

# Is Instructional Emphasis on the Use of Non-Mathematical Representations Worth the Effort?

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**Abstract.** A hallmark of physics is its rich use of representations. The most common types used by physicists are mathematical representations such as equations, but many problems are rendered more tractable through the use of other representations such as diagrams or graphs. Examples of representations include force diagrams in mechanics, state diagrams in thermodynamics, and motion graphs in kinematics. Most introductory physics courses teach students to use these representations as they apply physical models to problems. But does student representation use correlate with problem-solving success? In this paper we address this question by analyzing student representation usage during the first semester of an introductory physics course for biologists taught in an active-learning setting.

**Keywords:** physics education research, problem-solving, representations

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## INTRODUCTION

A fruitful area of physics education research has been the study of student problem-solving [1,2]. While significant work has been done in this area, few studies have focused on the broad effect of students' use of non-mathematical representations (NMRs) as part of their solution to problems [3-6]. This paper reports on the relationship between student use of NMRs and problem-solving success in introductory physics. The data used for this analysis were gathered in the first semester of an introductory calculus-based physics course for students majoring in the biological sciences at California State University, San Marcos. Preliminary analysis suggests that while student use of NMRs was necessary for problem-solving success, it was not sufficient to guarantee that success.

## BACKGROUND

The setting for this study was the first semester of a non-traditional introductory calculus-based physics course for students in the biological sciences during the spring of 2005. The course is based on the UC Davis reformed introductory physics course [7]. As such, it follows a non-traditional content sequence, beginning with energy conservation rather than kinematics, and has a reduced emphasis on mechanics. The class meets twice a week, and each class meeting

lasts 3 hours. The students work in an active-learning environment with reduced lecture time and increased time in small groups. In a typical class meeting, students participate in a number of activities that are about 1 hour in length. Students work together in groups to respond to a series of prompts and then report their responses to their peers in a whole class discussion. Lecture time is reduced to about 1 hour and 15 minutes per week and is used mainly to organize the students' ideas about the phenomena they consider in their group activities. The use of NMRs and a model-based approach to problem-solving are explicitly emphasized by the instructors.

Typically the students enrolled in this course are juniors and seniors, many of whom are transfer students from community colleges. Most of these students take physics later in their academic career than is suggested by their recommended course schedule. All the students have either taken calculus or are taking calculus concurrently. The students are ethnically diverse, and it is common at this institution for a student to be the first generation in their family to attend college. Of the 43 students enrolled at the end of the first week in the spring of 2005, 39 took the final exam. Of these 39 students, 16 were men and 23 were women.

## STUDY METHODS

The main source of data for this study was student responses to selected exam questions. Exam questions were included in the study if they fulfilled two conditions. First, the question could not explicitly ask the student to use a particular NMR (i.e. if a question about dynamics, the problem could not explicitly require a force diagram). Second, a correct solution to the problem should be achievable with or without the use of an NMR.

The problems selected for the study came from a range of different topic areas. Seven problems were chosen from the content areas of thermodynamic processes, dynamics, statics and mechanical energy conservation. None of these problems were specifically developed for use in this study, but represent problems that would typically be given in this course. All students were provided with a basic set of equations on all exams/quizzes.

Of the 7 problems used in this study, 4 of the problems were from biweekly quizzes. On each quiz students were allowed 45 minutes to complete two problems. The remaining 3 problems selected for the study were part of an 8 question final exam that the students were allowed 3 hours to complete. Of the 4 problems selected from quizzes, 2 of them (Problem 1 and Problem 3) were administered to just one section of the class ( $n \approx 17$ ). The exact problem statements can be found in the appendix.

To capture the effect of NMR use on the students' problem-solving success, student solutions were sorted based on their use of NMRs, and on whether their solution to the problem was correct or incorrect. A distinct NMR could be a drawing of the problem, a diagram such as an extended force diagram or an energy-system diagram [8], or a graphical representation such as a plot. When analyzing for correctness, solutions with only minor mistakes, such as calculation errors, simple mathematical errors, and transcription errors (copying the wrong information) were included as correct responses. Student grade on a problem was not used to determine problem-solving success in order to avoid the qualitative judgment required in determining partial credit (i.e. if one valued NMR use in a solution, students using those representations would be biased toward a higher grade).

## RESULTS

The first method of analyzing the data was to compare the success of NMR users on individual problems versus NMR non-users. Any kind of NMR use on a problem would result in a student being

classified as an NMR user. On all problems but one, students using NMRs had a higher percentage of correct answers (see Table 1). However in Problem 3, NMR non-users had a higher success rate than NMR users. This problem is the only single-step, qualitative question included in this study.

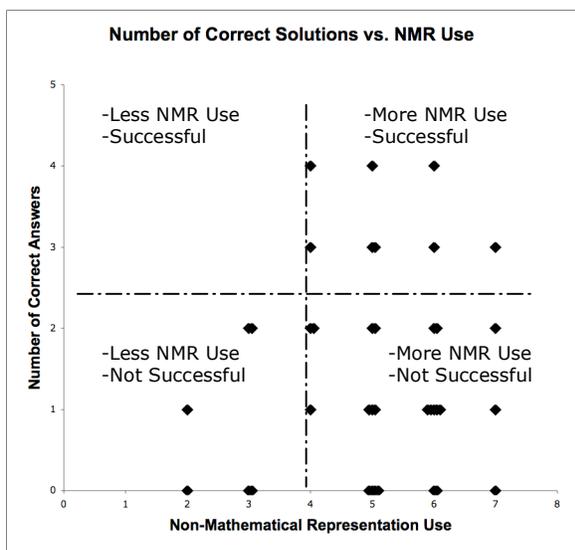
**TABLE 1.** Success rates of NMR users vs. NMR non-users.

| Problem #                            | NMR Users Success | Non-User Success |
|--------------------------------------|-------------------|------------------|
| 1 (Mechanical Energy) (n=17)         | 23 %              | 20 %             |
| 2 (Thermodynamic Work) (n=39)        | 65 %              | 46 %             |
| 3 (Force) (n=16)                     | 36 %              | 50 %             |
| 4 (Statics and Torque) (n=38)        | 31 %              | 0 %              |
| 5 (Heat and Temperature) (n=38)      | 51 %              | 36 %             |
| 6 (Newtonian Mechanics) (n=38)       | 39 %              | 0 %              |
| 7 (Mechanical Energy & Force) (n=38) | 6 %               | 0 %              |

To check if there was a significant correlation between students' success on problem-solving and their use of NMRs, a statistical correlation test was used to compare the number of correct answers versus the total number of NMRs. If students used multiple NMRs on a given problem, each NMR was counted individually. This resulted in a correlation coefficient of -0.03 for the subset of students that completed all 7 problems (16 students) and a correlation coefficient of 0.08 for the larger set (37 students) that participated in just 5 problems. Neither of these numbers suggests correlation between student success and NMR use.

To understand this lack of correlation between NMR use and problem-solving success better, the number of correct answers versus NMR use was plotted for those students completing all 7 problems and also for the larger number of students who completed just 5 problems (see Figure 1). Since both plots are similar, only the plot of the 5 problems is shown.

The plot in Figure 1 provides insight into the relation between NMR use and problem-solving success. While students are found in 3 of the quadrants, no students are found in the upper left hand portion of the plot. This suggests that at least some NMR use is necessary for student problem solving success on these types of problems. However the presence of a large number of students in the lower right hand quadrant suggests that NMR use is not sufficient by itself to guarantee student problem-solving success.



**FIGURE 1.** Each of the 37 points represents an individual student. The plot is divided into 4 quadrants to emphasize the different types of problem solvers. Note that some points are offset from integer values of NMR use to show how many students occupy a given point.

A preliminary qualitative analysis of student solutions suggests that the kinds of errors made by NMR users are different than those made by NMR non-users. In Problem 2, the students who used the PV diagram as part of their solution were almost all able to apply the idea that the work was related to the area under the PV curve. However, many of these students' errors resulted from incorrectly translating the information given in the problem statement onto a PV diagram. On the other hand, students who simply used an equation to solve for the work done often had difficulty determining the proper values for the pressure or volume. The resulting kinds of errors were different from those the NMR users made (i.e. the area under incorrect diagonal lines on a PV diagram versus incorrect values in  $W = -P\Delta V$ ).

## DISCUSSION

The results of this study suggest a connection between student problem-solving success and NMR use while also raising a number of questions. First, Figure 1 provides some insight into why there is a lack of correlation between NMR use and problem-solving success. For this particular population, a large number of students are using NMRs in their solutions, but not successfully solving the problem. This suggests that most students in this particular course are using NMRs, but are not yet proficient problem solvers. These trends may not persist in different settings, such

as a more traditionally taught course, or in the standard introductory course for physics majors and engineers. It may be that the stronger math background of students in the physical sciences and engineering would lead to a larger number of students in the upper left-hand quadrant. However, preliminary data suggest that the results in Figure 1 may in fact be robust across settings<sup>1</sup>.

It should also be noted that this particular study did not control for problem difficulty or use problems specifically designed for the study. As results from Problem 3 showed, NMR use may make it more difficult to achieve a correct answer in problems that involve only a few logical steps. Also, the quality of the NMRs was not evaluated in a systematic way. Rather, any NMR use was counted in a binary method, either used or unused. Future work could involve a study of the correlations between the quality of the NMRs used and problem solving success.

Further studies should investigate the differences in error type between NMR users and non-users. This may help to develop pedagogical strategies that can improve the success rate of NMR users who are not yet consistently successful problem solvers.

## CONCLUSION

This study looks at the relationship between student NMR use and problem-solving success. Results show that NMR use is critical to student problem solving success for the problems used in the study. Pedagogically this suggests that instructional emphasis on the importance of NMRs is useful, but (not surprisingly) is not a guarantee that they will be effectively used in a solution. Also, initial results suggest that there are interesting differences in the types of errors made by NMR users and non-users.

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## APPENDIX

1. Two masses are attached with a very light string (that does not stretch) as shown in the diagram to the right. When mass  $m_2$  is released it falls and pulls mass  $m_1$  to the right across the horizontal surface. The pulley has a very small mass, so its

<sup>1</sup> Students in the first quarter of a traditionally taught introductory course for scientists and engineers at the University of California, San Diego were given Problem 6 as part of a quiz. No students successfully answered the problem without an NMR.

rotational energy is negligibly small compared to other energies in the problem. When mass  $m_2$  has dropped a distance of 0.3 m from its initial resting position, its speed is measured to be 1.85 m/s. Mass  $m_1$  has a mass of 1.7 kg and mass  $m_2$  has a mass of 2.4 kg. Answer the question, "Is friction significant?" Support your answer quantitatively. (a diagram was included with this problem)

2. Consider 3 moles of an ideal diatomic gas (5 modes per molecule) that is enclosed in a cylinder. This gas undergoes a process that takes it from an initial state ( $V=0.04 \text{ m}^3$ ,  $P = 1.3 \times 10^5 \text{ Pa}$  and  $T = -64 \text{ }^\circ\text{C}$ ) to the final state ( $V=0.02 \text{ m}^3$ ,  $P=0.6 \times 10^5 \text{ Pa}$  and  $T = -224.8 \text{ }^\circ\text{C}$ ). The process has two steps. In step 1 of the process the gas undergoes a compression at a constant pressure of  $1.3 \times 10^5 \text{ Pa}$ , from a volume of  $0.04 \text{ m}^3$  to  $0.02 \text{ m}^3$  (see picture of step 1). In step 2 of the process the moveable piston is locked in place, and the gas is cooled so that the pressure drops to  $0.6 \times 10^5 \text{ Pa}$ . How much work is done during this process? Is the work done on or by the system? Show all your work. (a picture was included of a gas canister)
3. A ball falls from the top of a building. At first the ball's speed is changing, but after some time it reaches its terminal velocity and its speed no longer increases. During which of these two periods is the force of the earth on the ball the greatest, and how do you know? Fully justify your answer.
4. A bar of length 2 meters has three known forces on it, a 100 Newton force of someone pressing down on it 0.2 meters from the left end, an 50 Newton upward force of a string on it 0.6 meters from the left end, and a gravitational force downward at the center due to the 20 kg mass of the bar. To prevent this bar from moving, another force is required. Your job is to determine the direction, magnitude and point of application of the force required to keep the bar from moving or rotating.
5. A 400 gram chunk of iron at  $50^\circ\text{C}$  is dumped into an insulated cup containing 30 g of ice at  $0^\circ\text{C}$ . (alternate version 100  $^\circ\text{C}$  water) What is the temperature of the coffee/iron when they reach equilibrium?
6. Two boxes connected by a very light rope are pulled horizontally across a surface that has friction. The rear box has an unknown mass and the front box has a mass of 2 kg. It is also known that the frictional force of the floor on the rear box is 4 Newtons and that the frictional force of the floor on the front box is 2 Newtons. If front box is pulled forward by the rope with a force of 14 Newtons and both boxes are accelerating forward at a speed of  $2.0 \text{ m/s}^2$ , (note: the rope does not

stretch and has negligible mass) What is the force of the rope of between the boxes on the rear box?

7. Tarzan swings on a vine with a length of 6 meters. The vine can experience a tension of 1568 N before breaking. If Tarzan weighs 784 N (80 kg), what is the maximum initial height that he could start from without breaking the vine? Measure this height from the lowest point on the swing. (a second version gave the starting height and asked how long (L) vine could be)

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