Mathematical vs. Conceptual Understanding: Where Do We Draw The Line?

Homeyra Sadaghiani and Nicholas Aguilera

Department of Physics and Astronomy, California State Polytechnic University, Pomona, CA 91768

Abstract: This research involved high school physics students and how they learn to understand Newton's laws as they relate to falling bodies and projectile motion. Students in introductory, algebra-based, high school physics classes were evaluated based on their prior knowledge through a pretest, designed to assess their initial comprehension of the motion of falling bodies and projectiles. Groups were divided and taught separately with an emphasis on either mathematical derivation of equations, followed by brief conceptual discussions, or on thorough conceptual analysis, followed by a brief mathematical verification. After a posttest was given, an evaluation of the responses and explanations of each group of students was used to determine which method of instruction was more effective. Results indicate that after the conceptual group and math groups achieved similar scores on the pretest, the conceptual group obtained a slightly higher normalized gain of 25% on the posttest, compared to the mathematical group's normalized gain of 16% (unpaired two-tailed t-test P value for posttest results was 0.1037) and, while within standard deviations, also achieved higher overall scores on all posttest questions and higher normalized gains on all but one posttest question. Further, most students, even thoes in the mathematically-instructed group, were more inclined to give conceptually-based responses on postest questions than mathematically-based ones. In the context of this topic, the dominating difficulty for both groups was in analyzing two-dimensional projectile motion and, more specifically, the behavior of each one-dimensional component of such motion.

Keywords: Physics Education, High School, Conceptual versus Mathematical.
PACS: 01.40.ek, 01.40.Fk, 01.40.gb

INTRODUCTION

Physics is often described by students, especially those at the introductory levels, as one of the most difficult of the sciences to understand. Among many of these students, the difficulty in understanding physics is attributed to the high degree of mathematics that is required in physics, as compared to the other sciences [1]. As introductory physics is often the final science class that can be taken by high school students, and is usually not a requirement for high school graduation, many high school students simply refrain from taking any physics classes prior to entering college. Research studies suggest that this trend among high school students will likely have adverse effects on their performance in college physics courses [2].

There is much research on the learning patterns of introductory physics students done at the university level; however, such research is rarely conducted in high school classrooms. Studies on high school student learning of physics suggests that students have very strong incorrect ideas about the concepts of force and motion and that it takes much instruction and practice to convince them to alter these preconceptions [3]. This experiment was carried out in order to gain more of an insight into the conceptual versus mathematical learning processes of students before they enter college, and to assess their preconceptions and "conceptual primitives" [4] regarding falling bodies, so that educators may incorporate more effective teaching strategies in their high school physics classrooms.

METHODOLOGY

This study was conducted in two introductory, algebra-based, public high school physics classrooms that were taught by the same teacher and were at the same point in the curriculum for the school year, and had similar mathematical backgrounds. Classes had an initial introduction to Newton's laws, but had not yet applied them to falling bodies or projectile motion. One class was chosen randomly to be the "conceptual group" (N=28) and the other to be the "mathematical group" (N=36).

Each class was given an identical pretest in order to assess their initial understanding of the topics to be investigated. Instruction was then given by one of the investigator (NA), who was a senior undergraduate physics major pursuing a career in high school physics teaching, for one hour per day, over the course of one school week. Each class was instructed by conceptually or mathematically-emphasized means. Conceptually-emphasized instruction involved detailed analysis of the underlying concepts relating to a topic through discussions and exercises, followed then by a brief mathematical discussion or exercise to reinforce those concepts. Mathematically-emphasized instruction involved discussions and exercises relating to the mathematical derivation and analysis of a topic, followed then by a brief conceptual discussion or exercise to help explain what they had just derived or analyzed.

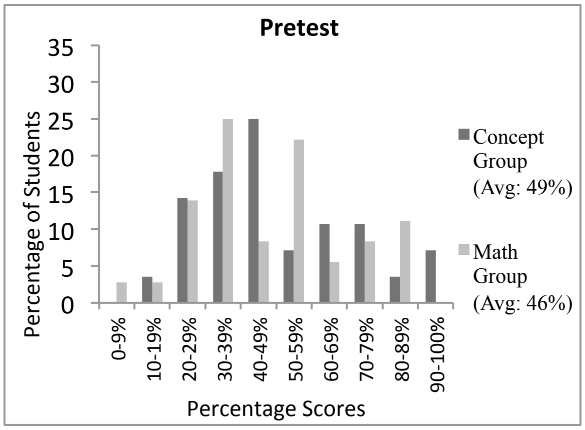

Figure 1. Overall pretest scores

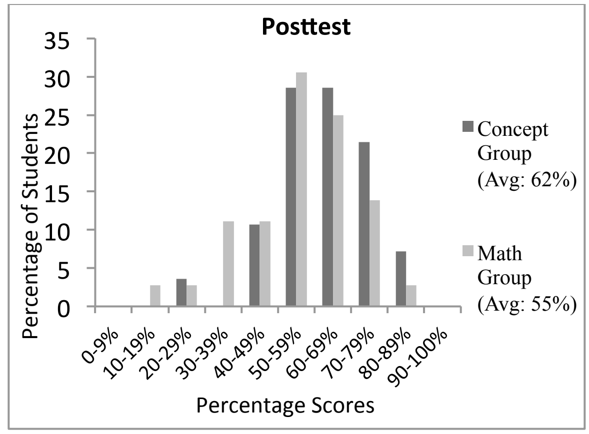

Figure 2. Overall posttest scores

As an example, suppose two classes, one using concept-based instruction and one using math-based instruction, were each given the scenario of a ball being dropped to the ground and taking three seconds to complete the fall. Upon learning the definition of acceleration as being the rate at which velocity changes over time, and learning that all objects fall with the same, constant acceleration due to gravity, the concept-based class might be asked to form small groups and draw vectors to represent how far the ball would fall during each second of free-fall (e.g., motion diagram) and discuss with their peers their reasoning behind their predictions. After such discussions, the class would be asked to compare their group's findings with the other groups in the class. The instructor would then support the students' conclusions by briefly walking the whole class through the mathematical calculations for the distance covered during each second of free-fall.

Given the same scenario and same background information regarding the question, the math-based class might be asked to form small groups and calculate the distance covered by the ball during each second of free-fall, using kinematic equations of motion. Groups would then come back together to discuss and compare their results with the rest of the class. The instructor would confirm the students' findings and add to the discussion as necessary, and then briefly walk the class through a conceptual exercise, such as drawing a vector diagram to represent the distance covered during each second of free-fall. (Overall, students would spend approximately 75% of their time analyzing topics with their specific instructional emphasis in small groups and as a whole class, and approximately 25% of their time discussing the alternative approach).

At the end of the one week instruction period, both classes were given identical posttests, in order to gauge their learning gains over the course of the experiment and to analyze how each group of students answered each question, based on their instructional emphasis. Through an alphanumeric labeling system, students who did not take both the pretest and the posttest were removed from the sample before analysis.

DATA

Before taking the pretest, students were asked to complete a short survey to determine their levels of previous mathematical preparation. It was found that both groups had similar backgrounds, with a fairly even split between students who had completed up through algebra II, and those who had completed pre-calculus or higher. Neither the instruction given, nor the pre/posttests required the use of calculus; either mathematical or conceptual approaches could lead to correct results. Thus, students' calculus preparations should not have had any direct bearing on the results of the study.

There were eight questions on the tests, which covered the topics of Newton Mechanics. The first four questions were multiple-choice with written reasoning to support each answer, while the last four were short answer/analysis questions. The format of the posttest was identical to that of the pretest, with the multiple choice section covering the same topics as the pretest, with only slight differences in the given scenarios in the problems, or simply in the numbers given for possible calculations, and the short answer/analysis question covering similar concepts, with slightly more difficult scenarios, such as solving a two-dimensional problem, as opposed to a one-dimensional problem. **Figure 1** shows overall percentage scores for the pretest in each class and **Figure 2** shows similar data for the posttest. On the pretest, the conceptual group achieved an average score of $(49\pm4)\%$ and the mathematical group achieved an average score of $(46\pm4)\%$. While the pretest results appear somewhat erratic, the posttest scores show a much more normalized point distribution with a relatively higher normalized gain of 25% for the conceptual group, compared to the mathematical group's 16%. In a 95% confidence interval, this difference is not considered to be statistically significant (unpaired two-tailed t-test $P = 0.1037$)

Table 1, below, describes the content of the pre/posttests and the concepts tested by each individual question. The percentages of correct responses for individual questions can be viewed graphically in **Figure 3**. Multiple-choice questions were graded based on whether or not the student was able to determine the correct choice, as well as if that student was able to provide a reasonable and correct explanation of his or her reasoning behind that choice. Short answer/analysis questions were graded on whether or not the student was able to determine the correct concepts or equations to apply, as well as if that student was able to correctly deduce or calculate the answer to that problem.

Table 1- The topics of pre/posttest questions

1) Newton's First Law in One-Dimension
2) Mass and Gravitational Acceleration
3) Constant Acceleration Due to Gravity
4) Ball Launched Horizontally from a Cliff
5) Displacement of Object going Up and Down with Gravity (with Horizontal Component for Posttest)
6) Velocity of Object going Up and Down with Gravity (with Horizontal Component for Posttest)
7) Acceleration of Object going Up and Down with Gravity (with Horizontal Component for Posttest)
8) Gravity on Objects with/without x-component (Including Upward motion for Posttest)

DISCUSSION

By viewing **Table 2**, below, one can closely examine how students went about answering each pre/posttest question, and the common difficulties and conceptual primitives that they faced when doing so. Many of the common difficulties found in this study agree with those found in previous research. Such common "conceptual primitives" [4] include the belief that gravity will pull a heavier object toward the ground at a greater rate than it would a lighter one, that velocity and acceleration are interchangeable [6], or simply that velocity implies acceleration, and that a force applied to a moving object perpendicular to its motion will have an effect on the velocity of that object in the direction of its original motion [7], among others.

On the pretest, both groups had similar errors in common, as was expected. More interesting data came from responses to the posttest. While both groups showed improvement in many of the same areas, and faced many of the same difficulties on the posttest, how they attempted to work through problems was, at times, surprising. For example, when asked whether a heavier or a lighter object would hit the ground first if dropped simultaniuously from the same height, the vast majority of students chose the correct answer on the posttest and gave some form of a conceptual explanation, stating that the acceleration due to gravity was the same for all objects. However, a small number of students in the mathematical group chose to explain their reasoning by writing down an equation for a body in freefall and simply stating that "mass is not included in this equation, therefore they should hit at the same time".

Another common problem in both groups, on both the pretest and the posttest, was the separation of x- and y-components in two-dimensional problems. Many students could visually identify a proper trajectory for a projectile, but had difficulties in analysing how the motion of that projectile should behave in each individual dimension. Given the relative complex nature of the problem, this difficulty is not unexpected, yet is not emphasised much in previous literature.

In addition, it was observed that, throughout the posttest, both groups tended to give more conceptually-based answers than mathematically-based ones, unless a calculation was absolutely necessary to reach a conclusion. The mathematical group did give more mathematically-based responses than the conceptual group, as would be expected, but even within that group, students tended to prefer to give conceptually-based responses when they were able to.

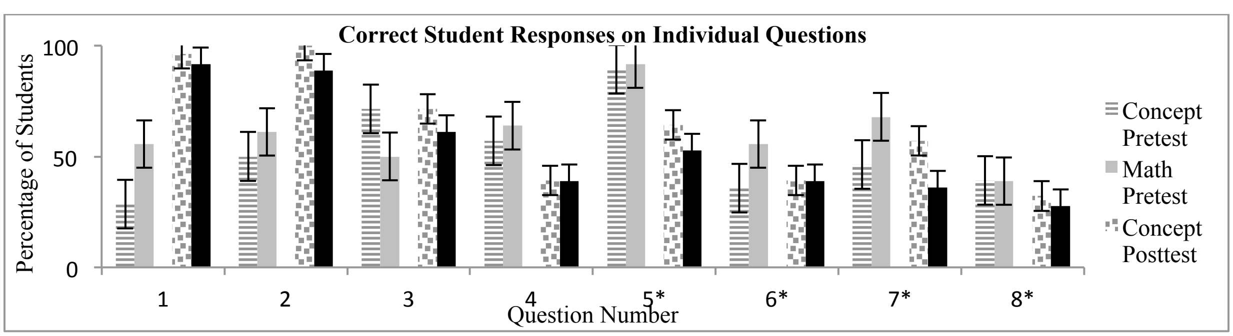

Figure 3. Correct student responses on pre/posttest questions. *Questions 5-8 were not multiple-choice questions and had a higher degree of difficulty on the posttest than on the pretest. (See notes in **Table 2**.)

Table 2. Common difficulties/conceptual primitives for each pre/posttest questions

	Pretest	Concept Posttest	Math Posttest
1	- Apply gravity in x-direction	- Nearly all gave conceptual analysis through words/pictures - Few math attempts were supplemented with conceptual description	- Mostly conceptual descriptions/pictures - Some unsolved equations in place of description
2	- Heavier object will fall faster - Applying air resistance	- Nearly all conceptual analysis - Very few mathematical attempts, supplemented with conceptual description	- Mostly conceptual answers - A few explained "mass is not included in this equation"
3	- Interchanging acceleration and velocity - Applying air resistance	- Interchanging acceleration and velocity - Trying to factor in terminal velocity when not appropriate	- Nearly all conceptual answers - Some abandoned mathematical attempts - Interchanging acceleration and velocity
4	- Many picked correct path shape, but chose wrong distance - Some chose incorrect path shape	- Nearly all incorrects chose correct shape/wrong distance - Very few mathematical attempts	- Nearly all incorrects chose correct shape/wrong distance - About same number math/concept attempts
5	- Generally understand displacement for simple 1-dimensional case	- Mostly used pictures to show trajectory/displacement - Small number of mathematical calculations - Difficulty with added 2nd dimension.	- Mostly used pictures to show trajectory/displacement - Mixed concept/math responses - Few math attempts achieved correct results - Difficulty with added 2nd dimension
6	- Confuse with acceleration - Many believe object gains velocity going up.	- Some still confuse with acceleration - Not separating x/y components (Difficulty with 2-dimensions)	- Some still confuse with acceleration - Mixed math/concept responses - Not separating x/y components
7	- Confuse acceleration with velocity - Think that acceleration changes	- Some still confuse with velocity - Most mentioned some constant acceleration in y-direction - Difficulty with added 2nd dimension.	- Some still confuse with velocity - Many had the number 9.8 written into equation but no explanation of what it was - Difficulty with added 2nd dimension.
8	- Belief that x-component will affect gravity - Disagreement on which will reach ground first (arguments for both cases)	- Nearly all conceptual - Assuming linear trajectory for bullet - Most students have difficulty with gravitational acceleration during added upward component of motion.	- Mostly conceptual answers - Not separating x/y components - Assuming linear trajectory for bullet - Difficulty with gravitational acceleration during added upward component of motion.

While mathematics is certainly an integral part of physics and should not be excluded from the learning process, examining of the data suggests that an emphasis on conceptual analysis and visualization of physics problems is beneficial, if only slightly, to the students' learning experience. This approach appears to be more in-line with the way in which students themselves, answer physics problems. As students graduate and gain a greater mathematical foundation at the college level, then the results of a similar study may change, but at this level, a fundamental, conceptual approach appears to be more appropriate, effective, and appealing to students.

ACKNOWLEDGEMENTS

We would like to thank the Cal Poly Pomona Physics and Astronomy Department for allowing undergraduate students the opportunity and the resources to conduct research in physics education. Such support could guide many talented physics students toward career paths in physics education and teaching.

REFERENCES

1. Meltzer, D.; "A Relationship between Mathematics Preparation and Conceptual Learning Gains in Physics: A possible "hidden variable" in diagnostic pretest scores", *Am. J. Phys.* ***70***, 2002
2. Sadler, P., Tai R.; "Success in Introductory College Physics: The Role of High School Preparation", John Wiley & Sons Inc.,2000
3. Hynd, R. et al.; "The Role of Instructional Variables in Conceptual Change in High School Physics", *J. of Res. in Sci. Teach.*, **31**, 9, 1994
4. Clement, J.; "Students' Preconceptions in Introductory Mechanics", *Am. J. Phys.* ***50***, 1982
5. Hake, R.; "Interactive Engagement versus Traditional Methods: A six-thousand-student survey of mechanics test data for introductory physics courses", *Am. J. Phys.* ***66***, 1998
6. Trowbridge D. & McDermott, ''Investigation of student understanding of the concept of velocity in one dimension,'' *Am. J. Phys.* **48**, 1980
7. Trowbridge D. & McDermott, ''Investigation of student understanding of the concept of acceleration in one dimension,'' *Am. J. Phys.* **49**, 1981