Using Pfund’s Method to find the Index of Refraction

Helene F. Perry , Department of Physics, Loyola College in Maryland

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A simple, sophomore-level lab exercise in error analysis

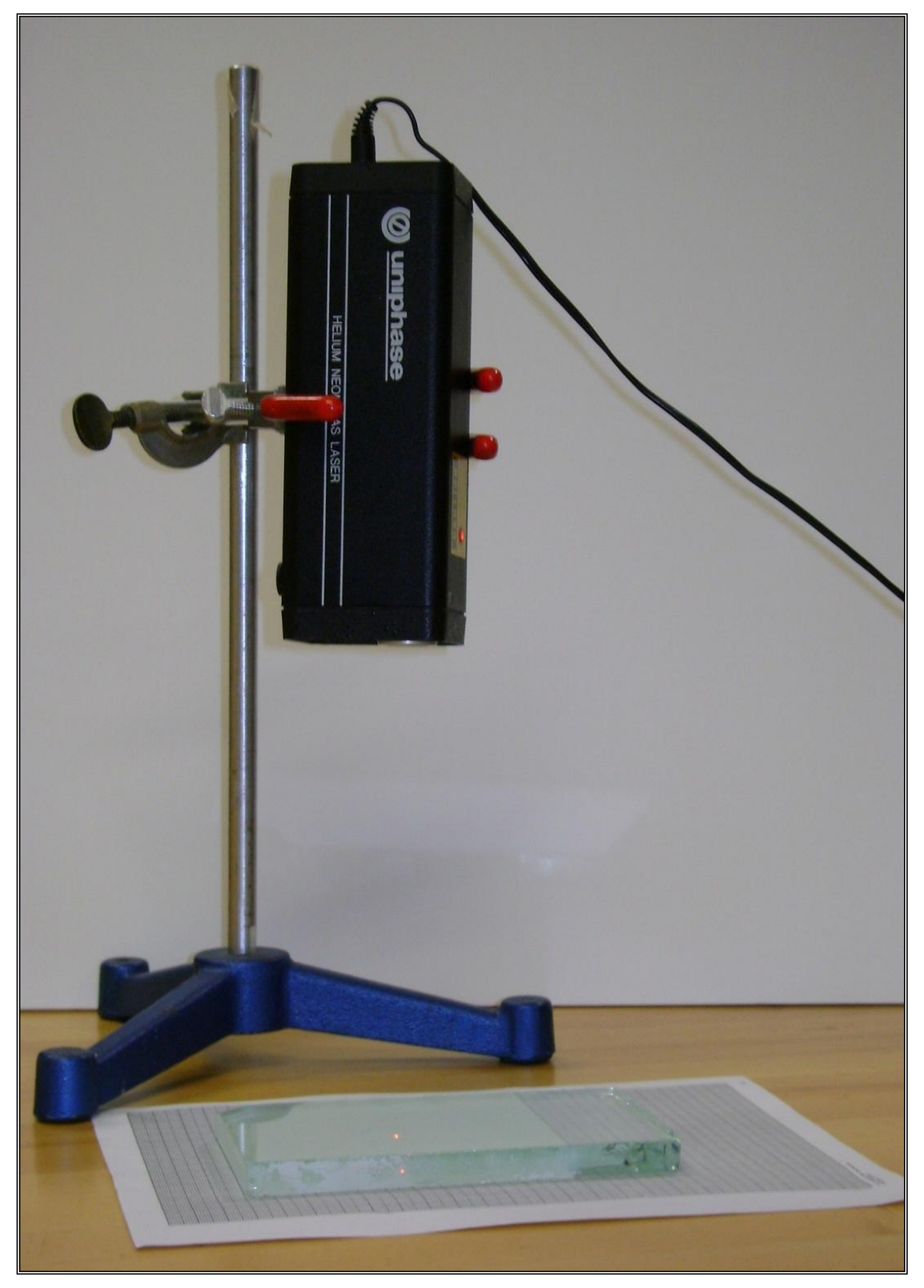
PFUND’S METHOD

Light rays that travel from a medium of higher optical density, such as glass, to a medium of lower density, such as air or a liquid, will have an angle of refraction greater than the angle of incidence.

Snell’s Law: $n_i \sin \Theta_i = n_r \sin \Theta_r$

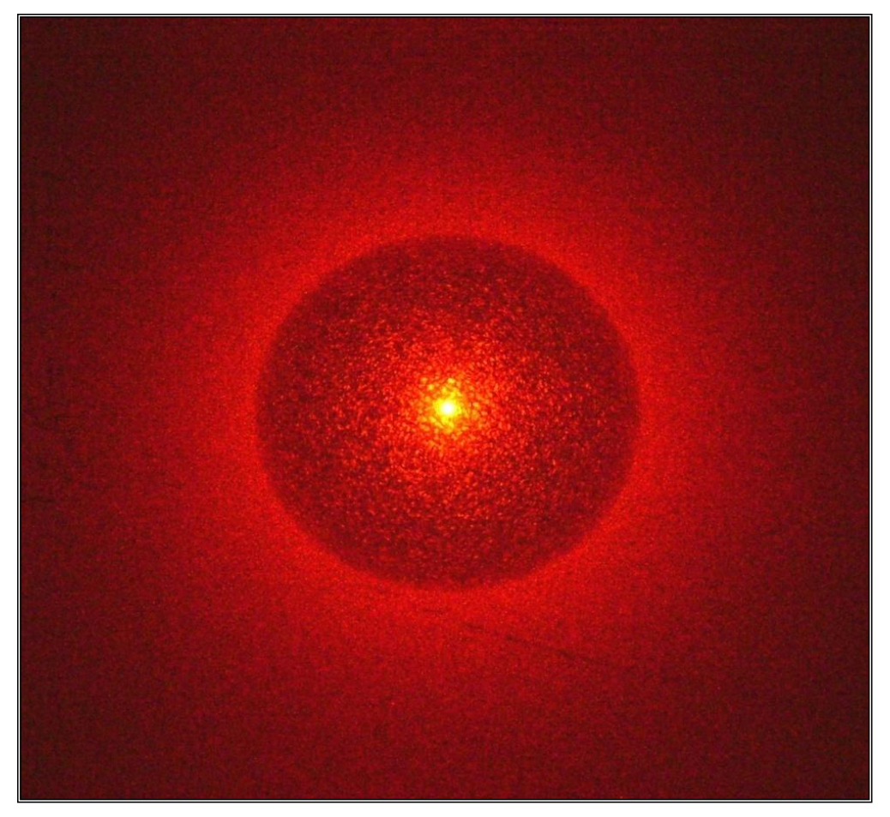
At a critical angle Θ_C the angle of refraction becomes 90° and the ray is reflected from the interface.

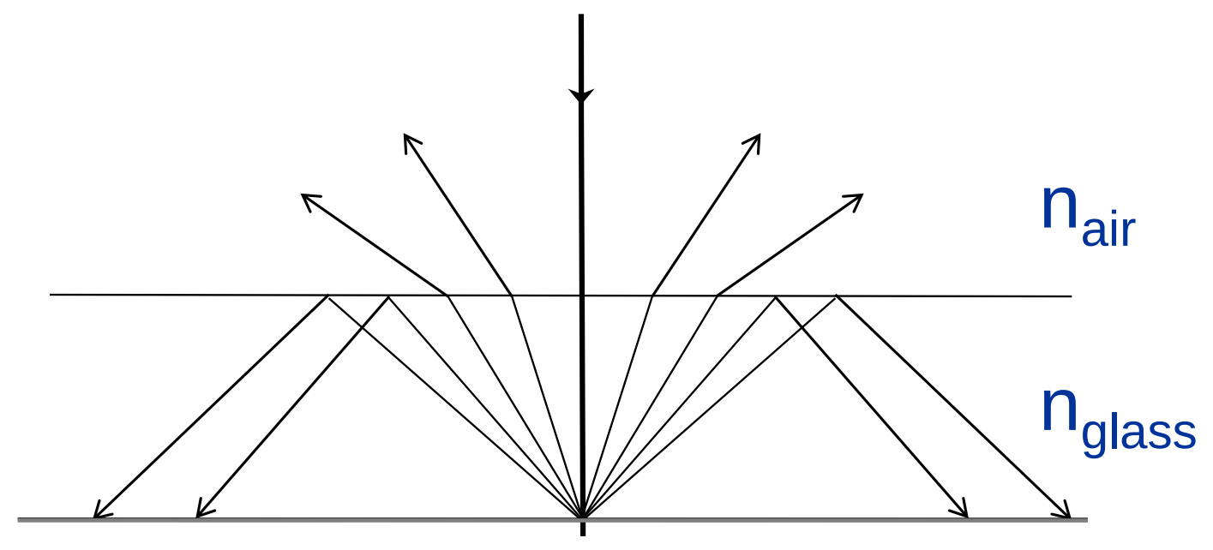
Pfund’s Method uses a determination of the critical angle at an interface to find the index of refraction of the media.



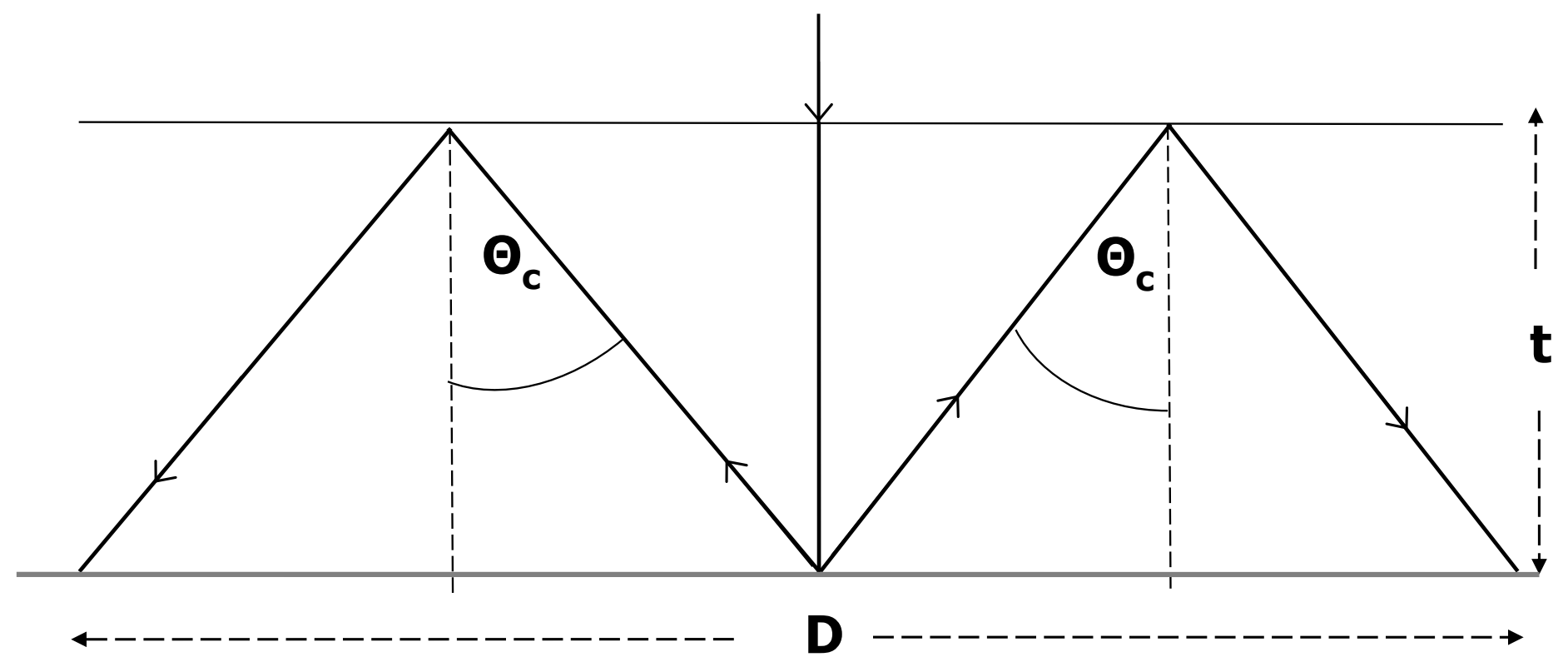
When a laser beam is incident on the upper surface of a flat, relatively thick glass plate sprayed on one side with white paint, the beam travels through the glass and is diffusely reflected from the spot of light on the bottom surface. Rays emerge uniformly in all directions, travel back to the upper surface and pass through it as long as the incident angle is less than the critical angle.

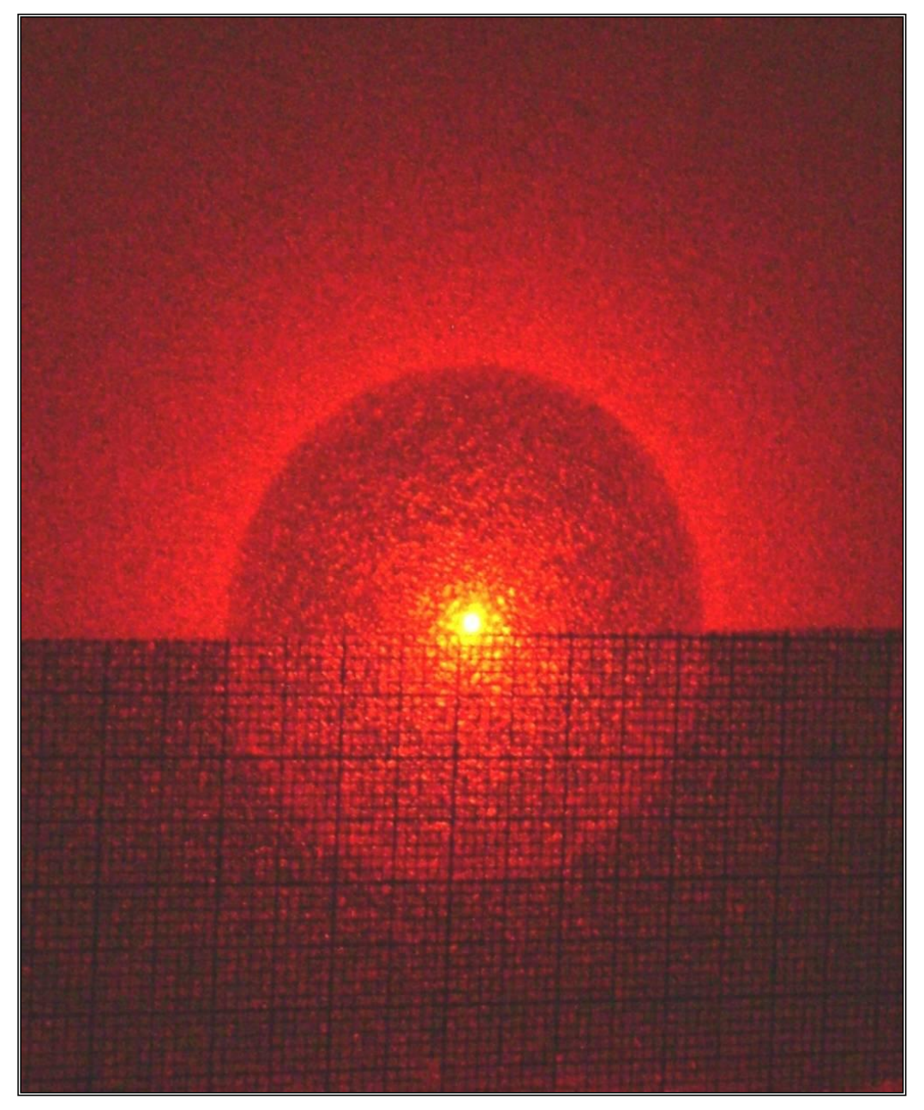
Rays incident at the critical angle will be reflected and form a sharp ring of light on the bottom painted surface.





Measurements of the diameter of this ring, D, and the thickness of the glass plate, t, allows the index of refraction of the glass relative to air to be found.





$$n_{glass\text{-}air} = 1/\sin \Theta_C$$

where

$$\sin \Theta_C = D/4 \Big/ \sqrt{t^2 + (D/4)^2}$$

so

$$n_{g\text{-}a} = \sqrt{1 + \left(\frac{4t}{D}\right)^2}$$

Both a HeNe laser and a green diode laser have been successfully used as a source. The green diode pattern is easier to read, however it is not as monochromatic as the laser and the edges are somewhat less well defined.

SAMPLE DATA

The thickness of the glass plate is measured with a micrometer caliper in places where it has not been painted (e.g. from 20 measurements with standard deviation):

Average glass thickness t = 1.2185 ±0.002 cm

Reading the ring diameter from a millimeter scale on the lower surface (5 measurements with the average deviation):

Average ring diameter = 4.194 ± 0.023 cm

Then

$$\boxed{n_{g\text{-}a} = 1.5332}$$

REFERENCES

S. Reich, “Measurement of refractive index in transparent plates with a piece of paper and a laser source”, Am. J. Phys. **51**(5), 469 (1982).

D.C. Look, Jr., “Novel demonstration of total internal reflection”, Am. J. Phys. **49**(8), 794 (1981).

C. H. Palmer, *Optics: Experiments & Demonstrations*, (The Johns Hopkins Press, 1962) Experiment A3.

R. W. Wood, *Physical Optics, 3rd ed.* (Macmillan, 1934) pp. 70-2.

ERROR ANALYSIS

Since the object of this lab is to have students engage in an exercise in error analysis, we can use one of the methods they have studied. This will often depend on when in the semester the lab is scheduled. However one way they usually find the uncertainty is from :

$$n_{g\text{-}a} = \sqrt{1 + \left(\frac{4t}{D}\right)^2}$$

letting
R = (4t/D),
$S = [4t/D]^2 = R^2$ and
P = 1 + S

$$n_{g\text{-}a} = \sqrt{1+S} = \sqrt{1+R^2} = \sqrt{P}$$

so $$\Delta n_{g\text{-}a}/n_{g\text{-}a} = \tfrac{1}{2}\,\Delta P/P = \tfrac{1}{2}\,\Delta S/P$$
$$= [R^2(\Delta R/R)/\,(1 + R^2)]$$

then
$\Delta R/R = (\Delta t/t + \Delta D/D)$
$\Delta S/S = 2\,\Delta R/R$ and
$\Delta P = \Delta S$

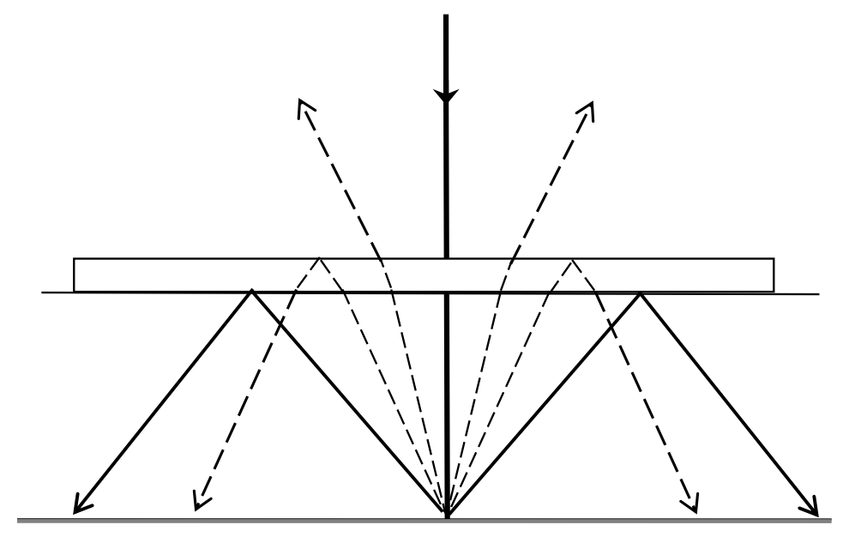
then

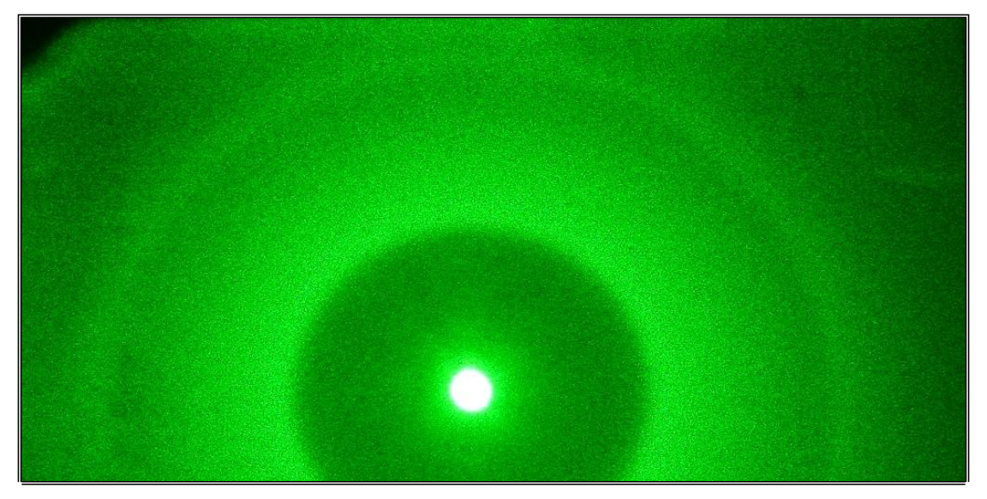
$$\Delta n_{g\text{-}a}/n_{g\text{-}a} = \left(\frac{(4t/D)^2}{1 + (4t/D)^2}\right)\left[\frac{\Delta t}{t} + \frac{\Delta D}{D}\right]$$

Using the sample data: $\Delta n_{g\text{-}a}/n_{g\text{-}a} = 0.26\%$

$$n_{g\text{-}a} = 1.533 \pm 0.004$$

INDEX OF REFRACTION OF A LIQUID





When a layer of liquid is placed on the glass surface, critical reflection occurs at both the liquid-air and glass-liquid interfaces. Two bright rings of light are produced; the first from the liquid-air interface, the second larger one from the glass-liquid interface forming a ring of diameter D_l. The index of refraction of the liquid is:

$$n_{liquid} = n_{glass} \sin\Theta_C = n_{glass} \Big/ \sqrt{1 + \left(\frac{4t}{D}\right)^2}$$

SAMPLE DATA & CALCULATIONS FOR WATER$_{20\ C}$

Glass thickness t = 1.2185   0.002 cm

Average ring diameter =8.860   0.032 cm

$$n_{water} = 1.339 \pm 0.006$$