Assessing Student Learning in Middle-division Classical Mechanics / Math Methods

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Introduction & Motivation

- Assessment tools helped drive the study of student learning in introductory physics[1-3] by:
  (1) ***identifying common and persistent student difficulties***
  (2) ***providing evidence for the effectiveness of transformed pedagogies and curricula***.
- Similar research into student learning in upper-level physics courses has begun.[2,4-10]
- At CU Boulder, we are transforming the first half of our classical mechanics sequence.[11,12]
- To assess the transformed course and to help further investigate student difficulties at this level, we have begun to develop the Colorado Classical Mechanics / Math Methods Instrument (CCMI).
- The CCMI is an 11-problem, open-ended instrument.

Designing the CCMI

Writing Questions:
- CU Faculty developed a set of consensus learning goals.
- Sixteen open-ended problems were initially written informed by observed difficulties.[8]

Expert Validation:
- Faculty (CU and elsewhere) reviewed the CCMI for content and clarity.
  (1) *Does each problem address the concepts and skills my students should master?*
  (2) *Is each problem written in a clear and concise manner?*
- Problems were improved iteratively leading to the 11-problem version (9 mandatory, 2 optional) that can be completed in 50 minutes.

Student Validation:
- Seventeen student interviews were conducted at CU to ensure the CCMI probed common difficulties and that students interpreted problems appropriately.

References

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Sample Problem: Energy Contour Map (Pre & Post-test)

Learning goals evaluated:
- Students should be able to predict direction and magnitude of the gradient given equipotential lines.
- Students should be able to determine the relative magnitude and direction of a force on a set of drawn equipotential lines.
- Students should be able to recognize equilibrium points on a plot of potential energy, U, and determine if these points are stable.

Problem Scoring Rubric (5 total points, ~15% of CCMI):

Part (a)

Full credit (1)	Correct	Identifying Stable equilibrium only (B)
No credit (0)	Incorrect	No credit for any other responses

Part (b)

Full credit (1)	Correct	Reasonable answer to correct response only, for example: • Lowest potential energy, Valley/well analogy, Stable against small pushes
No credit (0)	Incorrect	No credit for any other responses

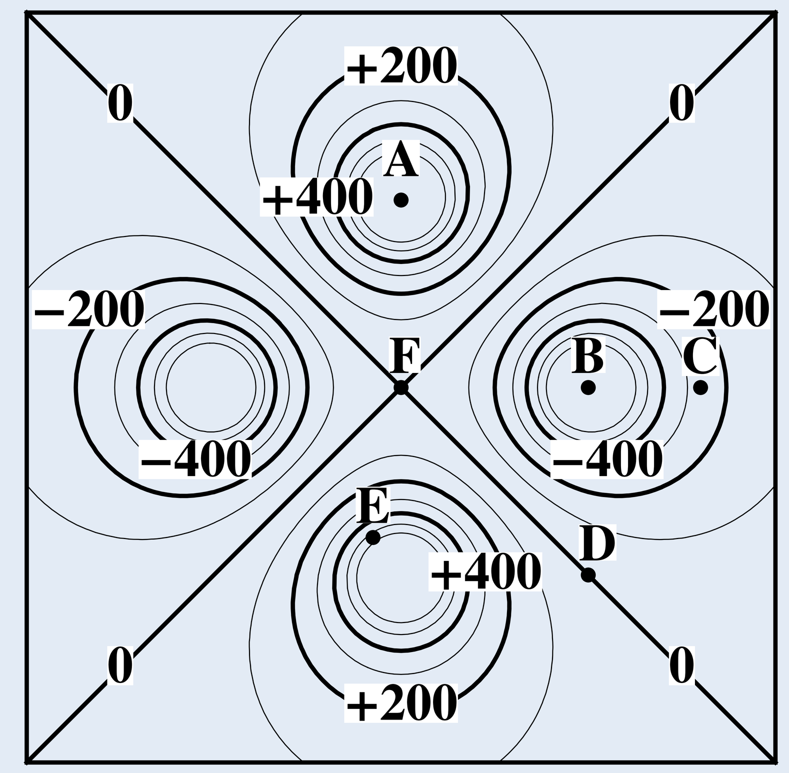
Part (c)

Full credit (1)	Correct	Either E>C>D>A=B=F or E>C>D=A=B=F (unclear if there's a non-zero gradient at D)
Half credit (0.5)	Incomplete answer	Correct answer but *1 location is missing* (e.g., E>C>A=B=F or E>C>D>A=B)
No credit (0)	Incorrect	No credit for any other responses

Part (d)

Full credit (2)	Correct	Perfect sketch at all points • All arrows in direction of decreasing potential • Correct relative magnitudes • Arrows perpendicular to contour lines • Zero at equilibria (A, B, F, and, possibly D)
Partial credit (1.75)	Magnitude error	Magnitude of arrows are *seriously* sketchy • Not obviously longer for larger gradients
No credit (0)	Incorrect	No credit for any other responses

Statement of the Problem (Appears on pre- and post-test):
Below is a plot of the potential energy, in Joules, of a particle free to move on a 2-d plane.

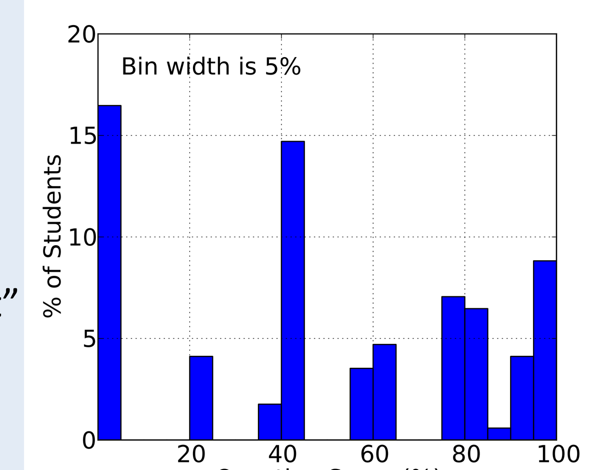


(a) For which of these points (A--F) is the particle in stable equilibrium?
(b) Please explain how you decided on the above answer:
(c) Rank the magnitude of the gradient at the above points, from largest to smallest. (If some points have equal gradients with equal magnitude, please make it clear in your answer.

(d) Draw vectors that represent the force, $\vec{F}$, at points A--F on the diagram above. Please make sure that the relative magnitude of your vectors is consistent among the points. Indicate clearly if $\vec{F} = 0$ at any of the points.

Student Performance:

Common & persistent difficulties:
(a) Identifying all equilibria (A,B,F)
(b) Including justification for A, F
(c) Ordering by energy "height" or absolute value of energy "height"
(d) Arrows at equilibria or arrows parallel to contour lines



Sample Problem: Writing a Differential Equation (Post-test only)

Learning goal evaluated:
- Students should be able to use Newton's laws to translate a given physical situation into a differential equation.

Problem Scoring Rubric (4 total points, ~12% of CCMI):

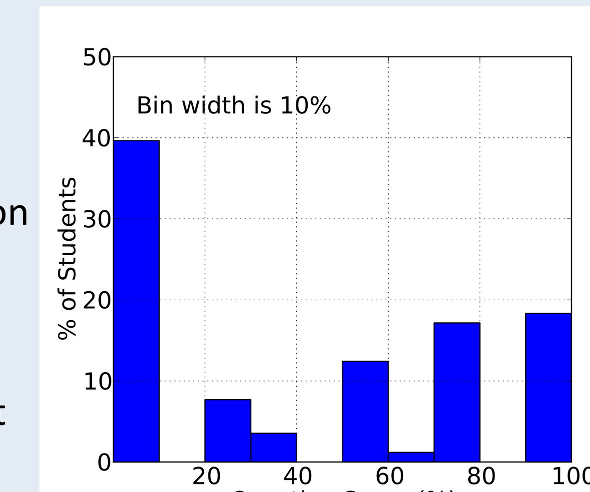
Full credit (4)	Correct	$m\ddot{x} = -c\,x^2 + k/(10-x)$ or an equivalent form
Minus 0.5 points	Neglected mass	Mass does not appear in the differential equation
Minus 0.5 point each (1 max)	Neglected coefficients	Constants (c, k) do not appear in the differential equation (1 point each)
Minus 1 point each (2 max)	Distance dependence error	Any errors in the distance dependence, e.g., • x instead of x^2 in the first term • 1/x instead of 1/(10-x) in the second term 1 point for each term
Minus 1 point each (2 max)	Sign error	Sign error in front of either term, e.g., • negative sign instead of positive sign for the second term as written above
No credit (0)	Incorrect	No credit for any other responses • First order equation in x • Differential equation equal to a constant • Not a differential equation

Statement of the Problem (Appears only on post-test):
A particle (mass, m) is confined to move on the x–axis between two objects that attract it. The particle does not leave the region between the two attractive objects.
- One object is located at x = 0 and the attractive force between the object and the particle is proportional to the square of the distance between them with proportionality constant c.
- The second object is located at x = 10 and the attractive force between the object and the particle is inversely proportional to the distance between them with proportionality constant k.

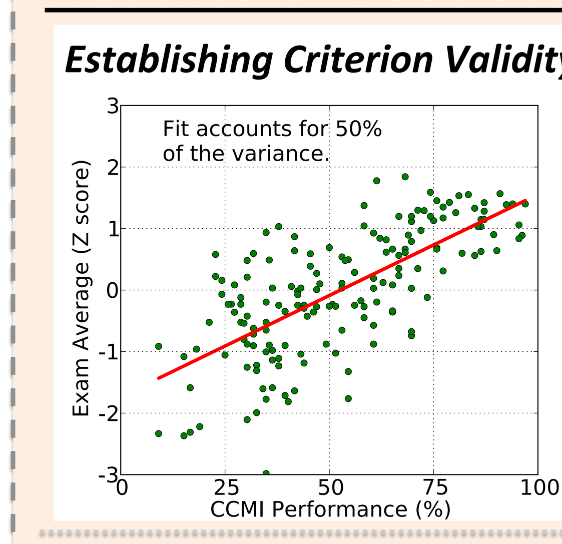
Write down a differential equation that describes the position of the particle as a function of time, x(t).

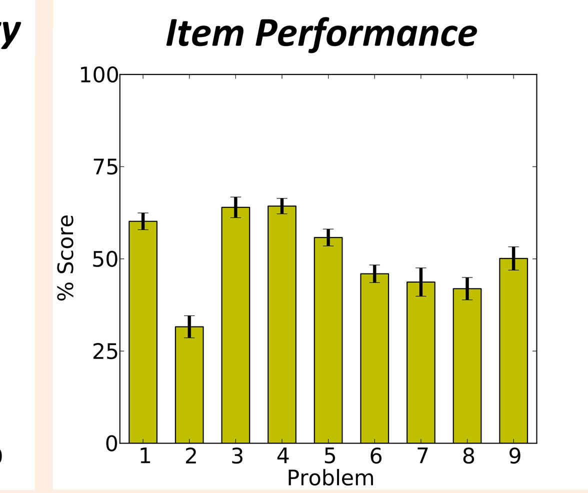
Student Performance:

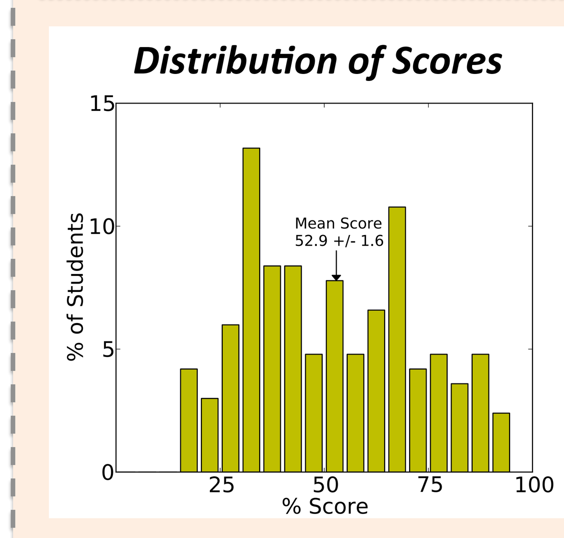
Common & persistent difficulties:
- Sign mistake on either term
- Incorrect distance dependence on either term
- Neglecting coefficients or constants
- Answering with an equation that is not a differential equation



Results & Test Statistics (N=167)







Item-test correlation
> 0.2 sufficient[13]
- *Q1, Q3-Q9: r= 0.45-0.53*
- *Q2: r = 0.31*

Q2 covers Taylor expansions; challenging for CU students[6]

Internal Consistency (α)
> 0.8 is sufficient[14]
- *α = 0.77*

More data needed to better estimate true α[14]

Preliminary Comparison of Student Performance at CU
3 recent semesters of pre/post data from CU Boulder.
Pedagogical codes: Clickers (CQ), in-class Tutorials (T), Group Problem sessions (GP), and Lecture (L)

Sem.	Faculty	Pedagogy	N	PRE	POST
1	PER	CQ, T, GP, L	62	34.8 ± 3.4	59.7 ± 2.8
2	TRAD	CQ, T, GP, L	41	25.3 ± 2.9	46.1 ± 3.0
3	TRAD	CQ, L	64	32.1 ± 2.7	51.0 ± 2.5

- A Kruskall-Wallis test detected a group difference[15]
- Pairwise Mann-Whitney tests[16,17] demonstrated that Sem. 1 students outperformed Sem. 2 & 3
- Learning gains appear higher in Sem. 1 and 2

Remarks

- The CCMI is a valid and reliable instrument for investigating student learning at CU.
- The CCMI can detect differences in instruction at CU.
- Student performance on the CCMI has informed the research agenda.[6]
- More data is needed from a variety of institutions and instructional contexts.

How can I help?

Willing to give the instrument to your students?
Interested in helping conduct interviews with physics students at your school?
Contact: caballero@pa.msu.edu if you are interested.