average force. (In fact, Bryce never defined what he meant by "average force.")

5. Why Bryce prefers "average height."

Bryce says of the two explanations: "I tend to believe things that are, kind of told to me, like in an equation or whatever. So I would personally take the jump to … from one to the average height. But for a lot of people that like wanna, like know like where you're coming from when you say this? [The layers approach] would be a much better way to explain it."

For those other people (not Bryce), the "layers" explanation is "like a more logical stepping stone from one place to the next […] it's I guess easier to like walk through it, like step by step. And make it […] makes sense that way."

6. Conclusions

- Bryce prefers a simpler explanation of the integral for *epistemological* reasons: he tends to believe things that are told to him in the form of an equation.
- However he does not always indicate that he believes equations without requiring an explanation. At one point, he attempts to explain why P = F / A and not P = F A.
- Could it be that multi-step derivations push Bryce away from sense-making? Such derivations contain "logical stepping stones" but are not interesting to Bryce, personally.