# COMPARING MATH AND PHYSICS PROBLEMS

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## Introduction and Methodology

- Part of a larger project studying the intersection of math and physics learning
  - Investigate how students view physics problems in terms of the required mathematical processes
- Students were given two sets of problems:
  - 10 math problems (5 pairs)
  - 10 physics problems (5 pairs)

  } *See table below*
- Data collected via 10 group interviews (3-4 students per group)
  - Winter 2012 term
- First semester, calculus-based, introductory physics students

- Mixed Methodological Study
  - Grounded theory approach[1] – no existing theory to impose on the data
  - Phenomonographic approach[2] – emphasis on the students' experience as the learner/participant

Problem Pairs:

| | | |
|---|---|---|
| Arithmetic | A-F | 5-9 |
| Algebra | B-J | 8-10 |
| Definite Integrals | D-E | 2-4 |
| Indefinite Integrals | C-K | 6-7 |
| Extrema/Derivatives | G-H | 1-3 |

| MATH PROBLEMS | PHYSICS PROBLEMS |
|---|---|
| A. What is 572 divided by thirteen? | 1. Two airplanes are flying in the sky – the displacement of plane A is $d = \sqrt{(5.8t^2 - 2)^2 + (6 - t^2)^2}$ relative to plane B. What is the closest distance that the two planes come to each other? |
| B. What is the value of $x$ as given by: $H(x, y, z) = xy + xz + yz^2$ if $H = 5.4$, $y = 2.3$ and $z = 0.80$? | 2. A force $\vec{F} = (3x^2)N\hat{\imath} + (4)N\hat{\jmath}$ with x in meters, acts on a particle, changing only the kinetic energy of the particle. How much work is done on the particle as it moves from coordinates (2m, 3m) to (3m, 0m)? |
| C. Evaluate the integral $\int(\sqrt{x} + 4x^2)\,dx$. | 3. A disk is rotating about its central axis like a merry-go-round. The angular position $\theta(t)$ of a reference line on the disk is given by $\theta(t) = -3.4 + 2.3t - 0.43t^2$ with t is seconds, $\theta$ in radians, and the initial angular position at zero. At what time does $\theta(t)$ reach its minimum value? |
| D. Evaluate the integral $\int_1^4 (3x^2 - 2x + 5)dx$. | 4. A time-varying force of $F(t) = 3(t^5 - 5t + 1)N$ pushes a block along a horizontal surface. How much work does the force exert on the block in the first 8m of the motion? |
| E. Calculate the area under the curve given by $y(x) = x\sqrt{9 - x^2}$ between x=0s and x=8s. | 5. A car travels at a velocity of $89m$ for 6.5s. Find how much distance the car traveled. |
| F. Perform the following calculation: $2423 \times 157$. | 6. If the acceleration of a jet ski is given by $\vec{a} = (4t^2 - 8)\frac{m}{s}\hat{\imath} + (6 - 7t^2)\frac{m}{s}\hat{\jmath}$, what is the equation for the jet ski's velocity? |
| G. What is the minimum value of a curve given by $f(x) = x + \frac{32}{x^2}$ over the interval [0,10]? | 7. The electric potential of a charged disk is given by $V = \frac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2 + R^2} - z\right)$. Write an expression for the electric field at any point from this disk. |
| H. A certain function is given by the equation $f(x) = 3x^2 - 4x^3 = 6$. At what points are there relative extrema? | 8. The average rate at which energy in conducted outward through the ground surface in North America is $54.0\frac{mW}{m^2}$, and the average thermal conductivity of the rocks near the surface is $2.50\frac{W}{m \cdot K}$. Assuming a surface temperature of 100°C, find the temperature at a depths of $35.0km$ below the surface. |
| J. The equation of a parabola is $(x + 3)^2 = 4p(y - 2)$, where $p$ is the focus of the parabola. Find the focus if a single point on the parabola is given by the coordinate (-7,-4). | 9. The mass of an iron atom is $9.27 \times 10^{-27}$kg, and its volume is $1.178 \times 10^{-30}$m$^3$. What is the density of an iron atom? Remember that density is defined as mass/volume. |
| K. What is the integral of the following function: $g(t) = t^4 + 3t^2 \sin(t) - 8t + 2$? | 10. A batter hits a ball when its center is 1.22m above the ground. The ball leaves the bat at an angle of $45^0$ with respect to the ground. With that launch angle, the ball should have a horizontal range (as measured at the 1.22m level) of 107m. Does the ball clear a 7.32m-high fence that is 97.5m from the launch site? |

## Results and Discussion

**Math Problems**

- Began the task quickly
- Indicated they were looking for "key words and symbols"
  - Ex. Integral symbol or the word 'extrema'
- Many groups initially made piles of 'calculus' and 'not calculus'

- When asked to create pairs, most groups did so quickly
  - 6 groups entirely correct
  - 1 group interchanged definite/indefinite integrals
  - 2 groups interchanged all calculus problems
  - 1 group struggled greatly and did not complete the activity

*"you read the problem, and you just know what math it wants you to do to get the answer"*

**Physics Problems**

- Took much longer to complete the task
  - Indicated that they had to read the problem entirely to determine what type it was
  - All group began by creating pairings (instead of broader groups), presumably because of the cuing from the first activity

- Creating pairings was a difficult task for most groups
  - 2 groups correctly paired the problems on the first try
  - 4 groups had some mismatched pairs, which they realized and corrected over the course of their verbal explanations
  - 4 remaining groups had a variety of mismatched pairs, with no noticeable trends in the pairings
    - One group insisted that they have two separate sets of pairings: a topic-based set and a solution-based set

*"just because you know what kind of problem it is doesn't mean you know how to solve it. You have to start working on the problem before you know what kind of math you have to do."*

**Combining Math and Physics**

- Difficult for all groups, particularly those with many mismatched parings
  - 3 groups ultimately completed the entire task correctly
    - All were groups who reconsidered their physics pairings
  - Integral confusion pervaded this task
    - Sparked a discussion of the mean of 'limits' in many of the groups (many students call them 'limitations')

- Future work: more fine-grained analysis of student discussions about their pairings and continuing the larger project's goal of understanding the role of math in student problem solving

## References


1. Charmaz, K., *Constructing Grounded Theory: A practical guide through qualitative analysis.* Sage: Thousand Oaks, 2006.
2. Marton, F., *Phenomenography – a research approach to investigate different understandings of reality.* Journal of Thought, 1986, **21**, 28-49.