Students' Difficulties in Interpreting the Torque Vector in a Physical Situation

Pablo Barniol, Genaro Zavala and Carlos Hinojosa

Physics Education Research and Innovation Group
Department of Physics, Tecnologico de Monterrey, Campus Monterrey,
E. Garza Sada 2501, Monterrey, N. L. 64849 Mexico

Abstract. In this article we investigate students' difficulties in interpreting the torque vector in a physical situation. To identify these difficulties, we carried out task-based interviews with undergraduate physics majors completing a junior level course in mechanics. In the task, we presented a drawing of a beam that is initially at equilibrium over a fulcrum. Later, a weight is hung from the left side. We detected an alternative conception in which students thought that the left side of the beam and the weight would have additional motion in the direction of the torque vector. To quantify students having this alternative conception, we designed and administered a multiple-choice question to undergraduate physics majors completing a sophomore-level modern physics course. We found that 18% of the students had this conception. Based on these results, we present some suggestions for instruction of the torque vector concept.

Keywords: Torque vector, physical situation, misconceptions, difficulties, cross product.
PACS: 01.40.Fk

INTRODUCTION

The tendency of a force to rotate an object around an axis is measured by a quantity called torque [1]. The torque ($\boldsymbol{\tau}$) is a vector quantity defined as $\boldsymbol{\tau} = \mathbf{r}\times\mathbf{F}$, so the direction of the torque is determined by the right-hand rule. The torque concept is studied in introductory and advanced university physics courses.

Some researchers [2, 3] have investigated students' understanding of the magnitude of torque, finding that they have difficulties in distinguishing torque and force. Others, such as Van Deventer [4], have analyzed students' understanding of the direction of torque, detecting that the main difficulty was in thinking that the torque of an exerted force on a pulley has the same direction as the exerted force. It is interesting to note that at the moment, there are no studies that focus on students' interpretation of the torque vector. This is the focus of our study, which is important to the PER community as it promotes conceptual understanding.

The two objectives of this investigation are: 1) to identify possible alternative conceptions for interpreting the torque vector in a specific physical situation, and 2) to quantify students having these alternative conceptions after instruction.

METHODOLOGY

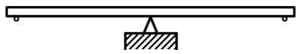
This research was conducted at a large private Mexican university. The study had two parts that were conducted over consecutive semesters. To address the first objective, during the first semester we videorecorded and then analyzed in-depth interviews with individual students using a think-aloud protocol [5] while they solved the task shown in Figure 1. We interviewed five self-selected undergraduate physics majors completing a junior-level course in mechanics. The course textbook was the text by Thornton & Marion [6]. (Note that among the interviewed students there were top and weak students).

To address the second objective, in the following semester we designed a multiple-choice option problem (fig. 2) based on the interviews´ results and administered it to 35 undergraduate physics majors who were completing a sophomore-level modern physics course. These students, before the modern physics course, had already taken all introductory level physics courses. Interviews and testing were conducted in Spanish.

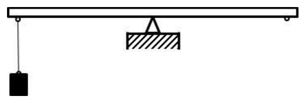
INTERVIEWS' RESULTS

We found that three students, from among the five interviewed, conveyed an alternative conception about the motion of the beam and weight. These students thought that the left side of the beam and the weight would have additional motion in the direction of the torque vector. The other two students interpreted the direction of the torque vector with the right-hand rule. Next, we analyzed the answers of the three students.

The weight and beam task. The figure below shows a beam of uniform mass density that is balanced over a fulcrum beneath the beam´s midpoint. The beam has two equal screw-eye bolts at both ends of the beam at a 0.5 m distance measured from the fulcrum. The beam is at equilibrium.



Later, holding the beam in a horizontal position, a weight is hung by a string from one of its extremes, as shown in the figure below.



Then, the beam and the weight are released. It is known that at that instant the tension in the string is 100N.

Questions: 1. Calculate the torque that is exerted around the fulcrum by the tension force of the string at the instant the beam and weight are released.

2. Is this torque a scalar quantity or a vector?

3. (If the student said that the torque is a vector): What is the direction of the torque vector?

4. What is the meaning of the direction of the torque vector in the described situation?

5. (If the student only interprets the direction of the torque vector with the right-hand rule): Besides this relation with the right-hand rule, does the direction of the torque vector have another meaning?

6. What is the meaning of the magnitude of the torque vector in the described situation?

FIGURE 1. Task used in the interviews.

Additional Motion Alternative Conception

Student A: Torque Vector as an "Angular Force"

In Question 1, student A correctly calculates the magnitude of the torque, but has difficulties with its units because he states that they are "Newtons". The student correctly answers Questions 2 & 3 about the torque's vector nature and its direction. By contrast, when answering Questions 4 and 6 on the meaning of the direction and magnitude of the torque, the student exhibits a misconception interpreting the torque vector. Next we transcribe these answers.

Interviewer (I): *What is the meaning of the direction of the torque vector in the described situation?* (Q.4).
Student A (A): *This one is hard. My impression is that…if this were three-dimensional….* (The student makes gestures and places an imaginary beam in front of him.)*…at the moment that the weight is placed, the beam, besides going downward* (downwards movement gesture), *would spin* (spinning movement gesture: the student makes a gesture to indicate that the beam spins, as if additional force were being exerted in the direction of the torque.) *…The torque is r cross F, the torque would be that way* (student points in the torque vector's direction) *so the beam would spin as well as move downward*
I: *And what is the meaning of the magnitude of the torque vector in the described situation?* (Q.6).
A: *I have always interpreted the torque as an angular force.*
I: *An angular force?*
A: *A force that is exerted in an angular direction. I don´t know why but that is my impression.*
I: (The interviewer places a pencil in front of him simulating the beam) *In this case, what would your angular force be?*
A: *It would be a force that would move the beam like this* (spinning movement gesture)*…and well…eventually it would fall because of the weight…*
I: *So, you relate the magnitude of the torque to a force going that way* (the interviewer points in the torque vector direction)*?*
A: *Yes.*

In the excerpt, we observe that student A has an alternative conception about the motion of the beam and weight. The student thinks that the left side of the beam and the weight will have an additional motion in the direction of the torque vector. We also observe that this student exhibits an alternative conception about the cause of this particular motion. The student mentions explicitly that the torque is an "angular force", and treats it as a force exerted on the eye bolt (where the weight is hung) that causes the additional motion.

Student B: The Possibility of Additional Motion although it goes against his "Intuition"

During the interview, student B also presents the possibility of additional motion of the beam and the weight. The student correctly answers the first three questions on torque calculation, vector nature and direction. In Question 4, the student interprets the direction of the torque vector with the right-hand rule. Then the student was asked in Question 5 about another possible meaning of the torque's vector direction. We transcribed this excerpt of his answer below.

Interviewer (I): *Besides this relation with the right-hand rule, does the direction of the torque vector have another meaning for you?* (Q. 5).
Student B (B): *Another thing that I could say, but I´m not sure and I would have to try it physically, is that if I do the forces analysis, I only have the weight going downwards and I don't have anything that makes the beam…* (spinning movement gesture). *I have the beam and it would do only this, that´s my opinion*

(downward movement gesture). *But if my vector is going out this way* (student points in the torque's vector direction), *it should be the cross product…Then I don't know if it would make it spin this way* (spinning movement gesture). *But I wouldn't dare say this, because I don't feel that this is what is physically happening. But since I have a vector going out* (student points in the torque's vector direction) *with that magnitude, then maybe...*

I: *So what you´re telling me is that it is difficult to you to interpret this vector?*

B: *If I´m asked about a physical description, yes.*

I: *You´re relating the vector to the motion of the beam?*

B: *Yes, to the motion of the beam.*

I: *So you think that maybe it affects the movement of the beam?*

B: *Yes.*

From the excerpt we can observe that student B presents in the interview the possibility of the additional motion of the beam and the weight as stated by student A. There are particular features of the reasoning process of this student. The first is that he doesn't establish this conception firmly, but only as a possibility (*"I could say, but I´m not sure"*.) It is interesting that he establishes that this motion is against his "intuition" about what is physically happening. The second is that he doesn´t mention explicitly that he interprets the torque vector as a force. We see that in the beginning of the excerpt he does the analysis of forces and considers only the weight, not the torque. However, when the student talks about the possibility of the additional motion, we see that he actually treats the torque vector as a force.

Student C: Incorrect Generalization of what happens in the Precessional Motion of a Top

Student C also presents the alternative conception of the motion of the beam and the weight. The student answers the first three questions correctly on torque calculation, vector nature and direction. In Question 4, he interprets the torque's direction with the right-hand rule. However, when the student is asked Question 5, he establishes that the beam will move with additional motion. Next we present the answer given by this student.

Interviewer (I): *Besides this relation with the right-hand rule, does the direction of the torque vector have another meaning for you?* (Q. 5).

Student C (C): *I was thinking about an example of a top that we saw in class. When the top is losing velocity it starts to fall because the force is going downward. In this case the torque indicates where the top is going to spin…But in the beam case* (the student makes gestures and places an imaginary beam in front of him), *I don't know if the beam is going to move also to this side* (spinning movement gesture)*...But I don't know...*

I: *What do you think?*

C: *That the weight would fall also towards me* (spinning movement gesture). *It will go downwards and also a little bit forward to the front.*

We see that this student also presents the additional motion alternative conception. The particular feature of the reasoning process of this student is that he establishes another misconception about the causes of this particular motion. This student generalizes, in an incorrect manner, what happens in the precessional motion of a top and applies it to the beam-weight system. Keep in mind that in the precessional motion of a top one can establish, using the equation $\boldsymbol{\tau}=d\mathbf{L}/dt$, that the movement of the crown matches the direction of the torque produced by the gravitational force [1]. The inaccuracy of this student is that he generalizes this fact and applies it to the beam-weight system. It is important to mention that at the end of the interview (not shown in the excerpt) the student changes his opinion (based on the fact that the beam doesn´t have angular velocity as the top) and states that the system will move only "downwards"; however it is interesting that during the interview this student also established the additional motion alternative conception and its causes.

Overall Analysis of Students' Answers

Students A, B and C present the alternative conception about the motion of the beam and the weight. However there are some differences between the reasoning processes of these students. Next, we present an overall analysis of these students' answers. Student A is the only one that mentions in an explicit way that he considers the torque vector as an "angular" force, and holds the motion alternative conception firmly. Student B establishes this alternative conception only as a possibility, although it is against his "intuition". Student C also initially presents the alternative conception, but at the end of the interview retracted his statement. As mentioned before, this student generalizes, incorrectly, what happens in the precessional motion of a top.

RESULTS OF TEST ADMINISTRATION

We administered a multiple-choice option problem (fig. 2) to address the second objective. The main result is that 18% of the students completing a modern physics course chose options A (12%) or C (6%) for question 2 of the problem. (Note that these options state the additional motion conception). Based on this result, we can establish that these students share the motion alternative conception described in the previous section.

Multiple-choice option problem.
For the next problem, use this key:

↓	⊙	⊗
Downwards	Out of the page	Into the page

(The wording and the figures of the multiple-choice option problem are the same as in the interview task. The only difference is that in the problem no value was given).

1. In the instant that the beam and weight are released, the torque vector that is exerted around the fulcrum by the tension force of the string has a direction:
(A) Out of the page **(B)** Downwards **(C)** Into the page

2. In this instant the left side of the beam and the weight move:
(A) Downwards and out of the page
(B) Downwards
(C) Downwards and into the page

FIGURE 2. Multiple-choice option problem**.**

CONCLUSIONS

In this article we detected an alternative conception in which students in a junior-level mechanics course believed that the beam and weight would have additional motion in the direction of the torque vector. Some of these students identify or treat the torque vector as an additional angular force vector; others believe that in a physical situation there is always a movement in the direction of the torque vector. We also found that in a sophomore-level modern physics course, 18% of the students held the additional motion alternative conception.

Next, we related the findings of this study to some previous studies. As mentioned before, some researchers [2,3] have analyzed students' understanding of the torque concept in problems in which it wasn't necessary to treat torque as a vector. For example, one problem asked them if the net torque would be zero in a situation in which two forces are applied in opposite directions at the ends of a rod. Rimoldini and Singh [3] found that in this type of problem, many students consider torque and force to be equivalent concepts and the authors established the following student reasoning as an example of this difficulty: "*The net torque is zero, because the forces would cancel out*". This specific difficulty was also detected by Ortiz et al. [2] (U. of Washington PEG group). By contrast, in our study we found that in problems in which it is necessary to treat torque as a vector, some students consider the torque vector as a force vector that affects the motion of the system.

Finally, we want to establish some suggestions for the instruction of the torque vector concept. From the results of our work, we note that some students are giving the torque vector concept some physical meaning (that it doesn't have), associating it with a displacement or a force vector. If we analyze the physics textbooks, we observe that the majority of them (for example [1]) focus on the calculation of the torque´s magnitude and on the description of its direction (right-hand rule), but do not present the reasons that establish the torque as a vector. We believe this point is very important in order to avoid the alternative conceptions analyzed in this article.

We can establish three instructional suggestions (The first two are based on the work of Feynmann et al. [7]): 1) Emphasize the fact that torque "has no a priori vector character", 2) Establish that after a mathematical analysis one can deduce that torque does "mathematically behave like a vector", 3) Establish that the torque vector is a vector that is different from the other vector concepts studied in a mechanics course, displacement and force, that have an a priori vector characteristic and therefore it is possible to assign a physical meaning to their direction. We believe that these three suggestions may decrease the alternative conception described in this article, because students will reflect on the vector nature of the torque.

ACKNOWLEDGMENTS

Authors acknowledge the support from Tecnologico de Monterrey through grant number CAT140.

REFERENCES

1. J. Jewett & R. Serway, *Physics for Scientists and Engineers. Volume 1.* China: Cengage Learning, 2010.
2. L.G. Ortiz, P.R. Heron, & P.S. Shaffer, *Am. J. Phys.*, **73** (6) 545-553 (2005).
3. L.G. Rimoldini & C. Singh, *Phys. Rev. ST. Phys. Educ. Res.*, **1** (2005).
4. J. Van Deventer, Master's Thesis, The University of Maine, 2008.
5. T. H. Chi, "Thinking aloud," in The Think Aloud Method, edited by M. van Someren, Y. Barnard, and J. Sandberg (Academic, Press, London, 1994), Chap. 1.
6. S. Thornton & J. Marion, *Classical dynamics of particles and systems.* Belmont: Thomson, 2004.
7. R. Feynman, R. Leighton & M. Sands, *The Feynman Lectures on Physics: Volume 1* (2nd Edition ed., Vol. 1). Boston: Addison-Wesley, 1963.