

# Understanding and Interpreting Calculus Graphs: Refining an Instrument

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**Abstract.** The objective of this ongoing study is to refine an instrument to evaluate conceptual understanding and graphical interpretation of a function and its derivative. The instrument is based on a modified version of the Test of Understanding Graphs in Kinematics (TUG-K) which consists of 26 items (7 dimensions). In the new instrument, Test of Understanding Graphs in Calculus (TUG-C), the kinematics context has been removed from the items creating a new context-free version. To favor the translation from kinematics to Calculus, the focus is on 5 out of the 7 original dimensions of the test, giving a 16-item test. A total of 526 students from a university level Introductory Physics course participated in the study. Half of the students were administered the kinematics test and the other half took the calculus test. This work will present data showing preliminary results of the instrument and new directions on improving the instrument.

**Keywords:** Graphical representation, calculus, evaluation instrument.

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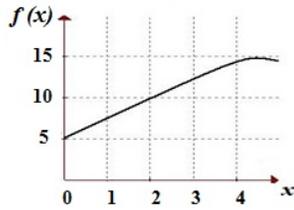
## INTRODUCTION

Student difficulties in reading kinematics graphs were well established years ago [1,2]. McDermott, Rosenquist and van Zee [1] categorized some common errors exhibited by students in interpreting kinematics graphs and proposed a few instructional strategies to address some of those students' difficulties. Some of the graphing errors documented by McDermott et al [1] are also addressed by Beichner in the Test of Understanding Graphs in Kinematics (TUG-K) [2]. Beichner's instrument is a validated tool for determining existing misconceptions in kinematics' graphs interpretation. The 21-items TUG-K has been used in numerous studies around the world, one example of this is the work done by Zavala et al [3] who modified and validated the TUG-K for Mexican engineering students, creating a test with 26 items and effective distractors for that population. Understanding the physical concepts involved in kinematics is not enough for successful interpretation of graphs; some literacy in graphical representation is required. However, this ability does not develop spontaneously; some form of instruction is required [1]. This raises the question of whether interpretations of mathematical graphs (context-free) encounter the same difficulties as kinematics graphs.

This ongoing study is being conducted at a private university in which engineering students are required to take four mathematical courses and three physics course as core requirements. Students, who fail the placement test, also take introductory courses before enrolling to the required ones. In the introductory calculus course as well as in the introductory physics course the graphical interpretation between a function and its successive rates of changes is discussed from the calculus perspective and from the kinematics respectively.

A present objective of this ongoing study is to develop a diagnostic test to evaluate students' understandings of the graphical relationship between a function and its derivatives. A further objective, as part of a larger project, is to evaluate the successfulness of the introductory calculus and physics courses to teach the understanding of graphs and investigate whether these courses complement each other or on the contrary, they do not help each other. To that end, the modified version of the TUG-K designed by Zavala et al [3] was rewritten to create a similar test in the context of calculus [4]. Each one of the 26 items was reworded so that the terminology used was the same the students studied in their Introductory Physics and Introductory Calculus courses. See Figure 1 for an example of this translation.

- 1.- The derivative of the function  $f(x)$  in  $x=2$  is:  
 (A) 0.5  
 (B) 8.5  
 (C) 2.5  
 (D) 5.0  
 (E) 10.0



- 1.- The velocity in the instant  $t=2$  is:  
 (A) 0.5 m/s  
 (B) 8.5 m/s  
 (C) 2.5 m/s  
 (D) 5.0 m/s  
 (E) 10.0 m/s

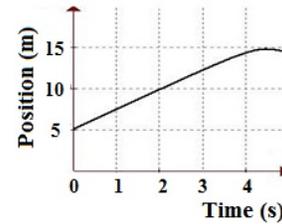


FIGURE 1. Example of the translation of question 5

Then, the final version was reviewed by professors who taught one or the other courses with the purpose of establishing content validity. Special care was placed in preserving the original structure of the problems. This test was named Test of Understanding Graphs in Calculus (TUG-C) and it was used to evaluate the entire population of students taking the Introductory Physics and Introductory Calculus in the spring of 2009.

TABLE 1. Equivalent objectives of the kinematics and the calculus version of the test considered for the analysis and the corresponding number problem in the short TUG-C version of the test.

| Obj. | TUG-K  | TUG-C  | Problems       |
|------|--|--|----------------|
| 1    | Given position-time graph, determine velocity                | Given the graph of $f(x)$ , determine $f'(x)$          | 1, 3, 6 & 15   |
| 2    | Given velocity-time graph, determine acceleration            | Given the graph of $f'(x)$ , determine $f''(x)$        | 7 & 11         |
| 3    | Given velocity-time graph, determine displacement            | Given the graph of $f'(x)$ , determine $\Delta f(x)$   | 5, 9 & 13      |
| 4    | Given acceleration-time graph, determine change in velocity  | Given the graph of $f''(x)$ , determine $\Delta f'(x)$ | 2, 4 & 12      |
| 5    | Given a kinematics graph, select another corresponding graph | Given a graph, select another corresponding graph      | 8, 10, 14 & 16 |

The results of a previous study [4] showed that while most of the problems in the test had an almost perfect “translation” from kinematics to calculus, there were some items that lost meaning or were too difficult for the students to answer. Those items corresponded to particular objectives of the TUG-K [2] which focused on the relationship of a kinematics graph to a textual description, and vice versa. Based on this analysis, it was decided to use only the 16 items on the test that correspond to the first five objectives. Table 1 shows the original objectives of the TUG-K, the corresponding objective in the calculus version of the test, and the item problem that addresses each objective.

## METHODOLOGY

The research was conducted in a large private Mexican university. The tests were administered to 526 students enrolled in the Introductory Physics course in that institution. A total of 265 students were given the TUG-C and 261 students responded the TUG-K. Both versions of the test consisted on the same questions (16 items) and were arranged in the same order. Both tests were administered in Spanish.

As seen in Table 1, item 3 on the TUG-C corresponds to the first objective. This item provides the graph of  $f(x)$  and asks information about  $f'(x)$ , whereas in the TUG-K the corresponding item provides the velocity graph and asks information about the acceleration, which corresponds to the second objective. This change was considered since removing the kinematics context from the question yield a question that would provide parallelism to question 15.

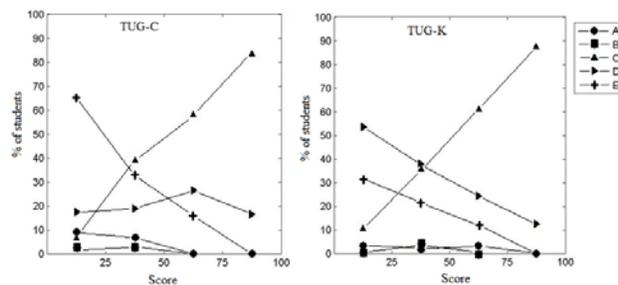
Both students performance and error distribution where analyzed and Item Response Curves, IRC [5] where calculated for all problems in both versions of the test. Pie graphs and IRC helped us to analyze differences and similarities between parallel questions.

## RESULTS AND DISCUSSION

This section is divided into five subsections addressing each of the five objectives of the test.

### 1. Given the graph of $f(x)$ , determine $f'(x)$

There are two kinds of problems in this objective: one that requires numerical calculations and one that requires making qualitative estimations. The first kind of problem shows a similar IRC for the correct answer. However, there is a noticeable difference in the error distribution among the incorrect responses between the TUG-K and the TUG-C versions (see Figure 2). Whereas in the kinematics version of the test the option corresponding to incorrectly calculating the velocity by dividing the value of position by time is a common error (option D), in the calculus version of the test the most attractive incorrect response corresponds to reading the value directly from the graph (option E).



**FIGURE 2.** Item Response Curves of question 1 showing the similar behavior of the correct answer for the TUG-C and TUG-K.

In the problems that require qualitative estimations (items 3 and 15) there is no noticeable difference between the results, correct answer and incorrect answers have similar IRC in both tests.

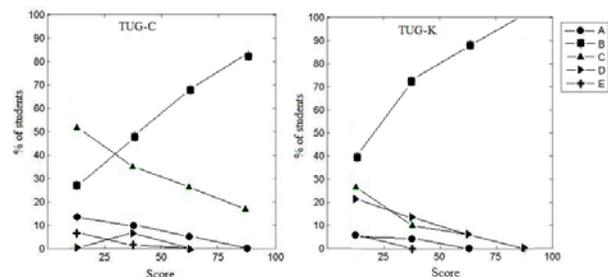
### 2. Given the graph of $f'(x)$ , determine $f''(x)$

Both problems in this objective require the student to make numerical calculations. Just like happened in the first objective, the correct answer had a similar behavior in both tests, and the most common

misconception in the calculus version for this type of problems consisted on reading the value directly from the graph, whereas in the kinematic version of the test students were more attracted to choosing the option corresponding to dividing the value of velocity by time. Given the parallelism between the relationship between a function and its first derivative and the relationship between the first derivative and the second derivative, the behavior of the responses for this objective were similar to the behavior of the responses for the first objective.

### 3. Given the graph of $f'(x)$ , determine $\Delta f(x)$

This objective has two items (questions 5 and 9) that involve numeric calculations. In both items the students are asked to calculate the change of value of the original function using the graph of its derivative. These two items were responded correctly more often in the kinematics version than in the calculus version of the test, as shown in Figure 3 corresponding to the IRC analysis for question 5. With respect to the incorrect responses, the option corresponding to calculate the change by multiplying the time and the value of the derivative in that point (option D in question 5) is a popular conception in the kinematics test (due to an incorrect interpretation of the definition of acceleration), but it is almost nonexistent in the calculus version (see Figure 3). On the other hand, the calculation corresponding to the slope in the given interval is a strong cognitive model in the calculus test (option C), but it is not so strong in the kinematics test. Observe that in the TUG-C, the curve for option C is a plausible answer even for students who performed very well on the test. On the other hand, option C rapidly decreases in the TUG-K meaning that only students with low performance chose that option.



**FIGURE 3.** Item Response Curves of question 5 showing the different incorrect cognitive models underlying each version of the TUG-C and TUG-K for this type of objective.

The other item of this objective (question 13) presents the student with the graphs of the derivatives of five different functions and asks to select the graph corresponding to the function with a greater increment in the interval. This is a very difficult problem in both versions of the test, only 11% of the students answered correctly in the TUG-C and 25% in TUG-K. Moreover, it is interesting to note that while the most popular incorrect response in the kinematics version (35%) corresponds to the graph with the greatest slope, in the calculus version the most popular incorrect response (34%) corresponds to the graph with a local maximum.

#### **4. Given the graph of $f'(x)$ , determine $\Delta f(x)$**

This objective has three items, two numerical problems and one of qualitative analysis. These three questions are parallel to the three items for the third objective. As expected, the behavior of the correct responses according to the IRC is similar to the curves obtained in the previous objective. This is due to the parallel relationship between involved functions in these two objectives.

#### **5. Given a graph, select another corresponding graph**

There are four items in this objective. All of them provide a graph and ask to determine which one of the 5 graphs keeps a specific relationship with the given graph. The relationship between the graphs goes from the function to the first derivative graph and vice versa, and from the first derivative graph to the second derivative graph and vice versa. That is, these items focus on the concepts addressed in the four previous objectives, but from a graphical representation.

The four items turned out to have similar IRC behavior for the correct response between the two versions of the test, but the calculus version seems to be a little harder. With respect the most common misconceptions, the IRC analysis shows that the cognitive models for both versions of the test behave in a similar fashion. That means that the calculus version represents a good “translation” of the kinematic test.

## **CONCLUSIONS**

We found that the context has an influence on the alternative cognitive models, but the correct answer works well for both versions of the test, the kinematics version and the calculus one. That is, the results indicate that the TUG-C has potential to become a strong instrument to measure conceptual understanding and graphical interpretation of a function and its derivative.

Further analysis will consist on reviewing the TUG-C to study whether there are alternative models which are not present in the kinematic test. To do that open ended questions and interviews will be carried on. Then, we will administer the TUG-C to a larger population and analyze its validity as an instrument with known techniques [6].

## **ACKNOWLEDGMENTS**

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