Student Understanding of Tunneling in Quantum Mechanics: Examining Interview and Survey Results for Clues to Student Reasoning

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Members of the University of Maine Physics Education Research Laboratory are studying student understanding of the phenomenon of quantum tunneling through a potential barrier, a standard topic in most introductory quantum physics courses. When a series of interviews revealed that many students believe energy is lost in the tunneling process, a survey was designed to investigate the prevalence of the energy-loss idea. This survey was administered to populations of physics majors at the sophomore and senior levels. Data indicate that interview results are shared by a somewhat larger population of students and give insight into additional models of reasoning (e.g. analogies to macroscopic tunnels) not found in the interviews.

Introduction

Compared with other areas of the physics curriculum, quantum mechanics has received little attention from physics education researchers.

We have continued a project investigating student understanding of tunneling in quantum mechanics [1,2]. Though this is a standard topic taught in most introductory quantum mechanics courses, we are finding that many students do not possess well-defined mental models of the tunneling process.

Student descriptions of energy loss

The project began with a series of interviews with physics majors during the 2002-2003 academic year. Two students enrolled in a senior level quantum and atomic physics course at the University of Maine were each interviewed twice during the Fall 2002 semester – once prior to instruction on tunneling, and once following the discussion of tunneling in class. Four students who completed a sophomore level introductory quantum physics course at the University of Maine during the Fall 2002 semester were each interviewed once during the Spring 2003 semester.

The interviews involved questions about a stream of charged particles tunneling through a square barrier. The barrier used in our interviews differed from that used by Bao [1] in that the potential energy on both sides of the barrier was equal, making the scenario simpler and allowing us to focus questions on student ideas about the affects of the barrier.

Though the protocol was refined for each set of interviews, a remarkable similarity emerged in the responses of the six students. Each of the six students articulated that energy was lost during the tunneling process.

An excerpt from an interview with Selena (all names are aliases) reveals a typical student response. Selena has been shown a square barrier (see Figure 1), and asked to reason about a stream of particles with kinetic energy half the value of the energy of the barrier.

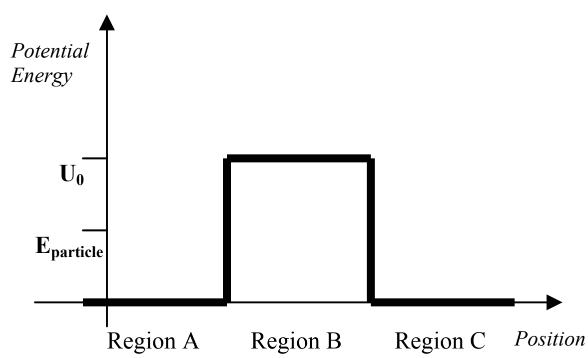

Figure 1 – Square Barrier

Interviewer: How does the electron's energy in Region C compare to its energy in Region A?
Selena: It's less.

I: It's less in Region C?
S: Mmm-hmm.
I: Ok, why is it less?
S: Uh, because it requires energy to go through this barrier.

Another student proposed the energy loss idea without being specifically questioned about energy.

Interviewer: Is there any chance the electron will ever be found in Region C?
Jack: Yes, there is.
I: There is? How do you know that?
J: Well I know because I was taught that… when the particle of some certain potential energy, or of some energy, encounters a potential barrier, there is a possibility, calculated through, well, wave equations and their integrals, that a particle will actually just go straight on through, losing energy as it does so, and come out on the other side of the potential barrier at a lower energy and continue on its path.

While examining the interview transcripts can provide some insight into student reasoning, examining an interview subject's hand motions and sketches provides additional clues. Two typical student sketches of the wave function in all three regions are shown in Figure 2.

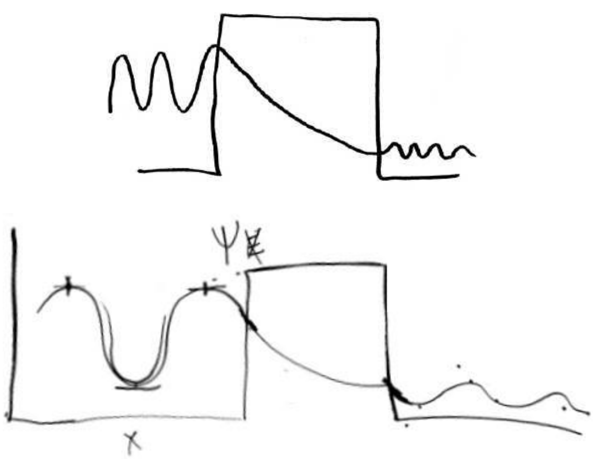

Figure 2: Student sketches of the wave function in Regions A, B, and C

In both sketches, the student has correctly drawn the wave function as sinusoidal in Regions A and C, as well as exponentially decaying in Region B. In addition, the amplitude of the wave function in Region C is smaller than the amplitude in Region A. During interviews, 5 of 6 students discussed the connection between the amplitude of the wave function and the probability of locating the particle. If a student is questioned about the probability of detecting a particle in Region C, examination of the amplitude of the wave function in Regions A and C will yield the correct response.

However, both students have incorrectly sketched the wave function as oscillating about a lower average vertical position in Region C as compared to Region A. If a student has this picture of the wave function in mind, sketched on an energy-versus-position graph (note the original label of "E" on the vertical axis of the lower sketch), he or she would likely reason that the particle has lost energy. Earlier results [2] indicate that students often use axis height to indicate particle energy in quantum tunneling situations.

The sketches reveal what may be a source of additional confusion for students. Both students have first sketched the square barrier, even though they were merely asked to sketch the wave function as a function of position. The vertical axis for the barrier diagram represents energy, but the vertical axis for the wave function sketch represents the amplitude of the wave function. Instructors often sketch the wave function on top of energy diagrams. Perhaps students, in this instructional environment, incorrectly link the amplitude of the wave function to the energy. Equipped with this model, it is reasonable that students examining the decaying wave function in Region B would conclude that the particle was losing energy.

Refining our understanding of student thinking

In order to investigate the prevalence of the notion that energy is lost during tunneling through a barrier, a survey was designed to ask students about energy loss and probability of detection for particles tunneling through square barriers.

The survey was administered during the Spring 2003 semester. Since neither quantum mechanics course is offered at the University of Maine during the spring semester, two other classes were selected in which to survey students. The survey was administered to 15 students in a sophomore-level classical mechanics course, and 4 students in an upper-level statistical mechanics course. Thirteen of the 15 (87%) classical mechanics students[1] had completed the sophomore level introductory quantum physics course. All four of the upper-level students[2] had completed a sophomore level quantum physics course[3], and three of the four (75%) had completed a senior level quantum physics course.

The survey begins by showing students a diagram of a square barrier (similar to Figure 1), and asking about the energy of particles that are detected in Region C. The results of question 1 are shown below:

Table 1: Responses to Question 1

Response *(correct is shaded)*	soph. level (n = 15)	upper level (n = 4)
Energy in Region C is the same as the energy in Region A	4 (27%)	3 (75%)
Energy in Region C is less than the energy in Region A	11 (73%)	1 (25%)

Students were then prompted to explain the reasoning used to determine their response. Answers from both sophomore and upper-level students who said energy was lost were consistent with the interview results. For example:

- "Some energy is dissipated as the particle tunnels through the potential barrier"
- "It will take some energy for the particles to penetrate the barrier in Region B"
- "Energy is 'lost' getting through the barrier"
- "The potential barrier Region B lessens the energy of the particles"
- "Particle should lose energy tunneling through a barrier"

We note that the more advanced students, though they performed better on the survey than the sophomore students, as expected, gave what seemed to be memorized or incomplete explanations of their reasoning. Responses included "particles are able to tunnel," and "the particles don't lose energy when they tunnel". These responses suggest that while advanced students have perhaps memorized the correct answers, their understanding of tunneling phenomena may be no better than that of sophomores.

The remainder of the survey dealt with the effect of modifying either the potential barrier (increasing width or height) or the particle energy (decreasing, increasing below the barrier energy level, or increasing above the barrier energy level). In each scenario, students were asked about (i) the probability of detection, and (ii) the energy of the particles in Region C.

Examining the responses of the sophomore-level students on the remainder of the survey (see Table 2), it is apparent that they perform better on questions about probability than on those about energy[4]. While this may indicate good mental models for probability, it may also suggest that students don't have well-defined connections between the various concepts involved in tunneling. Ambrose, et al. [3], have described student difficulties in reasoning about the behavior of waves. A student model based on the wave function portrayed in Figure 2 might suggest that everything – amplitude, energy, probability – decreases after tunneling. On the administered survey, such reasoning would suggest decreased probability (correct), as well as decreased energy (incorrect).

We present one result from the additional questions to illustrate the richness of student reasoning on quantum tunneling. Some students seem to think of physical, macroscopic tunnels when they reason about quantum mechanical tunneling.

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[1] All four students who were interviewed following completion of the sophomore-level course were part of this group.

[2] Neither of the interview subjects from the senior-level quantum and atomic physics course were part of this group.

[3] One of the students is a graduate student who completed undergraduate coursework at another institution.

[4] Three of the four upper-level students answered all questions correctly on the survey.

Table 2: Additional Survey Responses – Sophomore Level Students (n = 15)

Scenario	Probability Question Answered Correctly	Energy Question Answered Correctly
Barrier Width is Doubled	12 (80%)	4 (27%)
Barrier Height is Doubled	10 (67%)	3 (20%)
Particle Energy is Increased (but below barrier energy)	10 (67%)	4 (27%)
Particle Energy is Decreased	12 (80%)	3 (20%)

One interview subject discussed her mental picture of snowballs flying through snow banks when she thought about tunneling. Examining the survey responses from the sophomore level students seems to support this hypothesis. In the scenario where the *width* of the barrier is increased, the largest percentage (53%) answer that the energy loss is now greater. In the scenario where the *height* of the barrier is increased, the largest percentage (40%) also answer that the energy loss is unaffected by this increase. For us as researchers, this suggests an analogy to macroscopic tunneling; it does take more energy to tunnel through a *wider* mountain, but does not take more energy to tunnel through a *higher* mountain.

Conclusion

Our work in investigating student understanding of tunneling has revealed that students use many ideas both successfully and unsuccessfully. As examples of effective reasoning, we note that most students are comfortable enough with the idea of a wave function to be able to reason about its sinusoidal or exponential nature in domains where the particle's average energy is both greater than and less than the potential energy of the region. Students also apply the ideas of continuity to the wave function, as evidenced in interview responses as well as an examination of their sketches of the wave function in the three regions. Furthermore, students seem to accept the notion of tunneling; no student interviewed or surveyed responded that tunneling was impossible on the microscale.

Students also reasoned incorrectly in many areas. Our most common result was the idea that energy is lost by a tunneling particle. We believe that this is due to several reasons, including:

- misinterpretation of the graphical representations, specifically the vertical axis of the graph of the wave function;
- common sense ideas of objects passing through barriers; and
- an explicit analogy to macroscopic situations, such as building a tunnel through a mountain.

Further data indicate specific difficulties in interpreting elements of the wave function (such as amplitude or wavelength) that are consistent with the literature. Future research will investigate these and other results in similar and more applied contexts such as Scanning Tunneling Microscopes (STMs).

References

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