Addressing Student Difficulties with Concepts Related to Entropy, Heat Engines and the Carnot Cycle

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Abstract. We report the rationale behind and preliminary results from a guided-inquiry conceptual worksheet (a.k.a. tutorial) dealing with Carnot's efficiency and the Carnot cycle. The tutorial was administered in an upper-level thermodynamics course at the University of Maine. The tutorial was implemented as the third in a three-tutorial sequence designed to improve students' understanding of entropy and its applications. Initial pre- and post-tutorial assessment data suggest that student understanding of heat engines and the Carnot cycle improved as a result of tutorial instruction.

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INTRODUCTION

Considerable effort has been made over the past decade to document students' difficulties with topics relating to entropy.[1, 2, 3, 4] Christensen and Meltzer report on students' difficulties with the definition and concept of entropy as well as efforts to create tutorials to improve students' understanding.[1] Meltzer, and Bucy et al. discuss students' inabilities to reason about entropy as a state function.[2, 3] Cochran and Heron investigated students' understanding of entropy as it relates to heat engines.[4]

Cochran and Heron[4] report that after lecture-based instruction, about 25% of students ignore entropy and the Second Law of Thermodynamics (2nd Law) when answering questions related to heat engines in favor of reasoning based solely on the First Law of Thermodynamics (1st Law) and energy considerations. Another 15% of students state that an engine is physically possible if its efficiency is less than 100%. Other students categorize some engines as having an "improbably high" efficiency based on unstated thresholds not explicitly related to the Carnot efficiency.

Based on their results Cochran and Heron designed a guided-inquiry conceptual worksheet (a.k.a. tutorial) for use in an introductory physics course to help develop students' understanding of entropy and heat engines. Their tutorial focuses on the various forms of the 2nd Law (Clausius statement, Kelvin-Planck statement, $\Delta S_{universe} \geq 0$, etc.) and their applications. Up to 75% of students were able to reason correctly on questions regarding heat engines, refrigerators, and related devices after tutorial instruction.[4] For our upper-level course, we have developed a tutorial titled *Heat Engines* in which students derive both the Kelvin-Planck statement and Carnot's theorem using the entropy inequality form of

the 2nd Law ($\Delta S_{universe} \geq 0$). Our goal is to undeniably link the various statements of the 2nd Law by deriving one from the other rather than demonstrating their equivalence after they have been introduced.

In the following sections we briefly describe two entropy tutorials that we consider prerequisite to *Heat Engines*. We then discuss the motivation and design of *Heat Engines*. Next we report on the results of the first implementation of *Heat Engines* at the University of Maine (UMaine). Finally, we discuss revisions made to *Heat Engines* as a result of data from the first implementation as well as plans for future implementation and dissemination.

PREREQUISITE ENTROPY TUTORIALS

Christensen and Meltzer have developed two tutorials on entropy for use in an introductory physics course.[1] These tutorials were adapted for use in the upper-level thermodynamics course at UMaine in the fall of 2007 and the fall of 2008. These tutorials are essential to students' preparation for *Heat Engines*, as the entropy topics discussed therein are crucial to the reasoning outlined in *Heat Engines* and are reportedly not learned well through lecture-based instruction.[1]

The Entropy Two-Blocks Tutorial

The first entropy tutorial assumes that students have a working knowledge of the 1st Law and of the definitions of thermodynamic heat and work but assumes no prior knowledge of entropy or the 2nd Law.[1] The Entropy Two-Blocks Tutorial asks the students to consider several

examples of heat transfer between two thermal reservoirs (large, insulated, metal blocks). They are provided with the definition for the change in entropy due to a reversible process shown in Eq. 1 and the simplification for thermal reservoirs given in Eq. 2.

$$\Delta S = \int_{initial}^{final} \frac{dQ_{reversible}}{T}$$

$$\Delta S_{reservoir} = \frac{Q_{to\ reservoir}}{T_{reservoir}}$$
(2)

$$\Delta S_{reservoir} = \frac{Q_{to \ reservoir}}{T_{reservoir}} \tag{2}$$

By considering a situation in which energy is transferred from a lower temperature reservoir to a higher temperature reservoir through spontaneous heating, the students determine that the total entropy of the universe can never decrease: $\Delta S_{universe} \geq 0$.

The Entropy Two-Processes Tutorial

The second entropy tutorial examines the state function property of entropy. The ability to calculate a change in entropy due to an irreversible process by comparison to a reversible process with the same initial and final states is an incredibly powerful tool in thermodynamics that many students neglect to exploit.[2, 3] The Entropy Two-Processes Tutorial was developed by Christensen and Meltzer[1] based on a research task designed by Bucy et al.[3] The students are asked to consider two different processes that an ideal gas could undergo: a reversible isothermal expansion, and an irreversible free expansion into vacuum. The initial and final states of the gas are identical for both processes. The students are guided to use the idea that entropy is a state function to determine that they can use Eq. 1 to determine the change in entropy of the gas during the free expansion even though the process is inherently irreversible.

THE HEAT ENGINES TUTORIAL

Our goal with Heat Engines is for students to understand why the Carnot cycle is the most efficient heat engine, not just the formulaic expression for the Carnot efficiency. We start by assuming that the students have worked through the Two-Blocks and the Two-Processes tutorials and thus have gained a sufficient understanding of determining the change in entropy of a system during a specified process as well as the state function property of entropy. For a generic heat engine we define $Q_{\rm H}$ as the energy transfer through heating from a high temperature reservoir (at T_H) to the working substance of the heat engine, Q_L as the energy transfer through heating from the working substance to a low temperature reservoir (at $T_{\rm L}$), and W as the work done by the working substance over one complete cycle of the heat engine. Using these definitions and the fact that energy is a state function, we can write the 1st Law as

$$Q_{\rm H} + Q_{\rm L} - W = 0. (3)$$

In the first part of *Heat Engines* students are asked to consider the two limiting cases of heat engines in terms of efficiency, defined as in Eq. 4. Cycle 1 is defined as doing no work (W = 0, $\eta = 0$). Cycle 2 is defined as having no exhaust heat $(Q_L = 0, \eta = 1)$. They use the 1st Law and the definition of thermodynamic efficiency to calculate the efficiency of two different heat engines.

$$\eta = \frac{W}{O_{H}} \tag{4}$$

Students use the expression for the change in entropy of a thermal reservoir given in Eq. 2 and the state function property of entropy to calculate $\Delta S_{universe}$ for Cycle 1 and Cycle 2. Given these values for $\Delta S_{universe}$, students then invoke the entropy inequality form of the 2nd Law $(\Delta S_{universe} \ge 0)$ to determine whether or not each heat engine is physically possible. In realizing that Cycle 2 is impossible, students derive the Kelvin-Planck statement of the 2nd Law: It is impossible to construct a device that operates in a cycle and produces no other effect than the performance of work and the exchange of heat with a single reservoir.

In the second part of the tutorial the students are asked to combine the entropy inequality with Eqs. 2 and 4 to derive the constraint on thermodynamic efficiency due to the 2nd Law:

$$\eta \le 1 - \frac{T_{\rm L}}{T_{\rm H}}.\tag{5}$$

The students then examine the condition under which the equality in Eq. 5 holds, and thus determine that a reversible cycle is needed that can only be created using an alternating sequence of isothermal and adiabatic processes. While working through *Heat Engines* students derive the Carnot cycle as the cycle with the maximum possible efficiency (equality in Eq. 5) as well as invent the Carnot cycle as the only cycle that could achieve Carnot efficiency for any heat engine with an arbitrary working substance. The word "Carnot" is not used in the tutorial worksheet until after the students have derived this maximum efficiency and the corresponding cycle. No pictorial or graphical representations of heat engines are used during the tutorial.

For homework, the students consider the P-V diagram for a Carnot cycle in which the working substance is an ideal gas and calculate the efficiency using appropriate expressions for W and $Q_{\rm H}$. The students also consider two different heat engines (a Carnot cycle and an engine that For the following questions consider one complete cycle of a heat engine operating between two thermal reservoirs. The heat engine operates using an appropriate working substance that expands and compresses during each cycle.

As a result of one complete cycle of the Carnot engine, will the entropy of **the universe** *increase*, *decrease*, *remain the same*, or is this *not determinable with the given information*? **Explain your reasoning.**

As a result of one complete cycle of the Carnot engine, will the entropy of **the working substance** *increase*, *decrease*, *remain the same*, or is this *not determinable with the given information?* **Explain your reasoning.**

FIGURE 1. Portion of the engine-entropy question regarding entropy and the Carnot engine. The same questions were asked for an engine that is hypothetically more efficient than the Carnot engine.

does no work) operating between initially identical pairs of finite thermal reservoirs (i.e. not constant temperature) and determine the final equilibrium temperature for each reservoir in terms of the initial temperatures $T_{\rm H}$ and $T_{\rm L}$.

IMPLEMENTATION AND RESULTS

Heat Engines was administered after lecture-based instruction on heat engines to the upper-level undergraduate thermodynamics course at UMaine in the fall of 2008. Students had already worked through both the Two-Blocks and the Two-Processes tutorials. The class consisted of 10 students (primarily junior and senior physics majors and minors), 6 of whom completed Heat Engines. The entire tutorial session was video recorded. This was the first time that any students had seen or worked through Heat Engines.

The "engine-entropy" question (partially shown in Figure 1) was administered as a pretest at the beginning of the tutorial period and asked students about the change in entropy of the working substance and of the universe due to one complete Carnot cycle as well as one cycle of a heat engine more efficient than a Carnot engine. On the pretest some students demonstrated an understanding of the state function property of entropy and its implications for heat engines ("I think the entropy of the working substance stays the same because it returns to its original state after one cycle") but others did not ("From the equation $\Delta S = \frac{Q}{T}$, we know that the working substance will decrease in entropy since the change in entropy at low-T is greater"). Moreover, many students' responses indicated an understanding of state function properties of cycles, the reversibility of the Carnot cycle, or that a hypothetically more efficient engine than the Carnot cycle would violate the 2nd Law as stated by the entropy inequality.

The engine-entropy question was administered again approximately five weeks after tutorial instruction as part of an ungraded quiz. Students' responses on this ungraded quiz suggest that some students had a better understanding of the relationship between entropy and heat engines after tutorial instruction. Before tutorial instruction one student wrote, "The entropy of the working substance [for a Carnot engine] increases. By flowing through the heat engine the working substance loses internal energy and expands to a less ordered state." After tutorial instruction this same student stated that, "The entropy of the working substance will not change because the cycle ends with the working substance at its starting point." Before tutorial instruction a different student stated the entropy of the working substance for a Carnot cycle will "increase because it is an engine so heat flows into the substance and dQ is [positive] so $\frac{dQ}{T} > 0$, $\Delta S > 0$." After tutorial instruction, this second student stated that the entropy of the working substance will "remain the same [because] for a reversible cycle a state function doesn't change." These results indicate a marked improvement in some students' understanding of the relationship between entropy, heat engines, and the 2nd Law.

The "engine-feasibility" question (Figure 2) was included on a course examination approximately one month after the heat engines tutorial was administered and one week before the ungraded quiz. The engine-feasibility question, which asks students to determine whether a given heat engine is physically possible, was modeled after the ones developed by Cochran and Heron.[4] We chose to use this question to facilitate comparisons between our students' responses and the responses observed by Cochran and Heron[4] using similar questions in an introductory course. These comparisons are justified due to the similarities reported between introductory and advanced students in terms of their difficulties reasoning about topics relating to entropy and the 2nd Law.[1]

Consider the following heat engine. The high temperature and low temperature reservoirs are at 600 K and 400 K, respectively. The heat transfer from the high temperature reservoir to the working substance during one complete cycle is 600 J. The heat transfer from the working substance to the low temperature reservoir during one complete cycle is 350 J. The work done by the working substance during one complete cycle is 250 J. A diagram of this heat engine is shown at the right.



Determine whether or not the engine could possibly function. Explain your reasoning.

FIGURE 2. The engine-feasibility question (adapted from Cochran & Heron[4]), used on a course examination one month after tutorial instruction.

Student responses to the engine-feasibility question at UMaine can be categorized into three main types of reasoning:

- Compare efficiencies— Students calculate the efficiency of the engine and compare it to the efficiency of a Carnot cycle operating between the same two reservoirs.
- Calculate $\Delta S_{universe}$ Students calculate the change in entropy for each of the reservoirs to determine if $\Delta S_{universe} \ge 0$ is satisfied.
- Compare ratios Students calculate the ratios of heat transfers $(|\frac{Q_{\rm L}}{Q_{\rm H}}|)$ and temperatures $(\frac{T_{\rm L}}{T_{\rm H}})$ and make a comparison.

Of the six students who participated in the tutorial, three answered the engine-feasibility question correctly with correct reasoning (two used *compare efficiencies*, one used *calculate* $\Delta S_{universe}$). Three students (only one of whom participated in the tutorial session) used *compare ratios* incorrectly to get the correct answer. The two ratios shown above will only be equal for a Carnot engine, but $|\frac{Q_L}{Q_H}| \geq \frac{T_L}{T_H}$ is a valid feasibility test for any engine. However, all three students who used *compare ratios* only checked for equality; since the ratios were not equal, the students said that the engine would not function. The correct use of *compare ratios* was not observed in any of the students' responses to the engine-feasibility question.

DISCUSSION AND IMPLICATIONS FOR FUTURE RESEARCH

The results from our written data seem promising. Some students demonstrate an improvement in their understanding of entropy and heat engines from pre- to post-instruction. On the exam question, no student used only the 1st Law to determine the viability of the engine, evidence for which was reported by Cochran and Heron after lecture-based instruction. All students invoked the 2nd Law in some fashion or another.

The students who use *compare ratios* on the engine-feasibility question supply an interesting opportunity for further research. These students implied that a heat engine would not function if it was not a Carnot cycle. Would they come to the same conclusion if the proposed engine had a *lower* efficiency than the Carnot engine, i.e. if the ratio relationship was satisfied with the inequality but not the equality? Would these students even use the same method to solve this problem or would they calculate either the efficiency of the engine or $\Delta S_{universe}$ instead?

The video data from the tutorial sessions also provide valuable information as to the logistical aspects of administering the tutorial. Student discussions during the tutorial indicate some difficulties with understanding the definition of thermodynamic efficiency as well as its applicability. This difficulty manifested itself within the exam data when one student used an incorrect definition for thermodynamic efficiency ($\eta = \frac{W}{Q_{\rm H} + Q_{\rm L}}$) while solving the engine feasibility question. An elongated tutorial section on thermodynamic efficiency and its utility may prove beneficial in the next revision.

Other data suggest additional revisions in the structure of the tutorial, including a pre-tutorial homework assignment to be completed and brought to tutorial. This homework would facilitate the mental preparation for students to deeply engage in the tutorial. Upper-level undergraduate students are required to draw on several years of physics education, not just the everyday experiences that are often expected of introductory students. External mental preparation may be required for these upper-level students to appreciate the full implications of tutorial instruction. These revisions will be tested in the thermodynamics course at UMaine in the fall of 2009, and elsewhere over the next academic year.

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