Student ability to reason about basic quantum mechanics in the context of perturbation theory

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Introduction: We investigate student reasoning of basic, foundational ideas within the context of time dependent perturbation theory (TDPT). The foundational ideas of interest are

(1) energy measurements

(2) time dependence of quantum states

Student difficulties with these ideas hinder their ability to solve more complex, and often more physically interesting, problems.

Context for research: This research has been performed over several years during two quarters of junior-level QM at the University of Washington (UW). This research is guiding the development of QM Tutorials. Data is gathered using pre- and post-tutorial tests (both after lecture instruction) and individual student interviews.

Pre-tutorial data: The pre-tutorial question shown at right probes student understanding in the context of a TDPT problem. These questions are basic in nature, not requiring the use of complex TDPT mathematical equations, and thus they could have been asked very early in a QM course. The results are surprising as we found that many students struggle with ideas that are considered basic and foundational.

Tutorial instruction: Students worked through a tutorial on TDPT. Two distinct versions have been administered. An early version was designed to walk students through the derivation of the TDPT equation in addition to addressing some of the foundational difficulties as identified in the pretest. The second was modified to have less of a focus on the derivation and greater attention was given to examining the context and reflecting on how connect the equation to the context.

Post-tutorial data: Several post-tutorial exam questions have been administered. In 2013 (N=48) an exam question presented a TDPT scenario in which a perturbation was made to a harmonic oscillator potential. Students were asked about the probability of measuring a specific energy during and after the perturbation. The percentage of correct responses are shown:

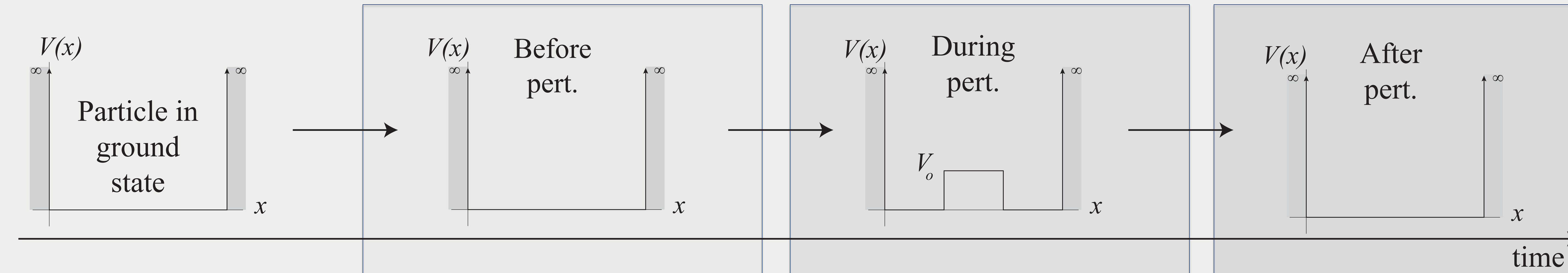
During perturbation	After perturbation
60%	40%

While we see some improvement over pretest results, explanations indicate many students use the same incorrect reasoning as on pretest.

Interview setup: Interviews were performed in order to gain insight into how students were thinking about these questions. Data presented here are from two top students (GPA of 4.0 and 3.8 in the relevant QM course) who had similar difficulties as demonstrated on the pretest. The interview consisted of the following steps:

1) Questions similar to the pretest (among others) were asked. The main difference was that the functional form of the potential was not given, which required students to think more generally about the problem and not rely on math alone.
2) Questions on the foundational ideas were asked within the context of a simple particle in a box problem (without any perturbation).
3) The original questions were asked again and participants were asked to reflect on their original answers.

A time dependent perturbation to the infinite square well



	Before	During	After
Question 1: What values could an energy measurement yield before, during and after a perturbation? This requires students to: • Think about what values of energy can be measured during a perturbation • Determine what energy values can be measured once a perturbation has been turned off	Correct reasoning: only the ground state can be measured, since nothing has happened that would evolve it out of the ground state. 50% answered correctly Common incorrect responses: • The perturbation hasn't been applied yet, so all energies are possible Note: all answers are considered correct only if accompanied with correct reasoning.	Correct reasoning: only energies of the perturbed well are allowed. These can be determined using perturbation theory. 55% answered correctly Common incorrect responses: • Linear combination of unperturbed energies • Calculate a range of possible energies by adding the ground state energy to V_o	Correct reasoning: the state evolved during the perturbation, so it is no longer in the ground state. Any energy of the infinite square well can be measured (dependent on the shape of the perturbation). 15% answered correctly Common incorrect responses: • The particle would return to whatever state it was prior to the perturbation, which in this case, is the ground state energy
Question 2: Does the spatial probability density depend on time? This requires students to: • Demonstrate knowledge of the spatial probability density • Determine whether or not the state is a superposition of energy eigenstates	Correct reasoning: The particle is in an energy eigenstate, so the probability density will not depend on time. 25% answered correctly Common correct answers with incorrect reasoning: • The wave function is time independent, so the probability density must be as well • Because the Hamiltonian is time independent, the probability density must be as well	Correct reasoning: The particle is no longer in an energy eigenstate, so the probability density will depend on time. 15% answered correctly Common incorrect responses: • The perturbation caused the Hamiltonian to be time dependent, so the probability density will depend on time	Correct reasoning: The particle evolved out of the ground state and into a superposition state during the perturbation, so the probability density depends on time. 15% answered correctly Common incorrect responses: • The wave function is time independent, so the probability density must be as well • Because the Hamiltonian is time independent, so must the probability density

These questions were administered to 71 students between 2005-07 near the end of a second quarter junior-level QM course.

Interview data: We expected students to struggle with the initial questions, based on what we have seen in pre and post-testing. However, we believed that after answering similar questions in the simple context, they would recognize their prior errors and correct their mistakes.

As anticipated, both students struggled with portions of the interview. In particular, they both incorrectly determined that the state of the system had no time dependence until the perturbation is turned on.

After correctly answering the particle in a box questions, they both returned to the TDPT scenario and corrected many or all of their previous incorrect answers.

Conclusion: Students struggle applying basic ideas to more complex scenarios. These ideas are proving to be persistent and are hindering the ability of many students to master more difficult topics. These results are informing the development of QM tutorials. Further research is required in order to better prepare students for further study in physics, or for applying these ideas to other areas.

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