Transforming Upper-Division Quantum Mechanics Learning Goals and Assessment

Steve Goldhaber[†‡], Steven Pollock[†], Mike Dubson[†], Paul Beale[†] and Katherine Perkins[†‡]

[†]Physics Dept., University of Colorado, Boulder, CO (per.colorado.edu), [‡]The Science Education Initiative (www.colorado.edu/sei)

Introduction

QM I Transformation at CU

- Approach:
  - Used results from PER[1–4]
  - Developed learning goals (see next column)
- Classroom Techniques:
  - Integrated concept tests into lecture
  - Modified Homework: justification of reasoning, estimation and math - physics connections
  - Hosted group homework sessions
  - Added tutorial sessions focusing on areas of known student difficulty[3,8]
- Assessment: Developed the QMAT

Learning Goals

- Faculty input via interviews and a series of meetings
- 18 faculty contributed their time
- Sample skill-oriented goal:

  "Students should be able to sketch the physical parameters of a problem (e.g., wave function, potential, probability distribution), as appropriate for a particular problem."
- Sample content-specific goal:

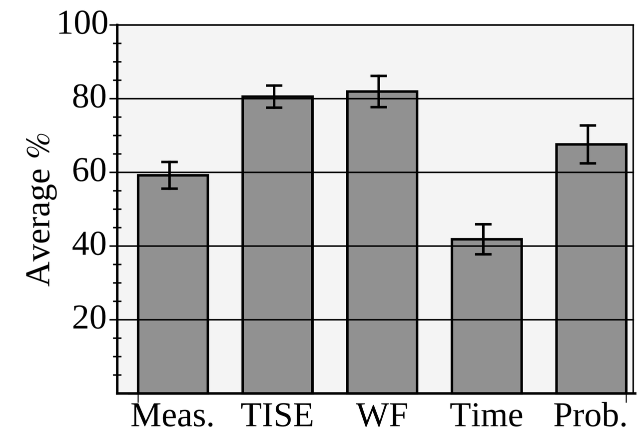
  "Given a wave function and an observable operator, students will be able to calculate that operator's expectation value"

Measuring Learning

- QMAT goals:
  - Reflect faculty learning goals
  - Assess student learning difficulties
  - Help faculty improve QM instruction
- Question development:
  - Student interviews:
    elicit learning difficulties (5)
    assess question validity (21)
- Administered:
  - Dec. 08 (N = 27), Apr. 09 (N = 36)
- QMAT has 14 questions, 28 separate question items (mostly free response)

Overall QMAT Results

Spring 2009 student results grouped into 5 categories of learning goals covered by the QMAT.



Meas. = Measurement
TISE = Time Independent Schrödinger Equation
WF = Wave Functions / boundary conditions
Time = Time Development
Prob. = Probability / Probability density
Error bars indicate the standard error of the mean.

The Quantum Mechanics Assessment Tool (QMAT)

Sample problems from the QMAT, some performance information, and sample student answers which reveal common student learning difficulties. Citations note problems which build on previous research.

Incompatible Measurements[3,5]

Consider a particle in a 1D, infinite square well with width a, centered at $a/2$. The normalized energy eigenstate wave functions are $u_n(x)$ with energies E_n ($n = 1$ is the ground state). The particle starts in a state given by

$$\Psi(x, t=0) = \left(\sqrt{\frac{4}{5}}u_1(x) + \sqrt{\frac{1}{5}}u_2(x)\right) .$$

a) You make an energy measurement on this system and find the maximum possible value for the energy. **What is the state, $\psi(x)$, of the system after this measurement?**

b) After the energy measurement, you make a position measurement. After this position measurement, you immediately re-measure the energy. **At this point, what value(s) could you get for energy?**

c) **Does your answer to part b depend on how long you wait between the position and energy measurements? Explain.**

Part (a): While 72% of students correctly noted that the state after the energy measurement is u_2, others made errors similar to the following representative quote: "u_1 because it has the highest probability"

Part (b): 31% recognized that all allowed energies were possible after the position measurement. The most common errors were from students who said that the only choices were E_1 or E_2 (the original energies, 42% of all answers). 19% said or implied that the position measurement would not alter the energy.

Part (c): Only 11% managed to convincingly describe why the amount of time before the second energy measurement did not matter. Others felt that energy eigenstates would evolve at different speeds or that the delta function resulting from the position measurement would spread out which would change the 'energy' of the state. • "Yes, if time elapses between measurements then the wavefunction has time to spread out." • "No. Measuring E sets the wave eq with that val forever. Collapses it to that u_n." • "No matter how long you wait you will measure E_1 or E_2."

Eigenstate Time Dev.[6]

Is the following statement true for all operators, $\hat{Q}$? **Explain briefly why you agree or disagree.**
A system which is in an eigenstate of $\hat{Q}$ will stay in that state until disturbed by measurement.

Only 36% of students gave a convincing (correct) explanation of why the statement was not true for an arbitrary operator. Common types of incorrect responses: "Agree → know it's a postulate, just not sure which one" • "Yes, Eigenstate = stationary state" • "Until measurement is made, we don't know what state the system is in."

Operator v.s. Measurement[7]

T or **F**: Acting on $|\psi\rangle$ with the Hamiltonian is the mathematical equivalent of making a measurement of the energy of that state.

Only 28% were able to explain the difference between applying the Hamiltonian operator to a quantum state and making a measurement. Most of the students who answered true gave reasons such as "$\hat{H}|\psi\rangle = E|\psi\rangle$" (32%) or stating or implying that they are equivalent because *any* quantum state has a well-defined energy (23%).

Hamiltonian and Time Dev.

T or **F**: Applying the Hamiltonian to $|\psi\rangle$ gives you information about how that state will evolve in time.

25% wrote a convincing explanation of how the Hamiltonian is related to time development. Others seem only to focus on the TISE: "$H|\psi\rangle$ tells you nothing about time" • "The Hamiltonian gives information about energy only" • "when we use $\hat{H}$, it's to solve the TISE, so $\hat{H}$ doesn't really tell us about time." • "$\hat{H} = \frac{p^2}{2m} + V$ this has no time dependence."

Summary and Discussion

- QMAT as an assessment tool:
  - Demonstrates that CU students are not achieving all of our learning goals, despite reforms that include clicker questions and other interactive techniques targeting these ideas
  - Exposes areas of common student difficulties (e.g., measurement and time development in QM)
  - Raises faculty awareness, and guides future reform efforts
- Some preliminary outcomes of the QMAT:
  - Across the QMAT, students frequently respond as though:
    - all quantum states (including superposition states) have a definite energy, and
    - time dependence only requires 'tacking on' a single term $\exp[-iEt/\hbar]$ to any quantum state (including superposition states).

    These observations are consistent with existing QM research literature.[2,3,5,6] Our preliminary interviews suggest that they are over-generalizing from the TISE, $\hat{H}|\psi\rangle = E|\psi\rangle$.
  - Students also frequently respond as though sequential measurements on a quantum state retain all original information encoded in the starting state, again consistent with literature.[3]
- The full set of learning goals, assessments, and other course materials, are available at http://www.colorado.edu/sei/departments/physics_3220.htm.
- **Use the QMAT in *your* QM I class! If you are interested, contact Steve Goldhaber at Steven.Goldhaber@colorado.edu.**

References and Acknowledgements

[1] B. A. Thacker, Editor, H. S. Leff, Editor, D. P. Jackson, and Editor, *Am. J. Phys.* **70**, 199–367 (2002).

[2] C. Singh, "Student Difficulties with Quantum Mechanics Formalism," in *2006 PERC Proc*, edited by L. McCullough, L. Hsu, and P. Heron, AIP, 2007, vol. 883, pp. 185–188.

[3] A. D. Crouse, *Research on Student Understanding of Quantum Mechanics as a Guide for Improving Instruction*, Ph.D. thesis, University of Washington (2007).

[4] L. C. McDermott, and E. F. Redish, *Am. J. Phys.* **67**, 755–767 (1999).

[5] C. Singh, *Am. J. Phys.* **76**, 277–287 (2008).

[6] C. Singh, *Am. J. Phys.* **69**, 885–896 (2001).

[7] E. Gire, and C. Manogue, "Resources Students Use to Understand Quantum Mechanical Operators," in *2008 PERC Proc*, edited by C. Henderson, M. Sabella, and L. Hsu, AIP Press, Melville, NY, 2008, pp. 115–118.

[8] P. Shaffer and the UW PEG, QM Tutorial materials via private communication (2008).

Funding

This work is funded by NSF-CCLI Grant # 0737118 and The CU Science Education Initiative.

Acknowledgements

We thank Carl Wieman and Sam McKagan for useful discussions. We also thank the physics faculty members who generously contributed to the creation of both the learning goals and the QMAT: A. Becker, T. DeGrand, O. DeWolfe, N. Finkelstein, C. Greene, A. Hasenfratz, E. Kinney, K. Mahanthappa, U. Nauenberg, J. Price, C. Rogers, K. Stenson, and E. Zimmerman.