

Procedural Resource Creation in Intermediate Mechanics

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Abstract. A problem in resource theory is describing the creation of new, high-level resources. We model resource creation by analyzing four student groups separating variables during a group quiz on air resistance. We assess each group's fluency and two observables: use of overt (such as divide, subtract, equals) and covert (such as moving, bringing, or pulling over) mathematical language and use of accompanying gestures (such as circling, grabbing, or sliding). For each group, the type of language and gesture used corresponds to how easily they carry out separation of variables. We create resource graphs for each group to organize our observations and use these graphs to model the creation of the procedural resource *Separate Variables*.

Keywords: Resources, Differential Equations, Intermediate Mechanics.

PACS: 01.40.Fk

INTRODUCTION

As part of our investigations into the conceptual, procedural, and epistemological resources students have in intermediate mechanics, we are looking at how these students solve first-order, separable differential equations (FOSDEs). Although the resources model [1,2,3] allows for the creation of new resources from existing resources, few specific examples exist in the literature. In this paper, we use observations of four groups of students to develop a model for the creation of the procedural resource *Separate Variables*.

Our data come from videotaped group quizzes on the topic of air resistance. In this testing format, students work in small groups of 3-4 students to solve the problem shown in Figure 1. Students in our course are familiar with group work, since approximately half of all class meetings make use of the Intermediate Mechanics Tutorials [3].

METHOD

Our students represent a broad range of experience and ability in mathematics and physics. They typically are physics or engineering physics majors or minors. Although Intermediate Mechanics is a sophomore level course, our students are fairly equally divided among sophomores, juniors and seniors, leading to a disparity in mathematics backgrounds -- about half have already taken a course in differential equations

while half take it concurrently. Although each group eventually accomplishes the same task, the methods and level of discussion required in each group are remarkably distinct. We focus on three aspects of the interactions: the (admittedly subjective) ease with which the separation is accomplished, the type of mathematical language the students use to describe their steps and the gesturing that accompanies the verbal discourse.

We divide the use of mathematical language into two categories: overt and covert. Overt language uses words with strict mathematical definitions, such as *equal*, *subtract*, *exponentiate*. Covert language is, in a sense, slang: *bring over* (for subtract or divide and cancel), or *kill* (for cancel), and context is frequently important in determining exactly what mathematical operations are indicated.

Gesture analysis gives additional insight into student actions. Scherr [4] defines gestures as “the spontaneous hand movements of individual speakers – movements that are directly tied to speech and created at the moment of speaking.” We note two types of gestures: *Grouping* gestures indicate terms, and are divided into *grabbing* (using the thumb and a finger to “pinch” around a term) and *circling* (circling a term or otherwise setting it off from the other writing). Circling is a gesture because it occurs separately from the writing down of the mathematical step, whether the student makes a physical mark or not.

You are at the top of a building with a beach ball. It is of a size that only the quadratic air resistance term exists, $F = -bv^2$. You throw the ball vertically downward. Because it's convenient, you choose down to be positive (so the ball is traveling in the positive direction). You observe that the ball *slows down* as it travels downward, eventually seeming to move at a constant speed until it hits the ground. Find the equation for the velocity of the ball as a function of height.

$$\sum F = ma = mg - bv^2$$

$$m \frac{dv}{dt} = mg - bv^2$$

Transform coordinates

$$mv \frac{dv}{dx} = mg - bv^2$$

FIGURE 1. Problem statement and solution path up to separation of variables.

Another gesture used is *sliding*, where, following a *grouping* gesture, the hand moves to indicate where the term will end up after the appropriate mathematics is done. Generally, grabbing is accompanied by sliding while circling is not.

THE GROUP INTERACTIONS

In Group 1, all the students work together to find a solution to the problem. Jared leads the discussion of separation of variables, working out aloud whether to use subtraction or division to move the $mg - bv^2$ term.

- 1 Jared: Now we can separate, right?
- 2 Keith: Yes.
- 3 Jared: Do, we, do we want to pull the whole entire term over, or... like divide by an entire --
- 4 Keith: I think we want to divide --
- 5 Brian: What term are we gonna have on one side? Just the v?
- 6 Jared: I think we want to pull this entire term over, divide by it.
- 7 Brian: Yeah.
- 8 Jared: 'Cause if, 'cause if --
- 9 Ann: Well, when --
- 10 Jared: Because if you subtract you're gonna have a zero, so there's no way to separate the variables after that.
(later)
- 11 Jared: So it's going to be m over mg minus bv squared dv dy equals one. Which is why we wanted to, because we wanted to have the one on the other side instead of a zero. So m dv over mg minus bv squared equals dy. Right?

Group 1 is the least adept; Jared seems unsure if subtraction or division is the appropriate operation to separate the variables, but decides on division. Since Jared must specifically distinguish between the use of division and subtraction, he must use at least some overt language. He also uses the covert "pulling", although context makes it unclear whether this is subtracting or dividing.

Although Group 1 generally uses few gestures, Jared does use a grouping gesture to indicate $mg - cv^2$

in line 6, as he says "pull this entire term over, divide by it." In this case, he draws parenthesis around the term in question, which we categorize as *circling*.

In Group 2, the students worked individually on the problem, occasionally comparing their solutions. Sarah first silently completes the separation of variables on her own paper and then helps her classmates Moe and Conrad to catch up.

- 12 Sarah: Alright, so where are you guys at?
- 13 Moe: I'm still trying to separate it.
- 14 Sarah: OK, um, one of the easiest ways is dividing the entire thing by this side, and then multiplying both sides by dx.
- 15 Moe: Yeah, yeah.
- 16 <long pause>
- 17 Sarah: Ok, alright. What are you doing right now?
- 18 Conrad: Alright, so we're at this point, right? We've got mv dv dx is equal to mg minus bv squared.
- 19 Sarah: Mm-hm.
- 20 Conrad: Alright. So what you're doing is just bringing this dx over?
- 21 Sarah: You bring the dx over and then divide both sides by this entire, um, expression. So it becomes, um, mv dv over mg minus c v-squared

Sarah, who is explicitly tutoring her group mates, makes most of the statements about separation of variables. Thus, the level at which she discusses the problem may not be that level at which she would solve the problem of her own accord. However, her group mates need her explanations to understand the solution, and so they are indicative of the kinds of resources required in the interaction.

Sarah is weighted heavily toward overt language. The only use of the covert "bring over" (line 30) seems to occur in response to Conrad's language. In her teaching role, Sarah seems to prefer the unambiguous overt language. She also uses circling in several places. This gesture is used with both the overt "divide" and the covert "bring over."

In Group 3, Simon, who is in control of the whiteboard marker, and Dan, discuss the separation.

Two other group members remain silent during the exchange.

- 22 Simon: So then we're gonna shuffle things around
23 Dan: Yes. dx over m –
24 Simon: d – what? dx over –
25 Dan: dx over m
26 Simon: You mean... d ... d ?
27 Dan: I just did this whole thing now, this exact same problem
28 Simon: Are you moving dx over there?
29 Dan: Yeah, you move dx over there, and them [$mg - c_2v^2$] over there and this [m] over there
30 Simon: Let's do it like this first.
31 Dan: It doesn't matter...
32 Simon: mv dv equals mg minus c -two v -squared dx
33 Dan: Yeah, if you want to write down that step.
34 Simon: And then we get the v on the other side
35 Dan: We move this over there and that over here.
36 Simon: Move the one with v over there?
37 Dan: Yeah... Move this whole term over there.
38 Simon: mv over mg minus c -two v -squared equals dx .

Group 3 uses only covert language; terms are “moved” instead of multiplied or divided by. This is possible because Simon and Dan use extensive grabbing and sliding. He reduces his verbalization by using “that” combined with an grouping gesture, in this case, grabbing, to replace reading “ mg minus c -two v -squared” aloud. He also avoids using overt language by using the sliding gesture to indicate what should be done with the term.

While Group 4 is exceptionally verbose in other discussions during the quiz [5], the discussion of separation of variables is limited to one short statement. The other group members, Max and Darryl, do not challenge Phil's procedure, which, although not verbally explicated, was nonetheless correct.

- 39 Phil: OK, now we want to separate variables. We have mv dv over mg minus bv squared equals <clicks tongue> uhh, dx .

Phil does not use any mathematical language to describe his separation of variables procedure. He simply states that he will separate variables and then verbalizes the result of the separation. Similarly, he doesn't use any gestures to indicate the procedure behind his separation.

Although it is clear that Phil considers the appropriate separation of variables procedure to be self-evident, the silence from his group mates could indicate either complete understanding or no

understanding at all and an unwillingness to ask for clarification.

A trend appears in these four episodes: less adept groups use more concrete language and circling gestures. As separation of variables becomes easier, covert language is accompanied by grabbing and sliding. Eventually, the separation is regarded as trivial and no discussion of the procedure is needed.

RESOURCE CREATION

Resources [1] are chunks of knowledge that students bring to bear on a situation. The knowledge may be of any type, including conceptual, epistemological, or procedural. Resources are often described as being “irreducible to the user” [2], as well as being created from other resources. For example, Sayre [6] says that resources are “used in whole, though they may have (tacit and unexplored) internal structure”, and Hammer *et al.* explain, “locally coherent sets of resources may, over time, become established as resources in their own right.” [7]. While we agree with the latter, the former requires some refinement. The irreducible nature of resources is sometimes taken to require that the internal structure of a resource *cannot* be accessed by the user. While this is certainly the case with primitives [8], it becomes less tenable with larger scale resources [9]. We say that a user has a resource if any internal structure *need not* be accessed, without making the claim that the user cannot access that structure.

Without following an individual over many episodes spaced in time, it is difficult to directly observe the development of any resource, much less a particular resource of interest. We seek to find alternative methods to overcome this technical issue. Just as one might use the observation of many students using a resource a single time rather than one student using it many times to justify the *existence* of a resource [8], we use observations of students at different levels to justify the *formation* of a resource. These observations do not describe the resource formation path of any particular student (since we use observations of many students) or of students in general (since any individual has a unique set of resources), but rather suggest a possible path that a student might take.

We submit the series of resource graphs [10] in Figure 2 as an example of the formation of *separate variables*. Each grouping contains the mathematical resources observed during the interaction, but for clarity, does not contain any conceptual or epistemological resources which may be in play. Of course, this is not the only possible process by which the “separate variables” procedural resource could be

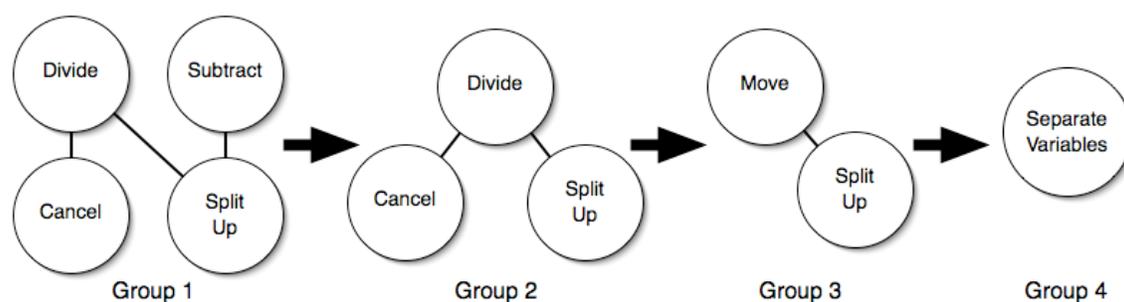


FIGURE 2. A model of the formation of the procedural mathematical resource *Separate Variables*.

created. In fact, considering that each individual has a distinct set of resources and resource graphs at his or her disposal, it is highly likely that the process by which any particular new resource is created for an individual is also distinct.

The earliest stage of resource formation we observed was seen in Group 1. Jared knows he needs to split up the variables, but is initially uncertain whether he should use division (with the required, but tacit, cancellation) or subtraction to accomplish this end. Thus, the first resource graph shows “divide” linked with “cancel”, and both “divide” and “subtract” linked with “split up”. In Group 2, subtraction is no longer considered as a needed operation (although, of course, there are many cases where subtraction *is* needed in separation of variables), leaving only “divide”, “cancel”, and “split up”.

In the third stage, we see a major shift away from overt language, leaving only the more general “move”. It is no longer important which operations are used to split up the variables; terms are simply moved to their correct locations. Finally, no individual steps are needed to explicitly describe the process. It simply occurs. At this stage, while the process may not be “irreducible” (in that the steps *could be* discussed), it clearly need not be reduced to be understood or used, and so we consider it a full-fledged procedural resource.

CONCLUSIONS

While the resources model allows for the creation of resources, specific examples are lacking. Direct observation of the formation of a specific resource in a particular individual can be exceedingly difficult. We model one possible path for the formation of the procedural resource *Separate Variables* using observations from four groups.

As the *Separate Variables* resource develops along this path, student language becomes less overt and gestures play a more important role. Eventually, the

individual steps of the procedure do not require explication. At this point, the internal structure of the resource need not be accessed and we can say the resource exists.

ACKNOWLEDGMENTS

Our thanks to Kate Hayes and Brandon Bucy for assistance in videorecording. This material is supported in part by the National Science Foundation under grants DUE-0441426, DUE-0442388, and REC-0633951.

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