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Understanding and Interpreting Calculus Graphs: Refining an Instrument

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Abstract

The objective of this ongoing study is to refine an instrument to evaluate conceptual understanding and graphical interpretation of a function and its derivative. The instrument is based on a modified version of the Test of Understanding Graphs in Kinematics (TUG-K) which consists of 26 items (7 objectives). In the new instrument, Test of Understanding Graphs in Calculus (TUG-C), the kinematics context has been removed from the items creating a new context-free version. To favor the translation from kinematics to Calculus, the focus is on 5 out of the 7 original objectives of the test, giving a 16-item test. A total of 526 students from a university level Introductory Physics course participated in the study. Half of the students were administered the kinematics test and the other half took the calculus test. This work will present data showing preliminary results of the instrument and new directions on improving the instrument

Introduction

The ability of extracting information from a graph or being able to condensate the behavior of relevant physical quantities in one is very important in both science and engineering fields. Because the principal difference between a novice and an expert is the way information is stored and organized in their cognitive structures, the interpretation and construction of graphs can be of help in teaching students how to learn physics and differentiate between the superficial and essential concepts involved in a phenomenon [1]. However, understanding the physical concepts involved in kinematics is not enough for successful interpretation of graphs, which raises the question of whether misconceptions in calculus have an impact in the resistance of misconceptions to instruction. Beichner's Test of Understanding Graphs in Kinematics (TUG-K) [2] was revised and modified by Zavala et al [3]. This study uses that revised version of the TUG-K to create a similar test in the context of Calculus [4]. This test was named Test of Understanding Graphs in Calculus (TUG-C) and used to evaluate the entire population of students taking the Introductory Physics and Introductory Calculus in the Spring of 2009 [4]. Based on that experience, it was decided to use only the 16 items on the test that correspond to the first five objectives (see Table 1).

Implementation

Instrument

To create the test, the modified version of TUG-K was reviewed and each of its 26 items was rewritten to remove the kinematics context both the text and the graphs included. Special care was given to write the new problems with the same mathematical language and notation used in the original TUG-K. Table 1 shows the objectives of both TUG-K and TUG-C and their correlation.

TABLE 1. Equivalent objectives of the kinematics and the calculus version of the test considered for the analysis and the corresponding number problem in the short TUG-C version of the test.

Obj.	TUG-K	TUG-C	Problems
1	Given position-time graph, determine velocity	Given the graph of f(x), determine f'(x)	1, 3, 6 & 15
2	Given velocity-time graph, determine acceleration	Given the graph of f'(x), determine f''(x)	7 & 11
3	Given velocity-time graph, determine displacement	Given the graph of f'(x), determine Δf(x)	5, 9 & 13
4	Given acceleration-time graph, determine change in velocity	Given the graph of f''(x), determine Δf'(x)	2, 4 & 12
5	Given a kinematics graph, select another corresponding graph	Given a graph, select another corresponding graph	8, 10, 14 & 16

Methodology

The research was conducted in a large private Mexican university. The tests were administered to 526 students enrolled in the Introductory Physics course in that institution. A total of 265 students were given the TUG-C and 261students responded the TUG-K. Both versions of the test consisted on the same questions (16 items) and were arranged in the same order. Both tests were administered in Spanish. Both students performance and error distribution where taken into account and the Item Response Curves [5] where calculated for all problems in both versions of the test.

Results

Objetive 1

There are two kinds of problems in this objective: one that requires numerical calculations and one that requires making qualitative estimations. The first kind of problem shows a similar IRC for the correct answer. However, there is a noticeable difference in the error distribution among the incorrect responses between the TUG-K and the TUG-C versions. Whereas in the kinematics version of the test the option corresponding to incorrectly calculating the velocity by dividing the value of position by time is a popular misconception (option D), in the calculus version of the test the most popular misconception corresponds to reading the value directly from the graph (option E).In the problems that require qualitative estimations there is no noticeable difference between the results, correct answer and incorrect answers have similar IRC in both tests.

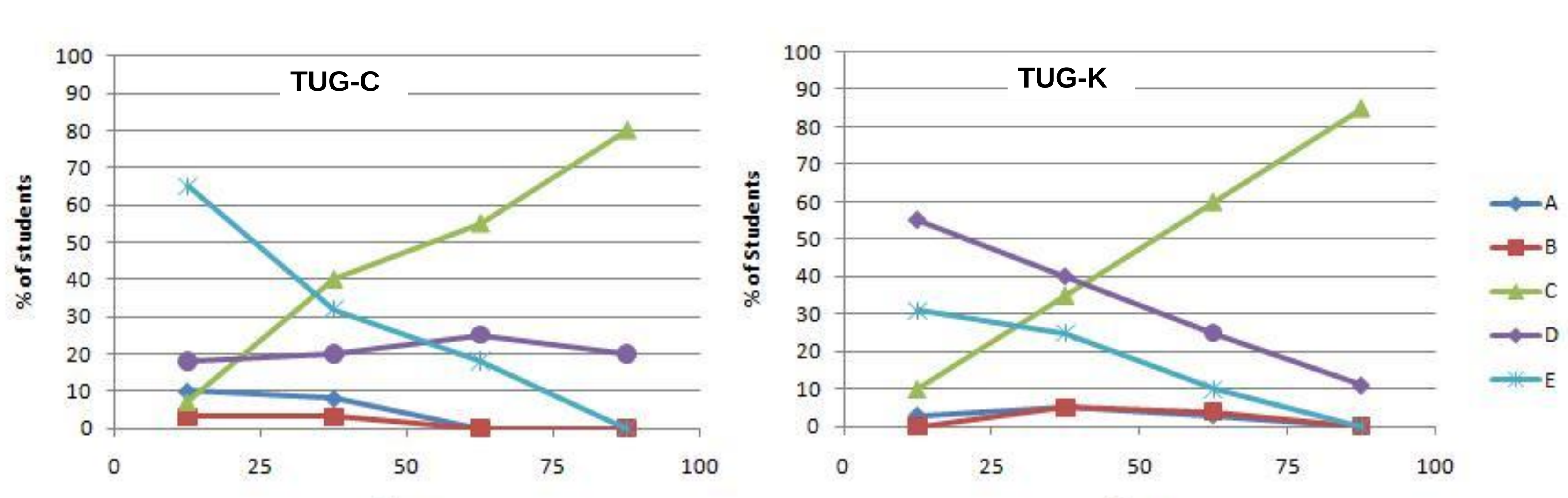


FIGURE 1. Item Response Curves of question 1 showing the similar behavior of the correct answer for the TUG-C and TUG-K, respectively.

Conclusions

- The context has an influence on the alternative cognitive models, but the correct answer works well for both versions of the test,
- The results indicate that the TUG-C has potential to become a strong instrument to measure conceptual understanding and graphical interpretation of a function and its derivative.
- Future research will focus on reviewing the TUG-C to double check that the questions are as clear as they could be, and that the cognitive models are the most common alternative models. After that, it will be administered to a larger population in order to validate it.

Results

Objective 2

Both problems in this objective require the student to make numerical calculations. As in the first objective, the correct answer had a similar behavior in both test. The most common misconception in the **TUG-C** version consisted on **reading the value directly** from the graph, whereas in the **TUG-K** version of the test students favored the value of **velocity divided by time**. Given the parallelism between f(x) <–> f '(x) and f '(x) <–> f ''(x), the behavior of the responses for this objective were similar to the behavior of the responses for the first objective.

Objective 3

In two items of this objective the students are asked to calculate the change of value of the original function using the graph of its derivative. These two items were responded correctly more often in the kinematics version than in the calculus version of the test. With respect to the incorrect responses, the option corresponding to calculate the change by multiplying the time and the value of the derivative in that point (option D in question 5) is a popular conception in the kinematics test (due to an incorrect interpretation of the definition of acceleration), but it is almost nonexistent in the calculus version (see Figure 2). The calculation corresponding to the slope in the given interval is a strong cognitive model only in the calculus test (option C).

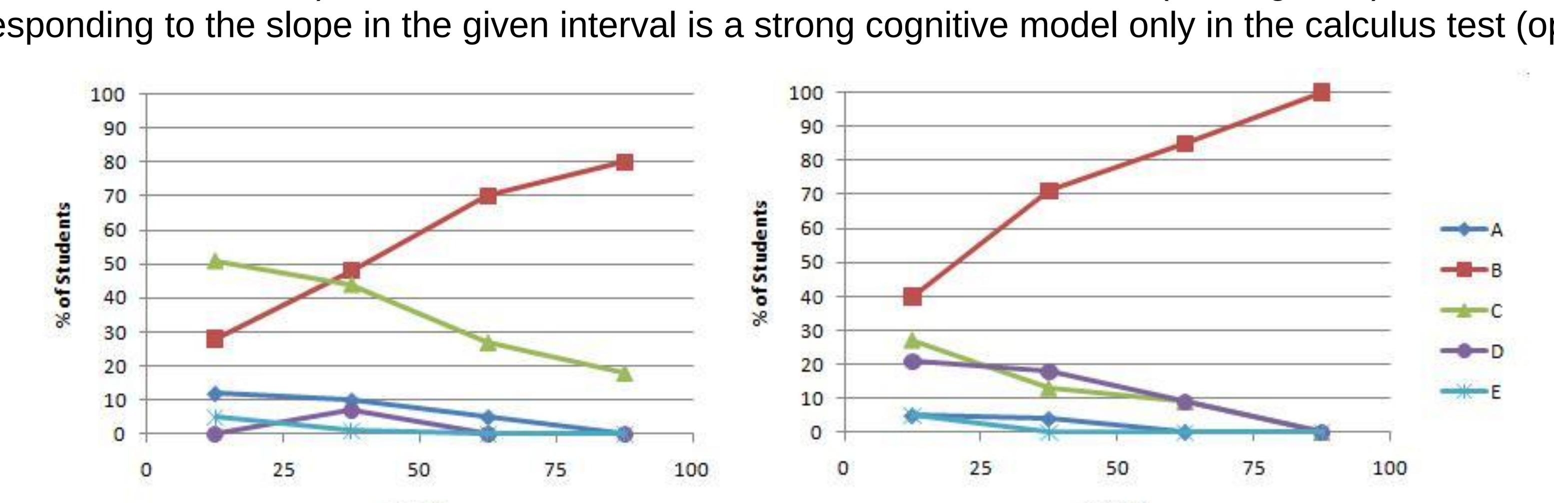


FIGURE 2. Item Response Curves of question 5 showing the different incorrect cognitive models underlying for the TUG-C and TUG-K, respectively.

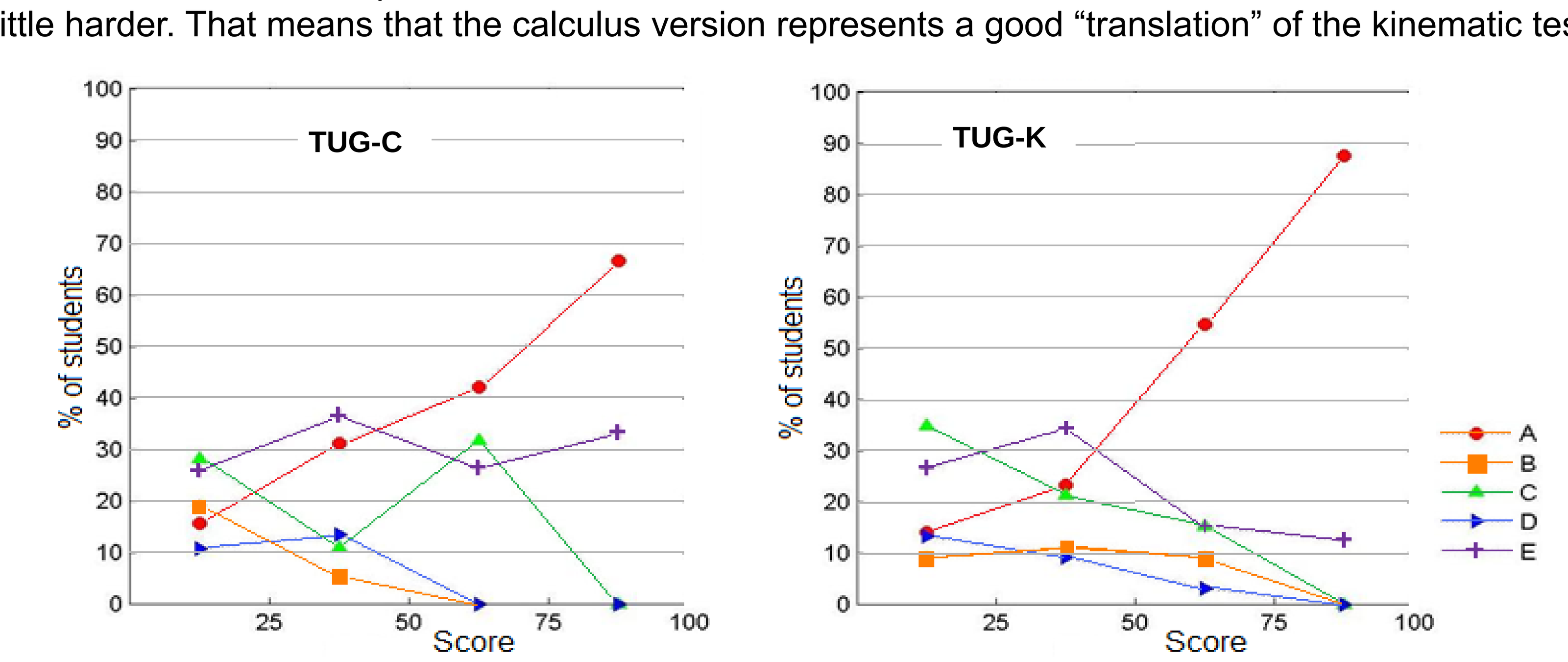
The other item of this objective presents the student with the graphs of the derivatives of five different functions and asks to select the graph corresponding to the function with a greater increment in the interval. This is a very difficult problem in both versions of the test, only 11% of the students answered correctly in the TUG-C and 25% in TUG-K. It is interesting to note that while the most popular incorrect response in the kinematics version (35%) corresponds to the graph with the greatest slope, in the calculus version the most popular incorrect response (34%) corresponds to the graph with a local maximum.

Objective 4

This objective has three items, two numerical problems and one of qualitative analysis. These three questions are parallel to the items for the third objective. Given the parallel relationship between the involved functions in these two objectives, the behavior of the correct responses according to the IRC is very similar to the curves obtained in the previous objective.

Objective 5

All of the items in this objective provide a graph and ask to determine which one of the graphs keeps a specific relationship with the given graph. These items focus on the concepts addressed in the four previous objectives, but from a graphical representation. The four items turned out to have similar IRC behavior for the correct and incorrect responses between the two versions of the test, but the calculus version seems to be a little harder. That means that the calculus version represents a good "translation" of the kinematic test.



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