

Student Understanding of Basic Probability Concepts in an Upper-Division Thermal Physics Course

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Abstract. As part of ongoing research on student understanding in upper-division thermal physics, we developed a number of simple diagnostic questions designed to probe understanding of basic probability concepts. Preliminary results showed that many students had difficulty in distinguishing the concepts of microstate and macrostate, and in applying mathematical relationships for multiplicity of simple systems. We have tested a tutorial sequence designed to address some of the difficulties. We will summarize previous results, show post-test results from the target courses, and describe aspects of the tutorial sequence that are likely in need of modification.

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INTRODUCTION

The field of Physics Education Research has provided a wealth of information on student learning of topics at the introductory level [1]. More recently, a number of researchers have expanded the field to include investigations of upper-division courses. This paper reports on one such project in the context of an upper-division course in thermal physics.

Thermodynamics and statistical physics are typically part of the core sequence in an undergraduate physics major. The exact proportion of these topics can vary from institution to institution, however, and some offer a hybrid course, typically described as thermal physics, that explicitly blends classical thermodynamics and statistical physics. In such courses, students must apply principles of statistics to physical systems. In the current project, we have investigated student understanding of probability in the context of a thermal physics course and developed instructional materials targeted toward improving student understanding.

BACKGROUND FOR THE RESEARCH

This work proceeds from the assumption that students construct understanding of scientific phenomena, in some cases developing ideas that are in contrast with accepted scientific viewpoints. We have sought to document student understanding of the target ideas using standard methods of physics education

research, particularly written conceptual questions for use in course assessments.

There has been little published work on student understanding of statistical physics. However, there are several previous studies in PER and other fields that are of interest. The work of Konold et al. on student understanding of probability and statistics influenced this study, and we will provide a specific example below [2]. That work in turn drew upon influential studies in psychology by Kahneman and Tversky [3]. Previous PER studies have investigated student learning of probability in other areas of physics, particularly quantum mechanics [4].

Research Questions

To what extent can students apply the binomial formula to determine probabilities in simple systems including coin flips or electron spins?

To what extent can students understand the distinction between microstates and macrostates for simple statistical systems?

Context for Research

This project was performed in several sections of an upper-division thermal physics course at California State University Fullerton (CSUF), a public comprehensive university serving a diverse student population of over 37,000 students. The course covers thermodynamics, statistical physics, and kinetic

theory, using a popular text [5]. It meets for two 75-minute blocks per week. The author has taught the course eight times, with enrollments between 6 and 19, and typically spends a significant portion of class time on small-group tutorial exercises.

Following the thermal physics approach, the course develops the ideas of entropy and the second law of thermodynamics through a statistical approach. Students apply simple probability to systems including the Einstein model for a solid and are taught what is known to physicists as the microcanonical ensemble formalism. Students learn the distinction between microstate (a complete specification of each of the components of a system, e.g., individual particles) and macrostate (a specification of the bulk properties of a system without regard to the individual particles). They are taught that all allowed microstates are equally probable, so the probability of a macrostate is proportional to its multiplicity, the number of microstates corresponding to the macrostate.

Most students in the course are physics majors who have completed introductory physics and several semesters of calculus. The CSUF introductory physics sequence does not include thermodynamics, but many students reported studying thermal physics in high school, in physics courses at other institutions, or in chemistry. A few students (10-20%) had completed a prior math course in probability and statistics.

Methods

All data in this paper were collected by analysis of student responses to written free-response questions posed on course examinations and ungraded quizzes. Students were asked to provide an answer as well as a written explanation.

INITIAL STUDENT UNDERSTANDING

For the initial phase of this project, the author developed and adopted simple probes of initial student understanding, shown in Fig. 1. The pretest in various forms has been given to seven sections of the thermal physics course ($N = 65$). Additional questions shown were added to later versions of the pretest, so student responses from these questions are limited to smaller numbers of students.

The initial version of the pretest contained only the questions in the first box. Students were asked several questions about probability in the context of the flipping of several distinguishable fair coins. This pretest was given after a short introductory lecture on probability, microstates and macrostates.

Essentially all students answered the first question correctly (three heads). The second question asks

Initial version:

1. Assume that you flip 6 fair coins.
 - A. What is the most likely no. of heads?
 - B. Approx. how likely is it that you will flip exactly 6 heads? (Estimate a %).
 - C. Is the probability of exactly 5 heads greater than, less than, or equal to the probability of exactly six heads?

Added questions:

2. Is the probability of flipping 6 fair coins and getting exactly 1 head greater than, less than, or equal to the probability of flipping 600 fair coins and getting exactly 100 heads?
3. Imagine that all families of 6 children in a certain state were surveyed. In 72 families the exact order of births of boys and girls was G B G B B G. What is your estimate of the number of families surveyed in which the exact order of births was B G B B B B?

FIGURE 1. Questions posed on pretest on probability. students to determine the probability of exactly six heads. There is one microstate, HHHHHH, corresponding to the macrostate ‘six heads,’ out of 2^6 total microstates possible, so the answer is 1 in 2^6 (about 1.6%). Around 55% (37 of 65) of the students answered this problem correctly. The remaining answers were usually offered without much explanation or work shown, but about 20% of the students answered 50% and about 10% answered 1/12 (which is 1/2 times six).

The third question asks students to compare the probability of flipping exactly five heads to that of flipping exactly six. Because there are six microstates with exactly five heads (HHHHHT, HHHHTH, etc.) the correct answer is that five heads is six times more probable. Most students (68%) answered that five heads is more probable. However, less than half of those gave a completely correct answer. Over a third of the overall group (23 of 65) gave incorrect estimates of how much more probable the five heads were. The most common incorrect response in this group has been that five heads is *twice* as probable as six heads, given by approximately 20% of all students.

Another common incorrect response on this comparison question was that the probabilities of five and six heads are equal. This response was given by 20% of the students. Most students gave very little explanation, typically simply restating their answer in sentence form: “equal. All probabilities have equal occurrences.” These answers are suggestive of a confusion between microstate and macrostate; the text and lecture emphasize that the fundamental assumption of statistical physics is that all allowed *microstates* are equally probable.

In order to make sense of the instructional sequence in the course, students must recognize how the distribution of statistical results changes as the number increases. Instructor experiences suggested that students had trouble with this idea. (See also Mountcastle et al. [6]). Therefore we added a question to the pretest (question 2 in Fig. 1) designed to examine how the probability changes with larger numbers. Students compare the probability of flipping exactly one head in six coins and to the probability of flipping exactly one hundred heads in six hundred coins. The probabilities are very different; the former is more probable by a factor of approximately 10^{64} . This problem has proven to be very difficult for students (six sections, $N = 60$). About 15% gave the correct answer with good or partially correct explanations. Another 15% made the correct comparison with incorrect or absent explanations. Almost all of the remaining 70% gave the same wrong answer, that the probabilities are equal. The fact that the ratios of heads to coins are the same appears to be very compelling for students, and explanations in support of this answer invariably cite this ratio.

In addition to the coin problems, we adapted a problem from the literature [refs 2, 3]. In the problem (#3 in Fig. 2), there are exactly 72 families in a certain region with birth order GBGBBG. Students are asked how many families will have birth order BGBBBB. For the current study, the problem was modified slightly, with students directed to make a numerical estimate, or if they could not, to state whether the result would be greater than, less than, or equal to 72. Out of four sections of the course ($N = 46$), about 40% of students answered correctly that the numbers should be the same, since each sequence is equally probable. The dominant wrong answer, given by 37% of students, was that the second sequence is less probable. These results are somewhat better than in previous reports, and may reflect the impact of a more selective student population or of the lectures. Correct answers included near-quotes from the lecture or text, though often these were tentative: “since all microstates have equal probability, right?”

Students answering incorrectly typically referred to the ratios of boys to girls in the two birth orders, or to the unlikelihood of the sequence of four boys in the second grouping: “less than, because of the lower probability of having a family with four consecutive boys being born in succession.” Answers of this sort were described by Kahneman and Tversky as illustrating the ‘representativeness heuristic,’ in which intuition suggests that a sample should be representative of the larger population. As the second sample has a different proportion of boys to girls than the overall population, it is perceived as being less probable. From a physicist’s point of view, these

responses reflect confusion between microstates and macrostates, and perhaps students misapplying an otherwise appropriate resource. A macrostate with five boys and one girl is indeed less probable than one with equal numbers of boys and girls, but a student must recognize that the phrasing of the question prompts for a comparison of microstates.

Student responses to the initial pretest questions suggested that most students had some intuition about probability. Very few questions were left blank, and many answers had elements suggesting some prior experience with the mathematics, e.g., factorials and other formal notation. However, the responses also indicated that it was not correct to assume that students had a solid prior understanding of this material.

DEVELOPMENT OF TUTORIAL

The tutorial *Counting States* [6] was developed after the initial version of the pretest and has been used seven times, with minor variations, in the thermal physics course (Spring 03 – 09). As pretest results suggested that students needed to improve their basic skills, the tutorial focused on the development and application of the binomial distribution formula. The tutorial asks students to consider states of several “state” quarters. (The different state symbols on the coins allow them to be distinguished.) After some initial questions to familiarize students with various assumptions (fair coins, total number of microstates), students are guided through the derivation of the binomial distribution formula for a set of five coins.

Students first determine the number of possible arrangements of coins that would give rise to exactly zero heads. They then imagine reversing one coin so that the resulting set of coins shows exactly one head, and consider the number of possible choices they have. This process continues with different numbers of heads, up to three. Students are then asked to generalize their results and construct the formula, then compare the results with the equation derived in the course text. This initial version of the tutorial was focused almost exclusively on the derivation and use of the formula, and the associated probabilities. Issues like the distinction between macrostate and microstate or the representativeness heuristic may have come up during student discussions, but neither the tutorial nor the associated homework were designed specifically to address student difficulties with these issues.

Counting States is typically used in the sixth week of the course, and is one of a sequence of three tutorials on probability and statistics used in the course. Subsequent tutorials address the problem of counting states in the Einstein solid, the multiplicities in two interacting Einstein solids, and the approach of

<p><u>Midterm exam problem:</u> Determine the probability, assuming fair coins, of:</p> <p>A. flipping a coin ten times and getting seven heads in a row followed by three tails in a row B. flipping a coin six times and getting exactly three heads</p> <p><u>Final exam problem:</u> State whether the probability is greater in case A, greater in case B, or equal in the two cases:</p> <p>A: flipping heads twice then five straight tails B: flipping seven straight tails</p>

FIGURE 2. Assessment questions on probability. the two solids to classical equilibrium. Throughout the sequence the ideas of micro- and macrostate are used, though in contexts other than coin flipping.

ASSESSMENT OF INITIAL TUTORIAL

In order to assess the initial version of the tutorial, we have posed quantitative and qualitative problems on course examinations. Additional feedback has come from students at CSUF and from instructors testing the initial version of the tutorial at other institutions. The clear signal is that the tutorial is only partially successful. Here we provide results from two problems posed to the same section of the CSUF thermal physics course, one on a midterm and one on the final, that illustrate the mixed results (see Fig. 2).

The more qualitative problem was from the final and asked students to compare the probability of two sequences of flips for a single coin. This problem is similar to the birth order problem described above but in a different context, and looks like the questions in the tutorial requiring use of the binomial formula. If students used that formula, they might have incorrectly compared the probabilities of *macrostates*. Almost all students (16 out of 18) answered correctly that probabilities were equal.

The more quantitative problem from the midterm shows a different pattern of student understanding. Students were asked to determine the probabilities of two outcomes. The first specifies a single microstate, so the probability is 2^{-10} . The second asks for the probability of a macrostate, requiring use of the binomial formula. Sixteen of the eighteen students answered the second part correctly (with small math errors in two cases). However, only half the class answered the first part correctly, compared to the 55% correct on pretest question 1B. The wrong answers on the first part were more sophisticated than pretest responses, invariably including use of the formula appropriate for a macrostate rather than a microstate.

There may be multiple explanations of the difference in student results on these problems. The

midterm came after all instruction on statistics, so better performance on the final is not due simply to additional instruction, though students may well have reflected on their mistakes on the midterm. The midterm results suggest that the tutorial alone does not address important issues for some students. In particular, it seems that many have difficulty in deciding when a description corresponds to a macrostate and when it corresponds to a microstate. Asking for a quantitative response prompted different behavior from some students.

DISCUSSION AND SUMMARY

Classroom testing and the post-test results described above suggest that the tutorial sequence is only partially successful. Students can apply the formula to calculate multiplicities, but often do not recognize when the formula is or is not appropriate. In particular there appears to be a need for additional effort to distinguish micro- and macrostates. It is not immediately clear why the different problems described in the previous section led to such different results, but it may be that the apparently quantitative problem led students to apply a formula in a fashion that proved inappropriate.

Additional feedback from instructors and students suggests that students may not understand the significance of the different ways of describing states. For example, in the assessment questions the sequence of events is used to illustrate distinguishability, but in the tutorial the state emblems on the quarters carry this information. Additional research continues, including student interviews, and we have begun to revise the tutorial to address these findings.

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