Salience of Representations and Analogies in Physics

Noah S. Podolefsky and Noah D. Finkelstein

University of Colorado at Boulder, 390 UCB, Boulder, CO 80309-0390

Abstract. This paper focuses on the dynamics as students reason using analogies. We describe analogical scaffolding, a model of cognitive processes by which students can use prior knowledge to learn new material, and apply this model to demonstrate its utility in describing the dynamics of student reasoning about EM waves in an interview. The present fine-grained analysis confirms prior large-scale findings, that representations play a key role in student use of analogy.

Keywords: analogy, representation, electromagnetic waves
PACS: 01.40.Fk

INTRODUCTION

This paper focuses on a fine-grained analysis of a student interview using the analogical scaffolding model [1] as an analytic tool. We describe this model and apply it to describe the dynamics of student reasoning about EM waves using wave on a string and sound wave analogies. The present fine-grained analysis confirms prior large scale findings, that (external) representations play a key role in student use of analogy. [1] [2] In this paper, we argue that an approach that treats representations as *part of* concepts can be extremely productive for understanding dynamic student reasoning. Though controversial, this view is articulated and supported elsewhere. [3]

A standard view posits an analogy as a mapping from a familiar conceptual domain, *base*, to an unfamiliar conceptual domain, *target*. Analogies in this sense can be productive when generated by the user, but students may not generate and/or use analogies productively. This general finding is paradoxical: how does the student know what mappings to make in an analogy if they do not have sufficient knowledge of the target *a priori*? (Not to mention the base.)[1]

This theoretical paradox is manifest empirically. Student interviews, such as the example analyzed in this paper, reveal that simply suggesting an analogy to students (e.g., stating "an EM wave is like a wave on a string") does not generally enhance these students' reasoning in any apparent way about the target. We have, however, found effective ways of promoting the

productive use of analogy by students. These findings called for an explanatory model, and based on student interviews, classroom observations, and large-scale (N>100) studies, we developed the analogical scaffolding model.

Prior empirical studies demonstrated the utility of analogical scaffolding. Students demonstrated significantly greater learning gains when taught about EM waves with multiple (vs. no) analogies [1] and with multiple (vs. single) representations. [2] Here, we extend these large-scale results, using a fine-grained approach to study the dynamics more directly.

ANALOGICAL SCAFFOLDING

A more detailed account of analogical scaffolding is provided in a prior paper. [1] We represent the relationship between a signifier, *sign*, the thing the sign refers to, *referent*, and a knowledge structure mediating the sign-referent relationship, *schema* (Figure 1). [4] In the case of a sound wave, the sign could be a *sine wave*, the referent *sound*, and the schema would include the elements *longitudinal* and *three dimensional (3D)*. Sign-referent-schema spaces can *blend*, producing new schemata and new sign-schema
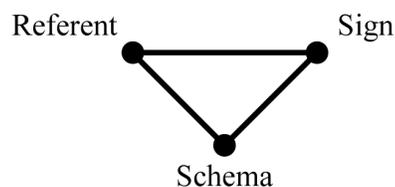

Figure 1. Sign-referent-schema diagram.

---

[1] This paradox is reminiscent of a problem elaborated by Plato. Namely, if one is to learn something that they do not already know, how does that person know what to learn?

associations. [5] For instance, a relatively abstract sign for an EM wave, a sine wave, can take on concrete features of a wave on a string, which uses the same sign (representation). Abstract ideas, such as an EM wave represented by a sine wave, gain particular meaning as result of a series of layered blends. [6] The interview analysis below provides a specific example of layering, building on string and sound to compile meaning into the canonical representation of EM waves.

REPRESENTATIONS AND SALIENCE

According to our framing of analogy, students presented with an unfamiliar (and perhaps challenging) problem can draw on existing mental structures to solve that problem by analogy. These mental structures may be cued by the student recognizing some similarity between the problem and prior experience (which may include another problem the student has solved previously). Two possible mechanisms for recognizing similarity, and hence making productive use of an analogy, are the following. If the two domains contain surface features (e.g., signs) that are similar, this can cue the student to make an analogical comparison. This mechanism explains why students may consistently solve problems that include inclined planes using kinematic equations (even for problems where the optimal solution method uses conservation of energy). [7] Note that in this case, students *do* use an analogy, albeit the base domain may be inappropriate. A second possible mechanism is that similarity is based on mental structures that transcend the surface features of a problem representation. This explains why physics experts can productively apply different solution methods to different problems which, say, all involve blocks on inclined planes. Still, *something* contained in the problem representation cues physicists to use one method or another, so the representation and mental structure cannot be completely separated.

A productive dimension for analyzing analogy use is *salience*, the strength of associations between sign features and schemata. [8] We adopt this notion of salience to argue that it is not the surface features of a sign that are salient *per se*, but the associations made with those surface features. Salience depends on the individual and context. A student presented with a sine wave may associate that sign with a material object (e.g., a wave on a string or a water wave). A physicist may associate the same sine wave with a graph (e.g., of electric field strength oscillating in time). Both the student and physicist cue on surface features – where the sine wave goes up, *something* goes up – but *what goes up* is very different.

How can the salient associations that students already use (often quite readily) lead to these students making associations that are salient to physicists? We seek a solution by proposing analogical scaffolding, focusing on representation use. This model suggests productive ways of scaffolding students' use of analogy by capitalizing on associations that are already salient for students and layering these associations towards more expert-like ideas.

DYNAMICS IN AN INTERVIEW

We have interviewed numerous introductory physics students in our studies of analogy. These think-aloud interviews generally involve students using curricular materials. Here, we focus on an interview with student "S". We describe several segments of this interview briefly and then apply analogical scaffolding in detail to a segment involving sound waves. At the time of the interview, S was enrolled in the first semester of a calculus-based introductory physics course. At this point in the semester, S told us the class was "doing oscillations and starting pressure."

In the interview, S was presented with the EM wave concept question in Figure 2 and asked to give his best answer.[2] S said he had "never seen anything like this before" and cycled through several answers over the next five minutes, finally settling on answer choice B (3>2>1=4). His reasoning was that "3 would be the first" and that "1 and 4 are on the same position on the two different waves." At one point, S stated that "x would…be the time."

We can already identify blends in S's reasoning. S appears to blend this picture, where *x* is position, with a graph in which the horizontal axis is *time* – hence 3 is "first". This idea may have been cued by the word "time" in the problem statement. In this case, a *position*
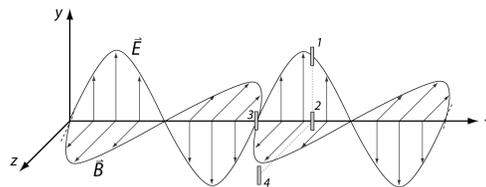
An electromagnetic *plane* wave propagates to the right in the figure above. Four antennas are labeled 1-4. The antennas are oriented vertically. Antennas 1, 2, and 3 lie in the *x-y* plane. Antennas 1, 2, and 4 have the same x-coordinate, but antenna 4 is located further out in the z-direction.

Which choice below is the best ranking of the *time averaged signals* received by each of the antennas. (*Hint*: the time averaged signal is the signal averaged over several cycles of the wave.)

A) 1=2=3>4   B) 3>2>1=4   C) 1=2=4>3   D) 1=2=3=4   E) 3>1=2=4

FIGURE 2. EM wave concept question. D is correct.

[2] Answering this question correctly requires a blend in which the EM wave sign is coupled to a schema including *3D* and *traveling (moving in time)* schema elements.

is time blend is salient for S. However, the peaks in the wave also cue a salient blend, along the lines of *a peak is a position* (S does not distinguish between the *E*- and *B-field* waves). Note that these salient blends are coupled to signs – the *x* axis, the two sine waves, and the word "time".

Next, the interviewer suggested analogies *verbally* and asked S if this helped with the EM wave concept question. First, the interviewer stated that an EM wave was like a wave on a string, but with no elaboration on how to use the analogy. S said that the string would "follow a similar pattern" to the EM wave, but that this would not help him answer the concept question. The interviewer then suggested that an EM wave was like a sound wave, again with no elaboration. S said immediately that, for making sense of the concept question, the sound wave analogy "wouldn't change too much." Note that in both cases, the sign is the verbal statement of the analogical comparison, but in these cases productive blends are not salient for S. We emphasize the distinction between not *salient* and not *existing*. As we will see, S does use several productive ideas about strings and sound, but these ideas become salient only under different conditions.

S was next presented with two new signs printed on a sheet of paper: one a sine wave and the other pictorial, depicting a hand at one end of a realistic representation of a string. He was told both represented a wave on a string. S stated that one representation was "a physical form of the other," implying a blend *sine wave is physical object*. S applied this blend to the EM wave, stating "2 is not really on…I guess none of them, other than 3, are on it." S also stated "as the hand moves it would follow the up and down with the hand," implying a blend *sine wave is moving object*. Note that the *moving object* element is not explicitly contained in the sign (a static picture), but becomes salient for S after he sees the two signs of a string.

Here, we reserve the detailed application of analogical scaffolding for sound waves. Selected transcript segments accompany a schematic representation of analogical scaffolding in Figure 3. Following the string discussion, S was presented with two different signs, a sine wave and a picture of a loud speaker with the "arrangement of air particles", and told these both represented a sound wave. These signs are shown in the topmost boxes of Figure 3. S first stated that the particles sign is a "physical representation of the sine", implying the blend *sine wave height is particle density*. In Figure 3, we place an additional node between sign and schema to indicate a prior blend contributing to a new sign-schema association. [1] S related quantities of the sine wave (e.g., "negative") with arrangements of particles (e.g., "grouping") and descriptive terms like "strong" and "weak" signal. S then applied these ideas to the EM

wave at time 17:37. Previously, S had said antennas 1 and 4 are less than 3 (answer B), but with the sound wave he said 1 and 4 are greater than 3 because they are where there are "more particles".

Now S has a new idea about antenna 2. Using the sound wave blend, S suggests that 2 is the same as 1 and 4, since the particles are "all the way down" (pointing at the dense region of particles in the pictorial sound representation, 17:59). He has applied ideas from the sound wave, now part of the *sine wave height is particle density* blend, to the EM wave.

S next tried to decide whether the particles extend in the *z* direction (in the picture, they are drawn only in the *x-y* plane). He created a new blend, drawing on experiential knowledge of sound – i.e., it is *3D* (18:37). Finally, S used a blend – *sine wave is 3D particle density* – to reason that 1, 2, and 4 are equal.

At this point, S proceeded to reason about point 3 by using the *sine wave is moving object* blend from the string. He could not deduce from the materials in front of him whether the wave at antenna 3 is stationary or moves as the wave propagates, (essentially whether the EM wave is a standing wave or a traveling wave) and he wavered between answers C and D in Figure 2. Nonetheless, S explicitly voiced the idea that a static picture of a string represents a moving object, and that an EM wave exhibits a similar property.

CONCLUSION

What does it mean to *know* a concept? S did not articulate significant knowledge of string or sound waves nor did he apply theses analogs productively to EM waves when these analogies were cued verbally. Presented with pictorial signs he did both. Did these signs cue schemata that S already had but simply did not articulate at first, or did S create new schemata during the blending process? We take a pragmatic position, framing concepts as observable through the reification of students' talk, gesture, and interactions with the environment. Our observations suggest that S's string, sound, and EM wave concepts changed dramatically under different conditions. The analogical scaffolding model captures this observed coupling between sign, schema, and referent. For S, concepts appear to depend strongly on the salience of sign-schema associations – for instance, he did not know to associate a sine wave, sound, and 3D, nor did he know to apply a sound wave blend to EM waves, without the signs in Figure 3. These associations became salient as S layered blend upon blend. We therefore argue that including signs as *part of* concepts can be extremely productive for understanding the dynamics of student learning. Furthermore, analogical scaffolding can be a useful tool for studying these dynamics.

Interviewer (I) directs student (S) to sound representations

16:48	S	This is saying, like, the signal almost, the signal is strong, well, I mean they're once again its another *physical representaiton* of the *sine*. So just looks like it's strongest here, [points to dense area of particles] so at the *high point it'd be strongest*. [points to peak of sine wave]
17:05	I	Mm hmm.
17:10	S	And, when you come *down negative there's the least particles*. [points to trough of sine wave] So it'd be weakest.

I directs S to EM wave concept question (Figure 2)

17:37	S	That, um, 1 and 4 are actually gonna be greater because this is gonna be a stronger signal. [points near dense particles on sound rep] Essentially like the sound wave. There's more particles into the peak. Greater signal.
17:52	I	OK. Greater signal than...
17:54	S	Than, like, 3 which is out here. [points near rarefied particles on sound rep] There'd be less particles.
17:59	S	2 I'm still kind of confused on. It seems like, well maybe it'd be the same 'cause if you look, if you look, um, at this high point [points at dense area in sound rep], there's a lot of *particles all the way down*. [sweeps pencil up/down over dense area of pictorial representation] Seems to follow it.

S self-directed to sound representations

18:24	S	Yeah. Because, like if you put them all here and, I don't know, is this saying that the *particles come out on the z-axis* as well or not?
18:35	I	Um, well what do you think?
18:37	S	I think they would. It'd be *three dimensional*.
18:39	I	What makes you think that?
18:40	S	Just have the *idea of sound*, so that it'd be a *big circle* [makes circle with both hands] of particles essentially.
18:47	I	Mm hmm.

S self-directed to EM wave concept question

18:48	S	But that would make me think since these are all along the same path, [sweeps pencil up/down over 1,2,4] they would all, and 4 and, yeah, *2's gonna be the same place as 1 and 4* as well, so they're all gonnafit in essentially into this *grouping of particles* in that case. [sweeps pencil up/down over dense particles]

Sound
- "Sine" [curve]
- "High point [is] strongest"
- "Down [is] negative"

Sound
- "Physical representation"
- "Particles all the way down"

Sound
- Up/down is more/less particles
- "Grouping of particles" depends on x

Sound
- "Idea of sound"
- "Particles come out on the z-axis"
- "Big circle"

EM
- Material object moving up/down
- 3 is "on" the wave
- 2 is not "on" the wave

Sound
- Up/down is more/less particles
- "Grouping of particles" depends on x
- "Three dimensional"

EM
- Up/down is stronger/weaker field
- Field (quantity) is oscillating
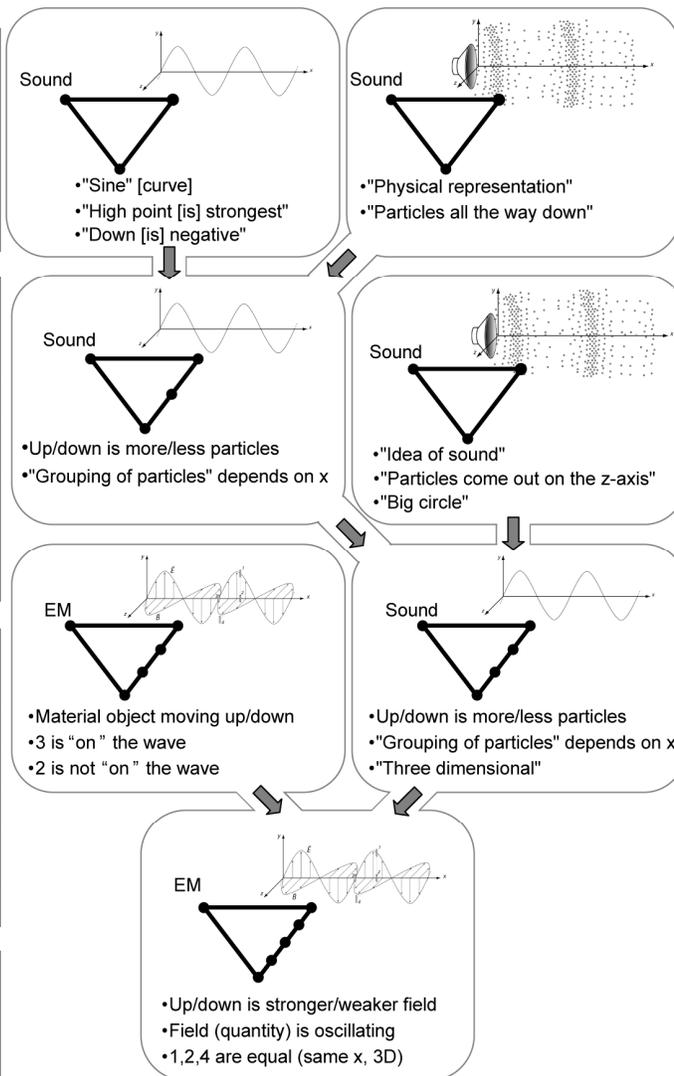- 1,2,4 are equal (same x, 3D)

FIGURE 3. Transcript (left) and analogical scaffolding analysis (right) for selected portions of transcript. In left table, timestamps in column 1; interviewer (I) and student (S) in column 2. Italics in column 3 indicate quotations used to code schema elements (repeated in quotations in the diagram on the right). For instance, the two sound spaces on the top (with sine wave and air particles signs) blend as shown by the arrows. A blend space can then become one of two inputs for another layered blend. The EM wave input space (3rd row from top, left) comes from a blend with a wave on a string earlier in the interview.

REFERENCES

1. N. S. Podolefsky and N. D. Finkelstein, *Phys. Rev. ST Phys. Educ. Res* 3, 010109 (2007)
2. N. S. Podolefsky and N. D. Finkelstein, in review.
3. D. Givry & W. M. Roth, *J. of Research in Science Teaching*, **43**(10):1086-1109 (2006)
4. W. M. Roth, & G.M. Bowen, *Learning and Instruction* **9**, 235 (1999)
5. G. Fauconnier & M. Turner, "*The Way We Think: Conceptual Blending and the Mind's Hidden Complexities*", Basic Books (2003)
6. G. Lakoff & R. Nunez, "*Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being*", Basic Books (2001)
7. M. T. H. Chi, P. J. Feltovich, & R. Glaser, *Cognitive Science* **5**, 121-152 (1981)
8. S. Vosniadou, "*Analogical reasoning as a mechanism in knowledge acquisition: a developmental perspective*" in *Similarity and analogical reasoning*, edited by S. Vosniadou and A. Ortony, Cambridge: Cambridge University Press, 1989, pp. 413-437