Upper-division Student Understanding of Coulomb's Law: Difficulties with Continuous Charge Distributions

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Introduction

Previous work has shown that ***students struggle to manipulate and make sense of the mathematics necessary to solve problems in physics*** [1]. This is of particular concern at the upper-division level where the physics content demands increasing mathematical sophistication.

As part of broader course transformations of junior-level Electricity and Magnetism (E&M) [2], we investigated ***upper-division students' difficulties using Coulomb's Law*** to calculate the electric field, *E*, or electric potential, *V*, from a continuous charge distribution.

The ***ACER Framework for the use of mathematics in physics*** [3] provided an organizational structure to identify students' difficulties manipulating the integral expression of Coulomb's Law.

The ACER Framework

ACER is a theoretical framework for the use of mathematical tools in context-rich physics problems which uses a researcher-guided outline describing the **Activation of the tool**, **Construction of the mathematical model**, **Execution of the mathematics**, and **Reflection on the result** (**ACER**).

Utilizing ACER

- The framework can be applied to a particular problem or a general class of problems using a specific mathematical tool.
- Here, we operationalize the framework for the exam question shown to the right.
- This outline can be extended to the broader category of problems involving the use of Coulomb's Law to calculate *E* or *V* from an arbitrary, static charge distribution.

The Course & Data Sources

Junior-level E&M I

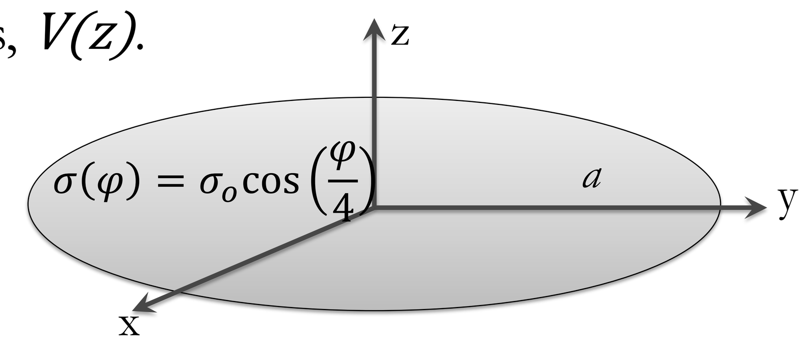
- Chapters 1-6 of Griffiths [4]
- Physics, engineering physics, and astrophysics majors.
- Transformed to include interactive engagement

Data Sources

- 4 semesters of student (N=172) solutions to a canonical exam question on continuous charge distributions (shown below)
- 5 one-on-one think-aloud interviews where students solved a slightly more advanced version of the exam question.

The Exam Question

Given a disk of radius a with surface charge density $\sigma(\varphi) = \sigma_o \cos\left(\frac{\varphi}{4}\right)$, determine the electric potential on the z-axis, *V(z)*.



Activation of the Tool

This component of the framework involves the selection of a solution method.

Elements

- The question asks for potential, *V*.
- The charge distribution is provided.
- Finite disk symmetry is not sufficient to use Gauss' Law.
- Direct calculation of *V* via Coulomb's Law is most efficient.

$$V(\vec{r}) = \frac{1}{4\pi\varepsilon_o} \iiint \frac{dq}{\vec{r} - \vec{r}'}$$

Findings

- Roughly a quarter of the students (N=46 of 172) attempted to find *V* by first calculating *E*.
- Of these, half (N=21) attempted to calculate *E* using Gauss' Law.
- Interviews did not investigate Activation.

Execution of the mathematics

This component of the framework deals with the mathematics required to compute a final expression.

Elements

- ε_o, σ_o are constants
- z is a parameter
- r' and φ' are variables

$$V(z) = \frac{1}{4\pi\varepsilon_o} \int_0^{2\pi} \int_0^{a} \frac{\sigma_o \cos\left(\frac{\varphi'}{4}\right)}{\sqrt{r'^2 + z^2}} r' dr' d\varphi'$$

$$= \frac{\sigma_o}{\pi\varepsilon_o}\left(\sqrt{a^2 + z^2} - z\right)$$

Findings

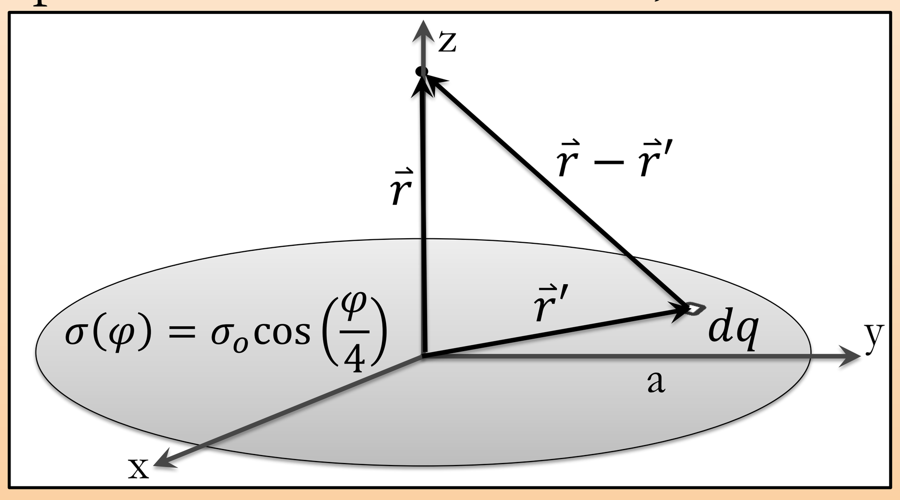
- Half of the students who progressed far enough to solve one or more integrals (N=53 of 103), made mathematical errors.
- Slightly more than half of these students (N=31) made significant errors such as pulling integration variables outside of integrals.
- Of the 4 interview participants who progressed far enough, 2 made significant mathematical errors.

Construction of the Model

This component of the framework deals with the math-physics connection.

Elements

- Select a coordinate system consistent with the geometry of the charge distribution.
- Express the differential charge element, dq
- Select limits consistent with dq and the physical system
- Express the difference vector, $\vec{r} - \vec{r}'$



$$V(z) = \frac{1}{4\pi\varepsilon_o} \int_0^{2\pi} \int_0^{a} \frac{\sigma_o \cos\left(\frac{\varphi'}{4}\right)}{\sqrt{r'^2 + z^2}} r' dr' d\varphi'$$

Findings

Common difficulties with dq (top) and script-r (bottom). Codes are not exclusive or exhaustive. Similar difficulties were observed in interviews.

Percentages are of just the students who had difficulty with dq (N=64 of 154).

Difficulty	N	Percent
Calculated total charge e.g. $dq = Q_{tot}\, dr\, dz\, r d\varphi$	10	16
Not integrating over charges e.g. $dq = \sigma\, dr\, dz\, r d\varphi$	39	61
Differential w/ wrong units e.g. $dq = \sigma\, dr\, d\varphi$	12	19
Other	10	17

Percentages are of just the students who had difficulty with script-r (N=72 of 154).

Difficulty	N	Percent
Ring of charge e.g. script-r = $\sqrt{a^2 + z^2}$	27	38
Distance to the source point i.e. script-r = r'	11	15
Distance to the field point i.e. script-r = z	10	14
Never expressed script-r	9	12
Other	15	21

Reflection on the Result

This component of the framework involves verifying that the result is consistent with expectations.

Elements

- Verify that the units are correct
- Check that the limiting behavior is consistent with Q_{tot} and geometry.

$$V(z \to large) \sim \frac{1}{4\pi\varepsilon_o} \frac{2\sigma_o a^2}{z^2} = \frac{1}{4\pi\varepsilon_o} \frac{Q_{tot}}{z^2}$$

Findings

- Only a small number of students (N=13 of 154) made explicit attempts to reflect on their final expressions.
- None of the interview participants made more than superficial attempts to reflect on their results.

Summary & Implications

We found that our upper-division students had difficulty ***activating Coulomb's Law*** as a solution method and ***spontaneously reflecting*** on the result. Additionally, they struggle to ***synthesize conceptual and mathematical resources to construct mathematical expressions consistent with the physical descriptors of the problem***.

To address these difficulties, we advocate ***explicit emphasis which aspects of the problem cue the students to activate Coulomb's law*** as well as ***how and when to reflect on their solutions***. Additionally, we recommend an increased focus on teaching students to ***construct the differential charge element and difference vector***.

References

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