"Surprisingly, there is an actual physical application…" Student understanding in Math Methods

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Abstract: Among the canonical physics core courses taken by most undergraduate majors is a course in mathematical methods. Physics education research has begun to explore upper division physics courses, as well as the use of mathematics throughout the physics curriculum. The math methods course is an especially opportune environment to study the development of conceptual understanding of key ideas in mathematics and physics as well as the development of broadly applicable skills and the sociocultural norms of physics. In this poster we will explore some of what happened in a particular math methods course, with attention to the development of student content understanding as well as the development of community norms.

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INTRODUCTION

Physics education research (PER) has begun to make significant inroads into courses beyond the introductory level, with researchers studying student learning in most of the core courses for physics majors. A course in mathematical methods is one such course that is a core requirement in many physics degree programs and that has received relatively little attention in PER.

A group at the University of Colorado has done work in a combined math methods / classical mechanics courses that has some overlap and also uses the Boas text [1]. Numerous studies have investigated student use of math in physics contexts, both at the introductory level [2-6] and in upper-division courses [7-12].

Context for Research

This work has taken place in the context of an upper-division course titled Mathematical Methods of Physics taught at California State University Fullerton (CSUF). CSUF is a large public comprehensive university serving a diverse student population. The Math Methods course, Physics 300, is required for physics majors and is a prerequisite for most of the other upper-division theory courses; for most students it is one of the first upper-division courses they take. The course uses the text by Boas [1] and covers a fairly standard list of topics. It meets for two 75-minute blocks per week. The course has as prerequisites three semesters of calculus, and most students have completed two or more semesters of introductory physics. The author has taught the course four times, with enrollments between 12 and 19, and typically spends a significant portion of class time on small-group tutorial exercises [11].

Research Methods

In this project we have sought to document student understanding of the target ideas using standard methods of PER, including written conceptual questions pre- and post-instruction, individual student interviews, classroom observations, and reflective essays.

CONCEPTUAL UNDERSTANDING

One key thread of this project has been to document student conceptual understanding of course content, both mathematical and physical. We have used written problems on graded and ungraded quizzes and course examinations; these questions have been qualitative or semi-qualitative in addition to more traditional quantitative problems.

For the purpose of this paper, we highlight two sample tasks that illuminate key aspects of the math methods course and the relationship between physics and mathematics that this course brings to the fore. First, we present an example of a mathematical idea that 'slips through the cracks,' and the implications for physics instruction. Secondly, we present an example of an application of 'physicist skills' employed in math problems.

Math that slips through the cracks: complex exponentials and oscillations

When the lead author was first assigned to the course, an instructor of a subsequent course noted that students seemed to have difficulty with complex exponentials. Subsequent investigation revealed some related previous work [13].

Students were given an ungraded quiz in which they were asked to sketch on a blank graph the real part of the function $f(t)=Ae^{i\omega t}$ as a function of time and identify any relevant points. The expectation was that students would sketch a cosine function with amplitude A (thus $f(0)=A$). This problem has been posed in four sections of the course (N=49), after reading and preliminary lecture on the topic. Students informally professed familiarity with Euler's equation ($e^{ix} = \cos x + i \sin x$) from previous courses, but appear not to have made the link to oscillations.

Only about a third of the students have sketched a function that is oscillatory with a nonzero value at t=0. A few more have sketched a sine function (zero value at t=0). Around ten percent have drawn an arrow, possibly a phasor in the complex plane rather than a sketch of $f(t)$. The most common answer, given by over 40% of respondents, has been to draw exponential growth.

This raises questions about whether these students will be able to apply this mathematical formalism to physical contexts without further assistance. That finding resonates with previous research; Sadaghiani [13] reported that 20% of students on one problem chose answers indicating that e^{kx} and e^{ikx} were solutions to the same differential equation, and that a similar number gave an answer to a potential well problem indicating oscillations when an answer for exponential growth/decay would be appropriate.

Physicist skills in math problems: series expansions

A second key type of task that we have used illustrates what we think of as 'physicist skills.' In this case, the problem (see Fig 1) requires that students take a mathematical expression and use a series expansion to determine an approximate result. This task is frequently required in physics courses given analytical results too difficult to solve exactly. It was posed on an ungraded quiz on the first day of class and then used as the basis of an interactive class discussion.

Although it can be found in introductory texbooks, this task is very challenging even for the Math Methods students. Only about 10% answered correctly, and around a third of the students left the problem completely blank despite having ample time to respond. Most of the students were unable to transform the expression given into a form suitable for the expansion, *i.e.,* $(1+x)^p$ with x dimensionless. (In this case the series will only converge and be truncated if $|x|<<1$.) Many students had difficulty in recognizing that the exponent p in the expression $(1+x)^p$ is -2 in this case, given that the squared expression is in the denominator.

Student responses illustrate interplay between math and physics. When asked what simplification is allowed by the fact that *y>>d,* about 20% of students stated that the electric field is zero if *y* is large. Another third of all students stated explicitly that the fact that *y>>d* allows one to set *d=0* (or set *y-d/2=y*). These statements lead to the same conclusion but are phrased in different terms, one physical and one more mathematical. The conclusion might be appropriate in certain circumstances, but in this case it essentially eliminates the interesting physics of the dipole configuration, in which the slight extra distance from the field point to one charge has important physical consequences. If *d=0* the two terms add to zero and there is no electric field, but do the students use the idea that *d=0* to predict that the field is zero, or do they use their intuition that the field vanishes (perhaps thinking of what happens at infinity) and force the math to support this intuition? It might also be students come to the same incorrect conclusion for different reasons. Similar issues have been documented in related problems [12].

This particular task is a useful one because it illustrates a case in which it is insufficient to say that *y>>d* means *d=0*. It is also an authentic physics example that builds upon introductory electricity and magnetism but can provide a bridge to higher-level content. A math course might provide students with a 'cleaned up' expression to simplify student calculations (and instructor assessment), but students need to use this technique 'in the wild,' with expressions that include constants and other parameters and thus have dimensionality.

NORMS AND EXPECTATIONS

While we have focused extensively on conceptual understanding, a significant goal of this project has been to document other aspects of the course. As one of the first upper-division courses for physics majors, this course plays an important role in establishing norms and forming expectations.

> Imagine that you are solving an E&M problem and you derive the expression $E = \frac{+kQ}{\left(y-\frac{d}{2}\right)^2} + \frac{-kQ}{\left(y+\frac{d}{2}\right)^2}$ for the electric field at a certain location, with y >> d.
>
> The first term of the expression is $\frac{+kQ}{\left(y-\frac{d}{2}\right)^2}$. Use the binomial expansion to construct the first three terms of a series for this term given that y>>d. Explicitly identify what *x* and *p* you are using. What simplification is allowed by the fact that y >> d?

FIGURE 1. A problem posed to students early in the Math Methods course. An algebraic version of the binomial expansion was given to students.

We believe that the process of enculturation plays a significant role in the development of physics majors and have examined this process in this course and others [14]. We have also examined student expectations in the course, particularly as it relates to the relationship between math and physics; we highlight one aspect of this latter question.

We present responses from a reflective essay assigned to students at the conclusion of the course. Students were asked to give advice to students taking the course in the future, and comment on what they saw as the important ideas of the course.

The responses to this prompt included a variety of statements ('don't leave your book in San Jose') but several reflect the underlying tension between math and physics:

> *Advice that I would give to a future PHYS 300 student would be not treat the course as just a math course. … be ready to use a lot of improvisation; I beleive [sic] that a major component of this course required that I try to correspond a mathematical method to a certain physics concept on the spot.*

A second student also articulated a difference between math and physics, and suggests that his understanding of the purpose of the course evolved through the semester.

The phrase 'bridge the gap' appears in many descriptions of this course and texts.

> *Throughout this entire course I feel that the core idea of this class was to bridge the gap between math and physics. This realization occured to me during the last two weeks when we covered eigenvalues. Surprisingly, there is an actual physical application.*

A third student was in the somewhat unusual situation of taking the course at the same time as he took junior level electricity and magnetism, for which math methods is normally a prerequisite. This student wrote:

> *I would emphasize that it is beneficial to have a clear understanding of these mathematical tools in 300, so that when problems appear later that require them, one can focus on the physics revealed by them rather than just trying to figure out how the math works.*

Each of these students make a point of distinguishing between 'math' and 'physics' and the language used is revealing ('gap'). Students write about math 'method' and 'tools,' in contrast to physics as 'concept' and an 'application.' The last student uses the provocative phrase 'the physics revealed by them [math tools].' We have additional evidence of this perceived divide and characterization of math and physics.

There are numerous lenses through which one might view these responses. One is by examining the process of transfer, and the beliefs students have about the roles that math plays in physics courses. We intend to explore this further and the implications it has for student learning. At the same time the students have adopted a certain stance that mirrors that of some of their faculty, suggesting a process of enculturation. How do students come to perceive differences between physics and math, and how do they come to identify with a field?

CONCLUSIONS

These preliminary results suggest that the Math Methods course is a fruitful ground for further research, in terms of student conceptual understanding of physics and math but also in terms of the ways in which students join the culture of physics. We have illustrated that even these relatively sophisticated students struggle in applying math concepts in the course, identified math that falls through the cracks in previous courses, and observed student perceptions of the differences between math and physics.

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