

# Can Multiple-Choice Questions Simulate Free-Response Questions?

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**Abstract.** We discuss a study to evaluate the extent to which free-response questions could be approximated by multiple-choice equivalents. Two carefully designed research-based multiple-choice questions were transformed into a free-response format and administered on the final exam in a calculus-based introductory physics course. The original multiple-choice questions were administered in another similar introductory physics course on final exam. Findings suggest that carefully designed multiple-choice questions can reflect the relative performance of the free-response questions while maintaining the benefits of ease of grading and quantitative analysis, especially if the different choices in the multiple-choice questions are weighted to reflect the different levels of understanding that students display.

**Keywords:** assessment, multiple-choice, free-response  
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## INTRODUCTION

When it comes to assessing students' learning in physics, there is always concern about the format of the assessment tool. While a multiple-choice (MC) test provides an efficient tool for assessment because it is easy to grade, instructors are often concerned about using it because a test in a free-response format facilitates a more accurate understanding of students' thought processes. In addition, free-response questions allow students to get partial credit for displaying the extent of their understanding of the subject matter tested, which is appreciated by many instructors and students. Thus, there appears to be a trade-off between the two assessment tools. If the instructors choose to implement a multiple choice test, they often feel that they are completely sacrificing the benefits that the free-response questions could provide.

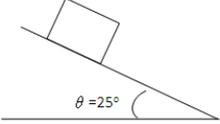
Research indicates that the difficulties students have related to a given topic can be classified into relatively few categories. If the choices in the MC questions are designed carefully to reflect the common difficulties students have, it is possible that the multiple choice questions will faithfully reflect the performance on the free-response questions while maintaining their benefits of ease of grading and comparison of classes taught using different instructional approaches. Here, we present a study designed to investigate the relationship between students' performance on quantitative free-response and MC questions. We converted two research-based MC questions into a free-response format and administered them on the final exam in a calculus-based introductory physics course (course A). The original MC questions were administered in an

equivalent introductory physics course (course B) on the final exam. Student performance in the two different courses is compared. Moreover, we investigate the correlation between students' actual performance on the free-response questions and the "simulated" multiple-choice performance had the problems been given in the MC format in course A.

## METHODOLOGY

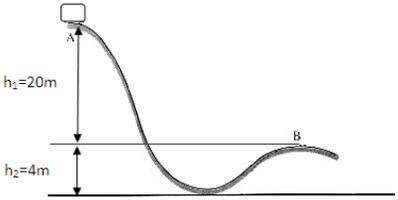
An object which weighs 20,000 N is at rest on a 25° incline as shown. The coefficient of static friction between the object and the inclined plane is 0.90, and the coefficient of kinetic friction is 0.80. Find the magnitude of frictional force on the object.  
Note: These trigonometric results might be useful:  $\sin 25^\circ=0.423$ ,  $\cos 25^\circ=0.906$

(a) 8460 N (b) 14496N (c) 16308N  
(d) 18120 N (e) none of the above



The diagram shows a rectangular block on an inclined plane. The angle of the incline with the horizontal is labeled as  $\theta = 25^\circ$ .

FIGURE 1. Problem Statement for Question 1.



The diagram shows a cart starting at point A, which is at a height  $h_1 = 20\text{m}$  above a horizontal line. The track curves down to a point, then loops up to a point B, which is at a height  $h_2 = 4\text{m}$  above the horizontal line. The track then continues horizontally.

A cart of mass 40kg is sliding down a frictionless track as shown in the figure. Close to point B, the track is part of a circle with a radius of 50 m. Assuming the cart starts from rest at point A, find the magnitude of the normal force on the car at point B.

(a) 392N (b) 705.6N (c) 313.6N (d) 15.68N (e) 78.4N

FIGURE 2. Problem Statement for Question 2.

Figures 1 and 2 present the MC questions that were administered in this study. Question 1 concerns an object at rest on an inclined plane. Students were asked to find the magnitude of the static friction acting on the object, which is equal to  $mg \sin \theta$  according to Newton's 2<sup>nd</sup> Law. Research [1,2] suggests that many students struggle with this question because they believe that the magnitude of static friction ( $f_s$ ) is always equal to its maximum value, the coefficient of static friction ( $\mu_s$ ) times the normal force ( $F_N$ ). This notion is not valid for this question because the maximum value of static friction exceeds the actual frictional force needed to hold the object at rest. Other student difficulties found from a pilot study include the confusion between static and kinetic friction and the challenge in decomposing the force correctly among others. The alternative choices in the MC question are designed to reflect these difficulties. Table 1 presents the choices in algebraic form where all symbols have their standard meaning (actual choices were numerical).

**TABLE 1.** The algebraic form and the assigned “weighted” score for each choice in question 1. The correct answer is indicated by the shaded background.

Choice	Question 1	Score
(a)	$mg \sin \theta$	1.0
(b)	$\mu_k mg \cos \theta$	0.3
(c)	$\mu_s mg \cos \theta$	0.5
(d)	$mg \cos \theta$	0.2
(e)	none of the above	0.0

Question 2 concerns a roller coaster cart on a frictionless track. The question asks for the normal force acting on the cart when it goes over a hump, which can be solved by using the principles of conservation of mechanical energy and Newton's 2<sup>nd</sup> law in the non-equilibrium situation (with centripetal acceleration involved). Previous research [3] indicates that a common difficulty introductory physics students have is that they think of a non-equilibrium situation which involves the centripetal acceleration as an equilibrium situation by treating the centripetal force as an additional force. The correct use of the centripetal acceleration and Newton's 2<sup>nd</sup> Law in this question should yield  $N - mg = -\frac{mv^2}{r}$ . However, students who treat it as an equilibrium question and believe that the centripetal force is an additional force obtain an answer of the type  $N - mg - \frac{mv^2}{r} = 0 \Rightarrow N = mg + \frac{mv^2}{r}$ , which has an incorrect sign. In addition to this common mistake, a pilot study indicates that some students incorrectly believe that the normal force is equal to the gravitational force ( $N = mg$ ) without contemplating the centripetal

acceleration. On the other hand, there are also students who completely skip the gravitational force and claim that  $N = \frac{mv^2}{r}$ . Moreover, some students have difficulty figuring out the speed of the cart at point B because they are confused by the two different heights provided. These common difficulties were incorporated into the design of the multiple-choice question. Table 2 presents the choices in algebraic form where all symbols have their standard meaning.

**TABLE 2.** The algebraic form and assigned “weighted” score for each choice in question 2. The correct answer is indicated by the shaded background. The speed at point B ( $v_B$ ) is calculated correctly using  $\sqrt{2gh_1}$  in choices (b), (c) and (e).

Choice	Question 2	Score
(a)	$N = mg$	0.2
(b)	$N = mg + m \frac{v_B^2}{r}$	0.8
(c)	$N = m \frac{v_B^2}{r}$	0.7
(d)	$N = mg - m \frac{v_B^2}{r}$ $v$ calculated using $\sqrt{2g(h_1 + h_2)}$	0.9
(e)	$N = mg - m \frac{v_B^2}{r}$	1.0

The MC and the corresponding free-response questions were administered on the final exam to the two introductory physics courses with 185 and 153 students. The Force Concept Inventory scores (pre/post-instruction) indicate that students in these two courses are comparable (no statistically significant difference). The free-response questions were the same as their counterparts in the MC format except that there were no choices provided. Students' performance on the free-response questions was graded using rubrics, with a full score of 1 for each question. The rubrics incorporated students' common difficulties. Partial scores were assigned based on the problem solving approach and the principles used. An example of a rubric can be found in [4].

To construct two types of “simulated” MC scores from the answers students provided to the free-response questions, student responses were first binned into different categories by comparing and matching their answers to the different choices in the MC questions. For the dichotomous MC simulation, a score of 1 (correct choice) or 0 (incorrect choice) was then assigned for the various categories. For the weighted MC simulation, to simulate the partial credit which is usually awarded for a free-response question, we assigned partial credit to different binned responses based upon approaches students used for the free-response questions. The scores assigned to each of the choices in this “weighted” MC simulation are shown

in Tables 1 and 2. The different weights for the choices reflect the different levels of understanding students displayed. The weights are commensurate with the rubrics used to grade the free-response questions.

To summarize, students who were given the free-response questions were graded using three different methods: using a rubric, using a dichotomous MC simulation, and using a weighted MC simulation. On the other hand, students who were given the MC questions were graded using two methods involving dichotomous or weighted scoring. Table 3 summarizes the different methods used to analyze student performance in the two courses. In order to compare student performance in the two courses, students in each of course were divided into groups 1 to 5 based on their overall performance on the final exam (with group 5 representing the group of students performing the best on the final exam, followed by those in group 4, etc.). For each group, students' average scores on each question were plotted to compare the trends in student performance in the two courses.

**TABLE 3.** Summary of Grading Methods in Classes.

Graded	Course A (free-response)	Course B (MC)
Rubric	Yes	--
Multiple-choice (dichotomous)	simulated*	Yes
Multiple-choice (weighted)	simulated*	Yes

\*Student responses were first binned into different categories by comparing and matching their answers to the different choices in the multiple-choice format and then assigning a score as discussed in the text.

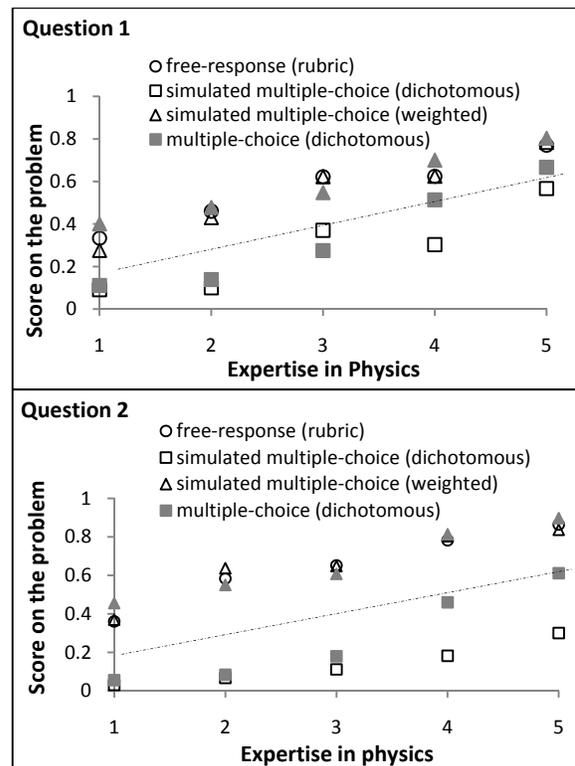
## FINDINGS

Table 4 presents the percentage of students who were binned into different categories by matching their free-response answers to the choices in the corresponding MC questions. We find that out of the 153 students involved, 84% and 88% of students' free-response answers could be matched to the a priori choices of the multiple-choice questions 1 and 2, respectively. Except for the mistake of using 1-D kinematics equations instead of the principle of conservation of mechanical energy to find the speed at point B in question 2 (which cannot be detected in the multiple-choice question because both methods yield the same numerical value for an option in the MC question) the findings suggest that a carefully designed research-based multiple choice question can reasonably reflect the distribution of common difficulties students have (as detected in their free-response answers).

**TABLE 4.** Percentage of students binned into different categories for simulated MC by comparing their free-response answers to the choices in both MC questions.

Choice	Question 1	%	Question 2	%
(a)	$mg\sin\theta$	28	$N=mg$	9
(b)	$\mu_k F_N$	7	$N = mg + m \frac{v_B^2}{r}$	31
(c)	$\mu_s F_N$	46	$N = m \frac{v_B^2}{r}$	28
(d)	$mg\cos\theta$	3	$N = mg - m \frac{v_B^2}{r}$ (v calculated using $\sqrt{2g(h_1 + h_2)}$ )	6
(e)	none of the above	16*	$N = mg - m \frac{v_B^2}{r}$	14
--	--	--	Other	12

\*Both choice (b) and choice (c) was found in one students' free-response answer in this category.



**FIGURE 3.** Students' average performance on questions 1 and 2. The white and black data labels are used to indicate students in course A (who were given the free-response question) and students in course B (who were given the multiple-choice questions), respectively. A dashed line is included on the figure to separate the data based for dichotomous case vs. the case where partial credits are assigned to the students.

Students' average performance on questions 1 and 2 is presented in Figure 3. The black and white labels are used to distinguish students in course B, who were

given the multiple-choice questions and students in course A, who were given the free-response questions. In general, we find that the trends for students' performance in the two courses are similar regardless of the question format they were given. For example, comparing students' rubric-graded free-response performance in course A to students' dichotomous multiple-choice performance in course B, we find that in both courses, students who displayed a higher level of expertise on the final exam (e.g., students in groups 4 and 5) on average typically scored higher on questions 1 and 2 in both formats than those who did not perform as well on the final exam.

Comparing students' performance in course A to course B, we also find that there is a better correspondence between students' performance on free-response questions in one class and the multiple-choice questions in another class (shown in Figure 3) if partial credits are awarded for both types of questions. The reason free-response performance for students in one class has a better match with the weighted multiple-choice performance in the other class than the dichotomous multiple-choice performance is that the weights for the weighted MC performance were similar to those used in the rubric to score the free-response questions.

**TABLE 5.** Correlation (N=153) between the free-response performance graded using the rubrics (FR) vs. the simulated multiple-choice performance for questions 1 and 2.

	FR vs. simulated multiple-choice			
	(dichotomous)		(weighted)	
	Q1	Q2	Q1	Q2
Correlation coefficient ( $r$ )	0.890	0.483	0.928	0.945
$p$ -value	0.000	0.000	0.000	0.000

Table 5 presents the correlation coefficients between students' free-response performance (graded using the rubrics) and the simulated multiple-choice performance (both dichotomous and weighted) in course A. It shows that the correlation coefficient is always higher for weighted multiple-choice simulation. The correlation between free-response and simulated dichotomous MC performance is higher for the question with a single stronger distracter choice (question 1) in MC compared to the question with several distracter choices (question 2) each of which represent different levels of understanding.

## DISCUSSION

We find that the trends in student performance on the research-based multiple-choice questions given to one class (in which the distracter choices correspond to students' common difficulties) and the free-

response questions given to another equivalent class are similar in that those who displayed a higher level of expertise on the final exam in each of the classes performed better on the questions than those who displayed a lower level of expertise regardless of the format of questions provided to them. Moreover, there is a good match between students' free-response answers in one class and the a priori choices in the MC questions administered to another class.

The findings suggest that research-based MC questions can reasonably reflect the relative performance of students on the free-response questions, especially if the answers for the MC questions are graded in a weighted manner by assigning partial credit to the different choices students selected. We note that (similar to the rubrics for the free-response questions) the weighting for the different alternative choices in the MC questions reflect the fact that some mistakes are not as "bad" as others despite the fact that they lead to students selecting an incorrect choice. In summary, if different scores are assigned to the different choices in the MC questions in the weighted model to reflect the different levels of understanding students display, there is a good overlap between students' MC performance in one class and the free-response performance in another class.

We re-emphasize that the fidelity of a MC question to a free-response performance depends strongly on the incorrect choices given [5]. If students' common difficulties found via research are incorporated, instructors can utilize MC questions without sacrificing accuracy in assessment of students' thinking processes. Free-response questions are useful only if they are graded carefully based on a good rubric. When they are graded leniently without a good rubric, the resulting scores will not typically reflect the appropriate level of understanding students' have. Weighted MC questions can be graded by a computer with weights corresponding to a good rubric for each distracter choice. Once the weights for the choices have been determined via research, MC questions can be as accurate for assessment purposes as rubric-based free-response questions without the time constraint.

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