Understanding Data Analysis from Multiple Viewpoints: An Example from Quantum Tunneling

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During individual clinical interviews, the interaction between researcher and interviewee leads to a specific set of data which can later be interpreted from several viewpoints. In this paper, we describe three analyses of a student's reasoning. First, we describe her "physics reasoning" in terms of the physical situation she describes and the "difficulties" she has in reasoning about the interview question. Second, we describe some "reasoning resources" that she uses. Finally, we describe "epistemological resources" that may influence her reasoning about quantum physics. We conclude with a discussion of implications about the practice of interviews and their analysis.

Introduction

While carrying out and again while later analyzing an interview, we have several possible interpretations of events and student responses that may guide our actions. Our goal in this paper is to emphasize the richness with which a single interview snippet can be analyzed, and to show that several different interpretations of the data may prove valuable, depending on the needs of the interviewer (Wittmann [1]).

In this paper, we describe three analyses of an interview in which a student, Selena (an alias), was asked to describe quantum tunneling through a square potential barrier. We analyze the results from three different perspectives. First, we give a possible "physics description" of Selena's reasoning in terms of a plausible physical situation that might have guided her reasoning. We describe her answers in terms of "difficulties" that she may have had with the material. Second, we outline some "reasoning resources" that she might have used as productive thinking tools. Finally, we describe epistemological resources which might guide her thinking about quantum physics.

The transcript given below and the accompanying video were presented during the data analysis consultation session of the Physics Education Research Conference, and the analysis that we present here is based partially on discussions during the sessions. We acknowledge the contributions of all participants and specifically thank (in alphabetical order): Gordon Aubrecht, Tom Carter, Andrew Crouse, Fred Goldberg, Peter Shaffer, Bruce Sherwood, Chandralekha Singh, Beth Ann Thacker, and DJ Wagner for their contributions.

Interview Question and Transcript

Selena was shown a square barrier (see Figure 1), and asked to reason about a stream of particles whose kinetic energy is half the energy of the barrier. Her sketch of the wave function (drawn before the discussion in the transcript) is shown in Figure 2.

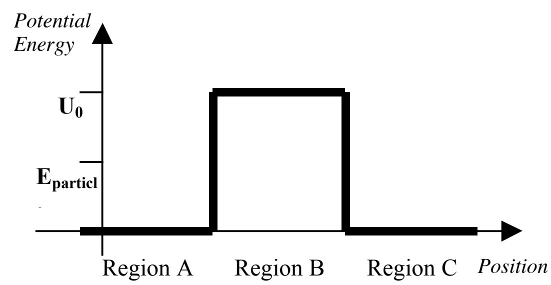

Figure 1 – Square Barrier

I: So you mentioned the word probability.
S: Mmm-hmm.
I: and that brings in to, in to my mind at least that it's, uh, perhaps more likely to be some place than some other place.
S: Mmm-hmm.
I: So where in Region A are you most likely to find an electron?
S: Um, well, from the goofy drawing I did, it would be here and here. [points to "peaks" of wave drawn in Region A]

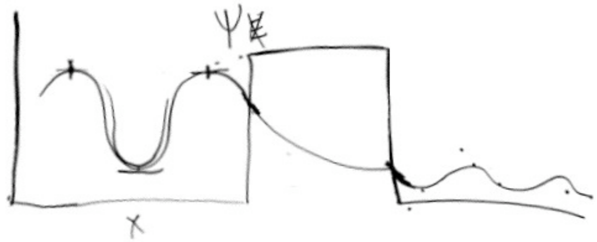

Figure 2: Selena's sketch of the wave function in Regions A, B, and C

I: And why would it be there – could you say what you're thinking about that?

S: Um, let's see, for the size of the boundary the, is it the… the *n* [writes '*n*'], I don't remember exactly what the, what the *n* was… The energy level of the particle is, determines the number of probability maxima in an area, so, say it stops here [sketches left-hand wall/axis on diagram]. The way I've drawn it, this is the *2* [writes '*=2*' after the previously written '*n*'], uh, second energy level, and the top of the peak [points to peaks of wave drawn in Region A] is indicating where it's most likely to be, and -- You know, if I'd drawn this right, this should be the bottom [indicates a shifting of the horizontal axis upward to coincide with the minimum position of the wave sketched in Region A] of the axis, it should be zero right there. [points to "trough" section of Region A].

I: Ok, you say it should be zero right there – you mean there's no probability of finding it in that region?

S: Mmm-hmm.

I: So the electron will never be in that region…

S: Uh, whether or not it actually is there we don't know, but we will never find it there.

I: Could you describe that a little bit further?

S: Uh, the math we have for describing these things is crappy, um, we don't actually know what's going on, we're assuming a whole lot of things, and, uh, according to the equations we have that work with observed stuff, we will not find it here.

I: So we could in principle take a measurement, you know, every second for the rest of our professional lives on this system, and never find the electron in this position.

S: Not if we're using these equations and the apparatus we've got, no.

Student reasoning: Difficulties with the physics

One way to analyze Selena's reasoning is in terms of the physics that she uses and difficulties she has in understanding the correct physics. Specifically, it seems that Selena focuses on a different situation than the interviewer asked about. Also, she interprets elements of the graphical representation inappropriately.

Bound particles. When asked about a beam of free particles incident on the square barrier from the left, she describes elements of a bound system. Having drawn a few oscillations in her sketch of the wave function, she interprets her diagram as indicative of the particle being in a bound state. When describing the particle in the "*n=2*" state, she seems to connect the number of nodes to the energy level of the particle.

Our interpretation of Selena thinking of bound states of a particle is reinforced by her drawing a line on the left edge of her diagram. This line is in the same place as the vertical axis on the given energy diagram. Selena seems to interpret it differently, though. She seems to interpret this line as evidence of a hard wall ("it stops here"). There is a hard wall on the left at x = 0 and a barrier keeping the particle bound until it tunnels through to the other side.

Our understanding of Selena's response is supported by evidence from other interview excerpts. Previously, Selena had talked about tunneling as a phenomenon like alpha decay. She had spoken of particles held in by a Coulomb force and tunneling out of a barrier.

> "This is the potential barrier, say like the, uh, Coulomb force of keeping a particle in a nucleus. There's an amount of energy that's required to get past that. The particle by itself generally doesn't have enough kinetic energy to leave the nucleus, because of the amount of energy holding it in, so, the uh… if the height of the… if the amount of energy of the potential is greater, it's going to take more energy for the particle to get through."

Thus, we see that Selena may be thinking of tunneling from a bound state to a free state throughout the course of the interview. She seems to interpret the left line of her graph to be where a particle stops and she misinterprets the mathematical representation of a vertical axis as a wall.

Axis height indicates energy of particle. Selena seems to use the height of the axis around which the wave function oscillates to indicate the energy of the particle. This is consistent with the way in which many textbooks draw wave functions (especially for bound states) on an axis created by the energy levels of the system, rather than on a separate graph with uniform vertical axes. Previous research has shown that many students use this reasoning in their interpretation of wave functions and tunneling (Wittmann [2]).

Conflating wave function and probability density. Selena fails to distinguish between wave function, ψ, and probability density, $|\psi|^2$. On the graph she has drawn, she focuses on peaks and troughs of the wave, as if she were looking at a probability graph and not a wave function graph. Her labels for the graph (both energy and wave function) are not consistent with her description of the graph. Nothing in her description indicates that she is interpreting the wave function she has sketched as indicating a free particle.

We note that there is no explicit coherence in the difficulties we describe. We make no claim about any coherent conceptions or misconceptions that Selena may have. We believe that few claims about a student's unitary model of the physics can be justified on the basis of a short interview snippet.

Student reasoning: Reasoning resources

In addition to describing Selena's reasoning in terms of difficulties and what she is unable to do, we also describe her reasoning based on useful tools that she brings into play. Describing productive elements of her thinking is consistent with Hammer's [3] and diSessa's [4, 5] work on resources and p-prims, respectively.

We can describe Selena's productive reasoning in terms of using a *compelling visual attribute* when Selena describes the relationship between energy and probability maxima. The idea is that the number of nodes of a bound state wave function are equivalent to the energy level of the particle. This trick is useful when describing bound state wave functions, but not free particles. It could be that Selena is over-extending her interpretation of the wave function and thinking of the particle as a bound state as a result. She is reading information out of the system based on peaks and troughs in her sketch, and connecting her initial ideas to others about probability. diSessa [5] has referred to the "span" of a readout strategy being over-extended.

We have previously discussed the idea that axis height indicates the energy of the particle. The idea that the height of the axis of oscillation indicates the energy level of the particle is consistent with reasoning about a different compelling visual attribute of a system than the one described above.

Selena also states that the number of maxima in the wave function indicates the number of the energy level of a particle. She misapplies this idea of *one to one* in the tunneling context. More seriously, her misinterpretation may guide her to think about the system she has drawn as a bound system and not a free particle system. Her use of the term "boundary" before discussing the $n = 2$ state implies that the idea was already in her head from before.

Student reasoning: Epistemological resources

Selena makes comments about the nature of our knowledge about quantum physics, with implied statements about what she wishes this knowledge were. We can analyze her comments based on the epistemological resources that Hammer and Elby have described [6].

Selena states "whether or not it actually is there we don't know, but we will never find it there." She has an appropriate idea of quantum physics: some information is either inexact or unknowable. We do not know whether she believes that the particle is actually in a specific location or not. We are as interested in what students think *can* be known in quantum physics as what students *do* know about quantum physics.

Selena views knowledge about quantum physics as something inexact ("we don't actually know what's going on"), described by "crappy" mathematics, and requiring assumptions about "a whole lot of things." We interpret this as being a possibly appropriate attitude about the physics, and also expressing Selena's frustration about understanding the physics. (Information from the video, not provided in the transcript, supports this interpretation.) Selena seems to believe that mathematics should provide a complete description of a phenomenon (consistent with an idea of *knowledge as completely describable)*, and is frustrated that it doesn't.

Selena also seems to believe that there is a correct and more exact picture of the quantum world, but "these equations and the apparatus we've got" don't allow us to find or describe it. Her attitude toward knowledge about quantum physics is both valuable in its skepticism and problematic in that she seems hesitant to use incomplete models of nature as a productive element of reasoning.

Discussion

In describing only three possible interpretations of the given interview snippet, we are leaving out several valuable elements of research in physics education. Our goal in this paper is to emphasize the richness with which a single interview snippet can be analyzed, and to show that one interpretation does not negate others which may prove valuable (Wittmann [1]).

We note that there are other interpretations of the interview that were not pursued. A single interview snippet in a single interview gives us no sense of the statistical relevance of Selena's answers. This would require additional interviews and possibly connected written questions that could be asked of larger populations. We make no claims about the strength of her beliefsnor the coherence of her reasoning during the interview snippet. Such an interpretation would require far more data from the single interview, along with data from other sources, such as additional interviews conducted at different times, written work that shows use of certain ideas, and so on. Other analyses not named here are obviously also possible and we do not mean to imply that our list is exhaustive.

Each of the interpretations of the data above, together with those not carried out, implies that data from individual clinical interviews should be used carefully and with specific attention to the goals of the researcher and to other possible interpretations that the reader might find useful. Presenting a single quote from an interview to support one's idea may be more misleading than helpful in many cases.

References

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