

# Student Views of Similarity between Math and Physics Problems

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**Abstract.** It is commonly known that students have difficulty connecting the techniques they learn in math classes with necessary steps for solving physics problems. In this study, introductory-level physics students were given a set of pure math problems and a set of physics problems that required them to use the exact same mathematical processes. The students were then asked to pair the analogous problems and explain the pairings. Presented here are the results of that study, which support previous findings that students have difficulty determining how the two are connected and give some insight into what can be done to help scaffold that connection in the future.

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## INTRODUCTION

Mathematics plays an integral role in physics – in fact, it is impossible to solve physics problems without what can at times be very difficult mathematical techniques. While content knowledge and problem solving skills have been studied in great detail in math and physics independently, there has been little research conducted on the interaction of the two. It has been frequently said that students who perform better in their mathematics classes are able to better perform in physics, yet there is little research on the reasons this is true. As such, there is a great need for scientific studies to be conducted at the intersection of these two disciplines.

The goal of the larger study will be to examine how students identify which math concepts are needed to solve a particular problem, and further examine how they implement that choice as they progress toward a solution.

In this initial pilot study, students were presented with a set of physics problems and an analogous set of ‘pure’ mathematics problems; students were asked to match the problems based on the similarity of their solutions and explain their pairings. Presented here are the results of those student-generated pairing explanations.

## LITERATURE REVIEW

Problem solving in physics is such a significant subset of research that resource letters have been written to help the research community see the big picture of what has been studied [1]. This research

has focused on student representations of problems, problem solving strategies, cognitive load, knowledge structures, expert-novice differences, alternative problem types, and many other aspects. Despite the buzz in this field, however, there is very little focus on the mathematics involved in problems solving.

“Problem Solving” is a phrase which involves many mental tasks: memory recall, representation creation, consistency monitoring, and many others. Some of the work on problem solving has been on a more theoretical level (studying cognitive load, examining expert/novice differences, and examining the effect of problem representations), while other work has been more application-based (creating instructional strategies and frameworks, creating alternative problem types).

Many argue that it is not possible to study problem solving as a whole, but rather that it is necessary to focus on the individual components. Many models of problem solving exist, some of which break the problem solving process down into discernable steps. For example, Anderson *et al.* argue that students must first transform information encoded in the problem into procedural knowledge before they begin to plan a solution [2]. Polya proposed four steps: understanding the problem, devising a plan, carrying out the plan, and looking back [3]. Similarly, Schoenfeld’s suggested that problem solving involved analysis, design, exploration, implementation, and verification [4]. Reif and Heller break down the problem solving process into three parts, one of which involves a search for a solution based on the constraints of the

problem [5], and earlier work by Larkin and Reif indicated that students who formally plan a solution before beginning are following a more expert-like approach to problem solving [6]. Yet the question as to how the role of mathematics plays into this process is still quite unanswered.

Some work has been done regarding math and physics, however, and much of it focuses on the theoretical aspects of problem solving. Bassok and Holyoak examined students' ability to transfer between physics and algebra problems as a function of which they learned first; they found that learning isomorphic algebra transferred efficiently to physics, while learning isomorphic physics problems first hindered the transfer to physics and led to context-dependent transfer [7]. Rebello *et al.* furthered the research on transfer between math and physics, highlighting the need to study various types of transfer when considering structured domains (math) along with less-structured domains (physics), and further indicate that students may not have improved problem-solving performance despite their ability to recognize similar problems [8]. Redish has discussed the ways in which physicists and mathematicians use variables differently and the implications that has for our students in terms of how they chose to 'use' their mathematics to solve physics problems [9]. Most recently, Bing and Redish investigated the ways in which students epistemologically frame the problem-solving

process, and found that they use four primary strategies: calculation, physical mapping, invoking authority, and mathematical consistency. These frames provide insight into student difficulties [10].

## METHODS

This pilot study, as well as the entire study as a whole, will assume a phenomenological approach to ensure coherence with the idea of a student-centered learning experience [11].

The data was collected over two terms at Mercyhurst College; both sets of data were from students enrolled in the first term of calculus-based introductory physics. There were 18 students who completed the activity during Fall 2010 term and 17 students during the Winter 2011 term.

Each student was asked to complete a written assignment in which they were to think about how math and physics are related. The entire assignment had three sections. In the first section, the students were presented with traditional textbook-style problems and asked to describe what mathematical techniques would be needed to carry out a solution. The second section provided students with a set of mathematical procedures and asked them to create a problem which would require those procedures. (It should be noted that this sort of problem-writing was used frequently throughout the course.) In the final

**Table 1. Problem sets provided to students**

MATH PROBLEMS	PHYSICS PROBLEMS
<p>A. A certain function is given by the equation <math>f(x) = 3x^2 - 4x^3 = 6</math>. At what points are there relative extrema?</p> <p>B. What is the value of <math>x</math> as given by: <math>H(x, y, z) = xy + \sin(z) + 4z^2</math> if <math>H = 5.4</math>, <math>y = 2.3</math> and <math>z = 0.80</math>?</p> <p>C. If the short side of a 30-60-90 triangle has a length of 3, what is the hypotenuse?</p> <p>D. Add the following two vectors: <math>\vec{s} = 6.6\hat{i} - 5.6\hat{j}</math> and <math>\vec{q} = -9.2\hat{i} - 1.5\hat{j}</math>.</p> <p>E. Calculate the area under the curve given by <math>y(x) = x\sqrt{9 - x^2}</math> between <math>x=0s</math> and <math>x=8s</math>.</p> <p>F. What is the integral of the following function: <math>g(t) = t^4 + 3t^2 \sin(t) - 8t + 2</math>?</p>	<p>a. A time-varying force of <math>F(t) = 3(t^5 - 5t + 1)N</math> pushes a block along a horizontal surface. How much work does the force exert on the block in the first 8m of the motion?</p> <p>b. If the acceleration of a jet ski is given by <math>\vec{a} = (4t^2 - 8)\frac{m}{s}\hat{i} + (6 - 7t^2)\frac{m}{s}\hat{j}</math>, find an expression for the jet ski's velocity at any point in time?</p> <p>c. A projectile is launched upward distance of 89m in 6.5s. At what initial velocity was the projectile launched?</p> <p>d. Two airplanes are flying in the sky – the displacement of plane A is <math>d = \sqrt{(5.8t^2 - 2)^2 + (6 - t^3)^2}</math> relative to plane B. What is the closest distance that the two planes come to each other?</p> <p>e. If the vertical component of a car's momentum is 5N and its total momentum is directed at <math>25^\circ</math> north of east, what is the magnitude of the total momentum?</p> <p>f. Two forces are acting on a block. They are given by <math>\vec{F}_1 = 3\hat{i} + 2\hat{j}</math> and <math>\vec{F}_2 = -1\hat{i}</math>. What is the net force acting on the block?</p>
<p><b>Answers: A-d, B-c, C-e, D-f, E-a, F-b</b></p>	

section, students were provided with a set of 12 problems: 6 ‘pure’ math problems and 6 physics problems. With the help of a mathematics colleague, the problems were designed such that there were analogous pairs of problems – for each physics problem, there was a ‘pure’ mathematics problem that embodied the same solution type. While it is impossible to create a perfect identical problem, we felt that, at least within the framework of the students’ understanding of math and physics, these problems were appropriately similar.

The students were asked to match the problems and to explain their pairings. The complete list of problems can be found in the table below. The directions for the assignment indicated that a full solution was not necessary and was given no points, but may prove useful. Only three students included written solutions in their submission, though it is not possible to say how many other students worked out the problems over the course of completing the assignment.

Due to the short nature of this paper, only data from the third (matching) task will be discussed in detail. A simple examination of what problems students paired together is presented. Further, the students’ written responses were also analyzed to find themes in the rationales for their pairings.

## FINDINGS

### Pairing Data

Table 2 presents the raw data from the student pairings, with the correct pairing shaded. A few trends are immediately noticeable from this data. First, the trigonometry pair (*C-e*) and the vector addition pair (*D-f*) were recognized and identified correctly by the vast majority of students. Likewise, many students were able to determine the correct pairings for simple algebra (*B-c*) and determining extrema (*A-d*); however, there was more variation in the incorrect answers given for these pairings. Perhaps not surprisingly, distinguishing between the two integration problems proved difficult for many students. Discussions with a mathematics colleague led to the inclusion of these problems as a means of examining the difference between definite (*E-a*) and indefinite (*F-b*) integration. As predicted, students frequently interchanged the two, though rarely paired integration with a different problem type.

While this gives an interesting starting point from which to examine the data, we must look to the student responses and their justifications for the pairings to gain insight into how the students paired the given problems.

**Table 2. Results from matching task, given as number of responses (out of 35)**

		Math					
		A	B	C	D	E	F
Physics	a	0	1	1	0	22	11
	b	3	1	0	0	11	19
	c	4	27	1	0	0	4
	d	27	3	3	0	2	0
	e	1	3	30	1	0	0
	f	0	0	0	33	0	1

### Written Explanations

The two pairs students identified most readily – trigonometry and algebra – also had very consistent reasoning patterns in the student responses. In terms of the trigonometry problem, the most frequent explanation is that both problems were asking them to solve for the hypotenuse of a triangle (13 of 35). Others gave less specifics, but still indicated that a triangle was required, which accounts for a total of 20 responses. Another 13 students indicated that they were paired simply because they both required knowledge of trigonometric functions such as sine and cosine. The only two remaining answers are the ones that were matched incorrectly – each of these students focused on the angle and its relation to the given physics problems. The vector addition pairing was also easily recognized and stated directly as such by 31 students.

The algebra pairing was typically recognized as a ‘plug-and-chug’ problem by the students. 23 of 35 students said that they paired the problems because the goal of each was to solve for the missing variable and noted that the rest were given. Four students said that each was a kinematics equation, and though they did not explain how the function was for kinematics, they all chose the correct pairing. The responses for the incorrect answers were scattered and had no apparent trends in this small sample size.

Perhaps surprisingly was how many of the students correctly identified the minimization problem. Anecdotally, that problem is one that has given students great difficulty on homework assignments in the past. However in this context, 18 students indicated that determining how close two objects could get involved minimization, which was a relative extremum. Another 6 students indicated that they were paired because both required that the first derivative of the function be set equal to zero to obtain a solution. However, 5 students indicated that the equation simply needed to be solved using the quadratic equation. Though these students were able

to match the problems correctly, they seemed to not understand the correct mathematical procedure.

Because of the difficulty students had with the integration problems, their written responses prove helpful in understanding their confusion. In fact, only three students even mentioned that one of the functions had limits. 18 of 35 students indicated that the area under the curve was the integral, and another 4 students mentioned that the area represented the 'total' of the function. However, not all of these students chose the correct pairing.

In terms of the physics problems, 11 students cited the integral relationship between acceleration and velocity. The relationship between force and work was apparently more difficult to identify, because not one student included it in their explanations. One student did include the formula for the work done by a force that is constant in time, but then proceeded to pair it with the trigonometry problem because of the necessary angle.

## CONCLUSIONS AND FUTURE WORK

As shown above in the pairings and reasoning explanations above, there are instances in which students are readily able to match the necessary mathematical techniques to a problem they are solving. This was shown to be true in the cases of algebra, trigonometry, vector addition, and even determining relative extrema. However, students seemed to have great difficulty distinguishing between integral pairs. While many students indicated that the integral could be interpreted as being the area under the curve, they did not seem to recognize the difference between definite and indefinite integrals or be able to attach a physical meaning in either case. Clearly, much work remains to continue to explore this and other issues related to the use of mathematics in physics problem solving.

There were two distinct limitations to this study: the small sample size and the use of written responses. By nature of conducting research at a small college, the sample size issues can only be overcome by repeated data collection over time. However, interviews can be used in conjunction with written responses, or even in place of them.

The goal of this pilot study was to test the feasibility of creating activities in which students must think about how mathematics techniques are used in physics problem solving. To that end, this study provided the ground work for creating a more in-depth and thorough study of how students utilize mathematics in physics. A trained mathematical eye may spot several similarities between the problems used in this study; for example the trigonometry

problem is, at the most basic level, a vector addition problem. We intend to further study student understanding of concepts to ascertain that those results are not affecting the larger goal of this study. The next step will be to work with a mathematician to create more sets of paired problems and refine the current pairs based on the results of this study. Once the new problems have been developed and a protocol is in place, semi-structured interviews will be conducted to allow for more in-depth probing of student reasoning.

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