Nesting in graphical representations in physics

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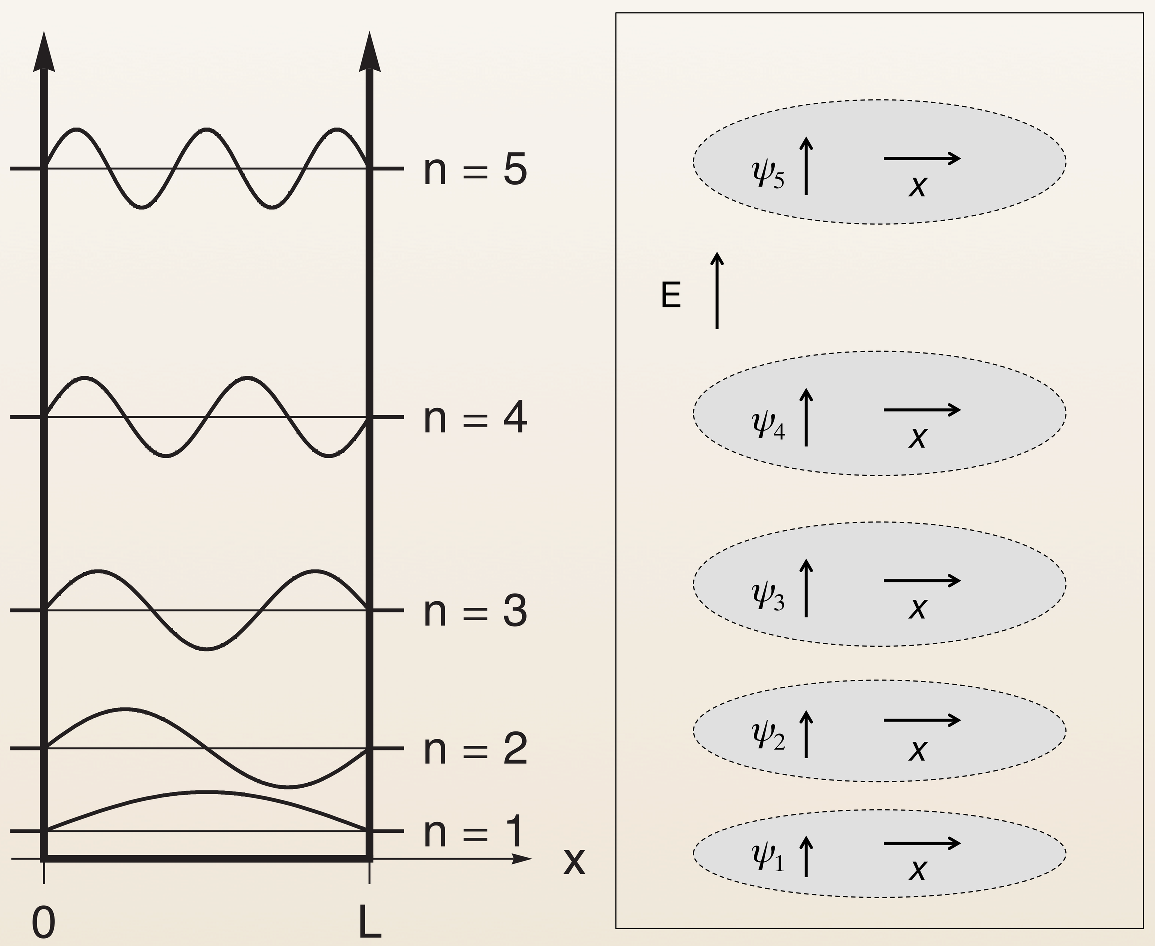
Existing Examples of Nesting in Physics Graphics

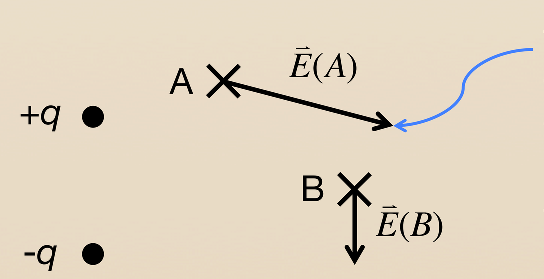
Quantum Infinite Square Well



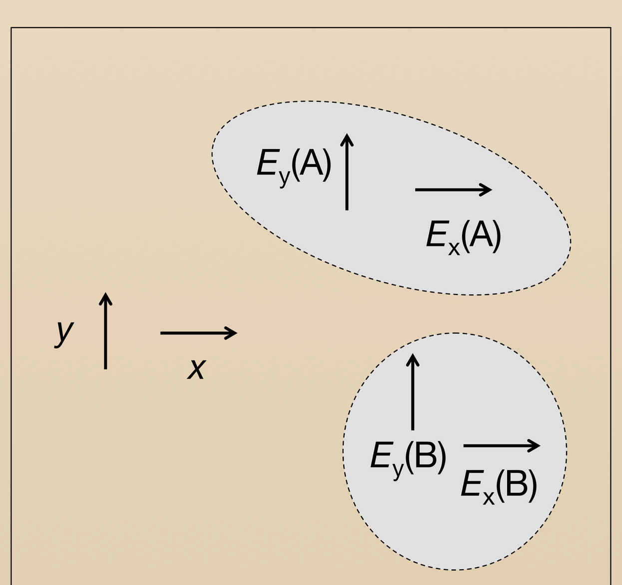
The diagram (from Ref. 2) on the LEFT depicts the five lowest energy eigenvalues and their eigenfunctions for a particle in a one-dimensional infinite square well. The diagram involves nested coordinates, with energy as the **outer vertical coordinate**, and the amplitude of the quantum wave function as the **inner vertical coordinate**. Figure reproduced with permission. On the RIGHT is a distilled version of the graphic, showing the relationships between inner and outer coordinates.

Electric Field Vector Diagram

Multiple occupancy: Locations *in the graphic* can refer to locations in configuration space or in electric-field space. The meaning of location in the graphic is therefore not *exactly* unambiguous.



The arrow representing electric field does not point to another field location. We predict that student difficulty with interpreting the multiple occupancy of the graphic will vary according to the proximity of the end of this arrow to another marked field location, like point B.

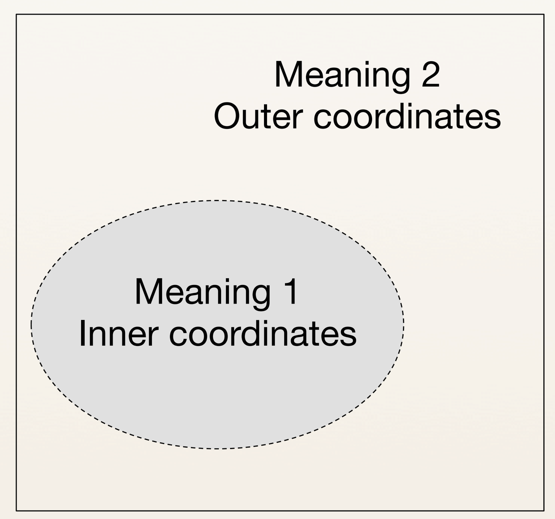


The traditional electric field vector diagram is a **nested graphic.** The inner coordinates are components of the electric field, while the outer coordinates are location.

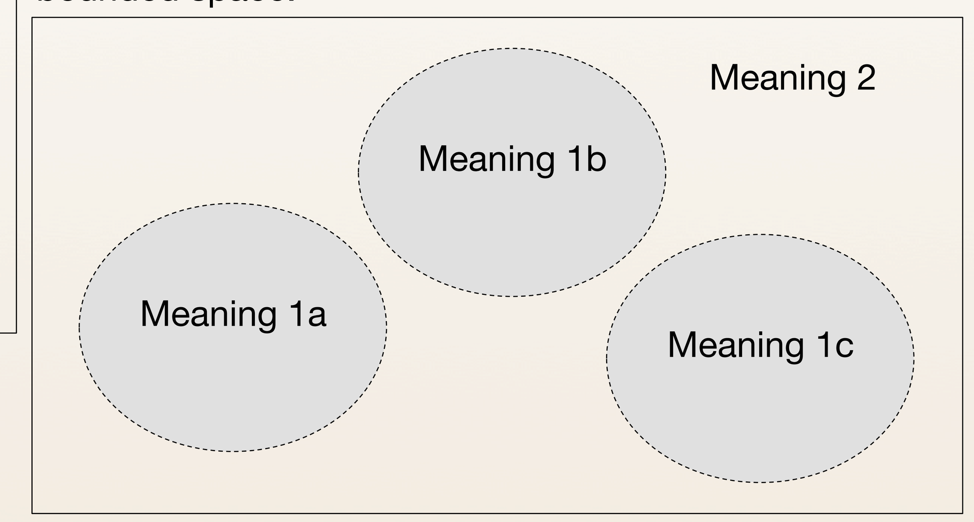
The diagram also uses **small multiples,** like the quantum infinite square well diagram, in which the inner coordinates are repeated several times.

Nesting involves a **discrete** (not continuous) number of "nests." The discrete nature of some quantum systems makes nesting a natural fit for representing them. Systems with continuous data (like classical electric fields) must be **sampled** if nesting is used.

Nesting Explained Abstractly

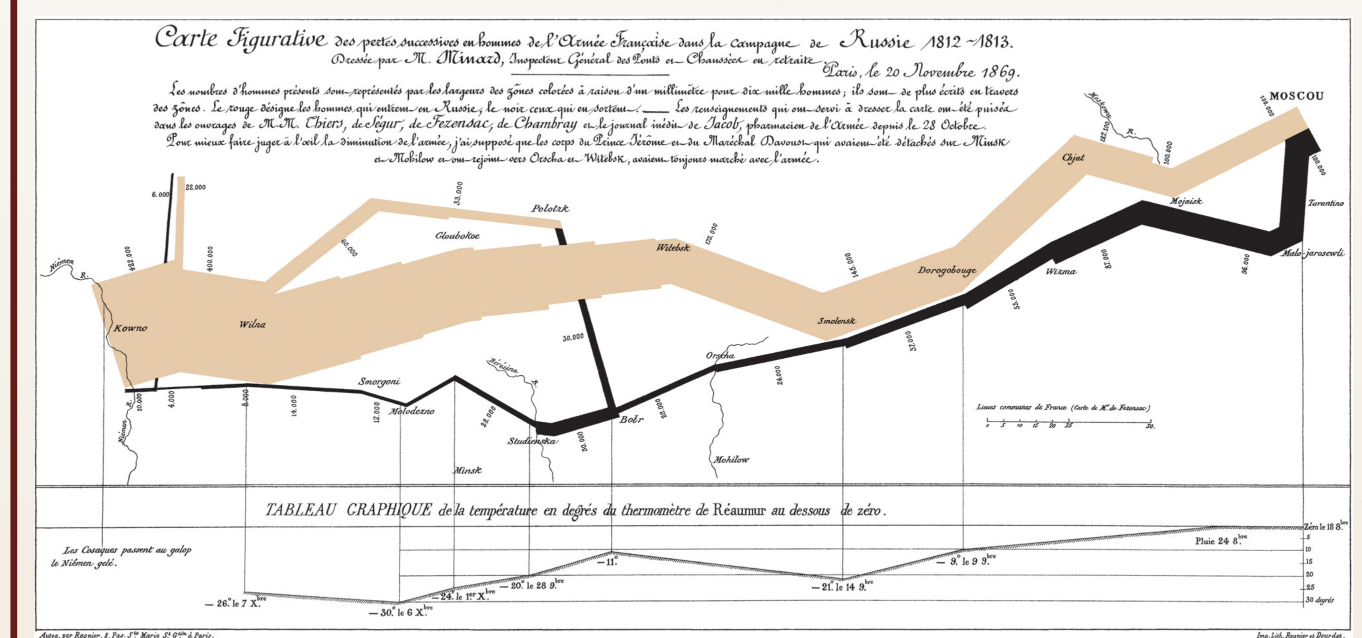


(LEFT) Generic nesting diagram. The meaning attributed to location within the bounded space is of a different character from that attributed to location outside the bounded space.



(RIGHT) "Small multiples," in which the meaning of location on the inside of several bounded spaces is analogous.

Principles of Graphical Excellence

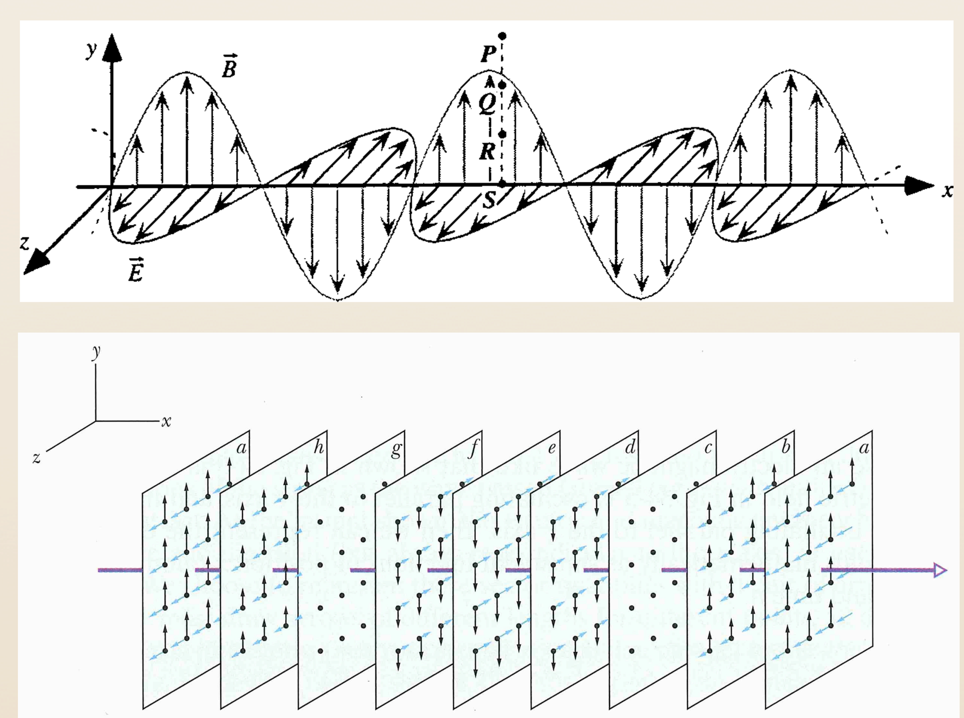


According to Tufte (Ref. 1), principles of graphical excellence include:

- Show multivariate data to encourage multivariate reasoning
- Show causality / mechanism / explanation / systematic structure

This is Minard's 1869 graphic of the French Invasion of Russia in 1812 and shows 6 dimensions of information (see Ref. 1). The diagram uses nesting, with the number of troops as an **inner coordinate** and geographic location as the **outer coordinates.**

Application to PER on Electromagnetic Waves



Traditional graphic
- Used in formal PER empirical study (Ref. 3). Students rank points P, Q, R, S according to magnitude of E and B fields.
- Graphic space is **doubly-occupied** (with location and field) but **not nested.**
- Field at multiple points in y-z plane not represented
- Graphic shows labeled points as visually different though they are physically identical
- Ref. 3: "The most common error has been to ascribe to the plane EM wave a finite spatial extent in the plane perpendicular to the direction of propagation." This error is predicted by the nesting model because of double-occupancy without use of nesting.

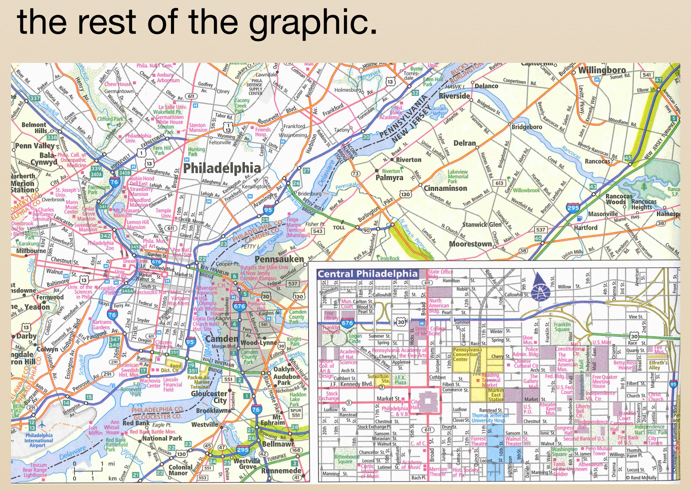
Nested graphic
- Used in PER-based textbook (Ref. 4), but not yet in any formal PER empirical study
- Nesting is accomplished by limiting extent of field vectors in graphic to distance less than that between **sampled** field locations
- Shows *systematic structure* (independence of fields on y and z coordinates)

Application to Future Gesture Research

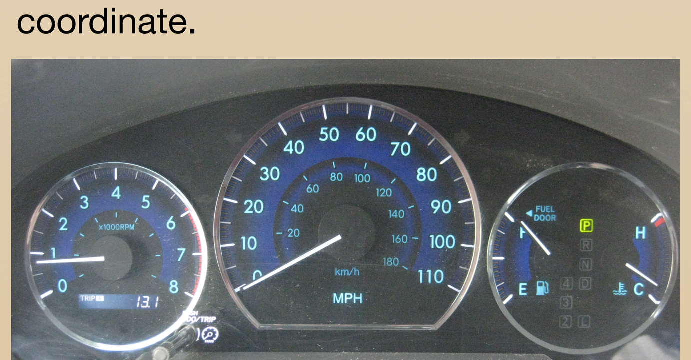
Learners sometimes gesture in the context of a **mathematical gesture space,** where gestures derive part of their meaning through coordination with other gestures in space and time (Ref. 6). The nesting idea, if it can be observed in gesture in future study, suggests that two gestures, or two parts of a gesture, may function together in a nested relationship, with one gesture acting with an **inner meaning** and another one acting with an **outer meaning.** Without the nesting idea, we might interpret the gestures as otherwise merely sequential. Nesting also suggests that one way gestures or gesture parts may be mutually coordinated is through their relative size: the "smaller" inner meaning may be expressed with the **fingers** while the "larger" outer meaning is expressed with the **arm,** or alternatively, scaling up, with the **arms** and the **whole body,** respectively.

Everyday Examples

In a printed road atlas, it is routine to show some regions twice: once with its surroundings, and once with zoomed-in detail, with the detail as an inset. It is understood that one cannot travel into the inset along the same path as one would trace a finger across the graphic. This map in this inset is also slightly rotated, which further decouples it from the rest of the graphic.



In each of the dials on the dashboard, the location of the needle has different meaning. Each meaning is bound within a region of space. This binding is accomplished in part by each needle having a constant length, with no meaning assigned to an inner radial coordinate.

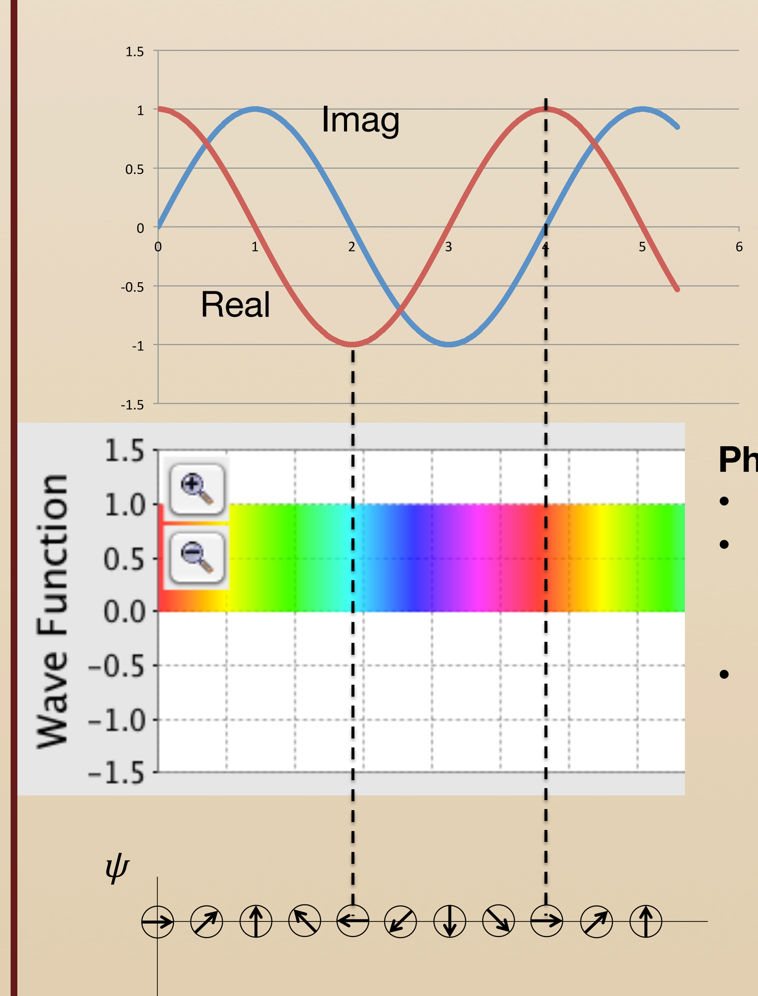


Explicit Use of Nesting to Create New Graphics in Physics

Complex Scalar Plane Wave

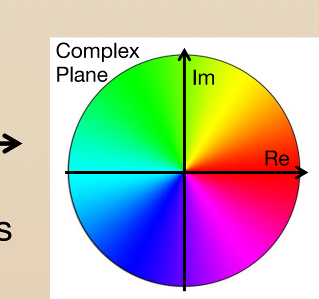
$$\psi(x) = Ae^{ikx} = A\cos kx + iA\sin kx$$

Real part — Imaginary part



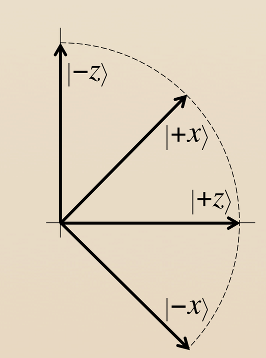
Traditional graphic uses **doubly-occupied** vertical coordinate, with real and imaginary parts separated.
- Constant magnitude obscured
- Relationship of parts looks like translation symmetry rather than mutually orthogonal projections
- No visual connection to familiar complex plane

PhET graphic (See Ref. 5)
- Shows constant magnitude
- Color field "hyperlinks" spatial information in the legend (not provided by PhET)
- Shows complex phase as continuous



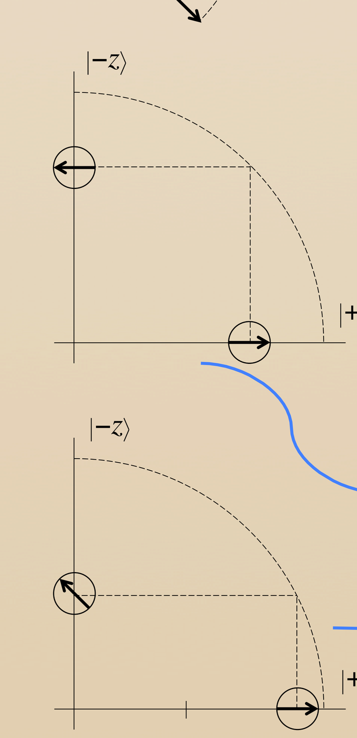
Nested graphic
- Shows constant magnitude
- Easier to write and draw than color graphic
- Highlights "mechanism" of rotation for Re and Im parts
- Visually resembles familiar complex plane
- Shows complex phase in **discrete samples**

Spin-1/2 Quantum States



Non-nested graphic showing four quantum states for a spin-1/2 system: spin-up and spin-down in the z and x directions. The state vectors are arranged to highlight their various (real) inner products. The imaginary parts are suppressed.

Not all possible states can be represented.



Nested graphic showing two quantum states (LEFT and BELOW LEFT) for spin-1/2 systems in the +z/-z basis. The large circular arc shows that the states are normalized. The outer coordinates are the magnitudes of the coefficients for the +z and −z components of the state. The inner coordinates are the complex phases of those coefficients.

All possible states can be represented.

$$|\psi\rangle = \frac{e^{i0}}{\sqrt{2}}|+z\rangle + \frac{e^{i\pi}}{\sqrt{2}}|-z\rangle = \frac{1}{\sqrt{2}}\left(|+z\rangle - |-z\rangle\right) = |-x\rangle$$

$$|\psi\rangle = ?$$

Check your understanding of the graphic!

1. Tufte, E. R. (2006) Beautiful Evidence. Graphics Press, Cheshire, CT.
2. McIntyre, D. H. (2012) Quantum Mechanics. Pearson, Boston.
3. Ambrose, B. S., P. R. L. Heron, S. Vokos, L. C. McDermott (1999) "Student understanding of light as an electromagnetic wave: Relating the formalism to physical phenomena," *Am. J. Phys.* **67** (10) 891-898.
4. Cummings, K. et al. (2004) Understanding Physics. John Wiley & Sons, Inc., New York.
5. http://phet.colorado.edu/en/simulation/quantum-tunneling (retrieved July 4, 2012)
6. Yoon, C. (2011) Grounded blends and mathematical gesture spaces: developing mathematical understandings via gestures, *Educ. Stud. Math.*, 78:371-393