Expt. 2: Light Intensity, Blackbody Radiation and the Stefan-Boltzmann Law

Goals: Determine the emission spectrum from a heated blackbody object (filament lamp) as a function of temperature using a photo-diode and a set of interference filters. Compare the results with the classical theory of Rayleigh-Jeans and the quantum theories of Stefan-Boltzmann and Planck. Use the latter to determine the temperature of an unknown high-temperature source (halogen lamp).

Suggestion: Bring a USB memory stick to class – it can be used for copying the Excel® and Vernier *Logger Pro*® spreadsheets used for the lab.

Apparatus and Demonstration List

1. Hewlett-Packard model 974A and 34401A digital multi-meters
2. Hewlett-Packard model E3632A low voltage DC power supply
3. Test leads with banana plugs
4. Photodiode boxes with small and large detectors (one opened for inspection)
5. Photodiode box mounted on a rotating platform
6. Coax cable for connecting photodiode box to digital voltmeter
7. BNC to banana plug adapter for voltmeters
8. Tungsten lamp and holder
9. 120 volt Halogen lamp for use as an unknown black-body emitter
10. Optical bench assembly
11. Three optical bench mounting clamps
12. Aluminum light baffles
13. Six or more mounted interference filters from 450 nm to near infra-red
14. Hand-held spectrometer
15. Metric tape measure
16. Two banana plugs - alligator clip adapters
17. Solar photovoltaic cell
18. Opaque white viewing cards
19. Precision voltage and resistor reference sources for calibrations (check on availability)
20. Student-supplied digital camera for pictures of set up (optional)

    DEMO: Infra-red night-vision viewer
    DEMO: Infra-red non-contact thermometer

References: Physics 240 and 340 texts
                HyperPhysics (http://hyperphysics.phy-astr.gsu.edu/hbase/HFrame.html)
                Wikipedia: Blackbody Radiation

2.1 Introduction

In Experiment 1, we explored the behavior of simple gases such as helium. An "ideal gas" thermometer works particularly well with helium because the interaction between atoms is very weak. As a consequence, helium is extremely hard to liquefy – its temperature must be reduced to 4.2 K before it can condense. Another simple physical system is a photon "gas" in equilibrium with its surroundings as in a so-called "blackbody". Such a system has some unusual properties: the "particles" are, by definition, relativistic, and quantum mechanical effects are significant. In fact, the failure of classical mechanics to provide a finite value for thermal emission of electromagnetic radiation led to the discovery that light and other forms of electromagnetic (EM) radiation consist of discrete energy packets, aka light quanta or photons. In this experiment, you will measure the EM spectrum produced by a tungsten filament at various temperatures and compare the results with both the classical and quantum-mechanical predictions.

To get familiar with the equipment, you first will make some measurements of light intensity as a function of the distance from the source and the angle of the sensor with respect to the light beam. The effects you will observe are only geometrical in nature, but they are basic to many aspects of physics. They are responsible, for instance, for the seasonal variations in average temperature that mark the difference between summer and winter. These are particularly pronounced at high geographical latitudes, as will be discussed as part of this lab.

2.2 Measuring Light Intensity

The basic device for measuring light intensity in these experiments is a *reverse-biased silicon diode*. Recall what the current-voltage diagram for a diode looks like. A silicon diode is the most elementary semiconductor structure one can construct. Schematically it looks like the sketch shown in Figure 2.1. In the upper region of the diode, electrical current is transported by positive charge carriers called "holes"; in the lower region, the current carriers are negatively charged electrons. If this device is "reverse-biased" so that the upper electrode is more negative than the lower one, the charge carriers will separate, as shown in Figure 2.2. In this case, the positive holes will be attracted to the top and the electrons towards the bottom, leaving a "*depletion layer*" in the middle. Since almost no free electrical carriers are present there, the current through the diode when it is in the dark is effectively zero (typically nanoamperes or less). Under these circumstances, a continuous incident light beam (or x-ray beam, or charged particle beam) will produce electron-hole pairs in the depletion region at a certain rate. These pairs separate due to the electric field present in the depletion layer and get carried away to the battery terminal, amounting to a light-induced current. The current is then *directly proportional* to the number of incident photons absorbed per time For a well-designed photodiode, the average number of extracted electron-hole pairs per incident light photon (the quantum efficiency) is fairly close to 1 (typically 0.3 to 0.8 in the visible light region). The photodiodes used in the present lab are mounted in small blue boxes and wired as shown in Figure 2.3. A 1 kΩ protection resistor has been added in series with the diode as a provision to limit the current, in the unlikely case that the battery is inserted in the wrong way (see Figure 2.3). The protection resistor will not affect your measurements as long as the current is kept below 200 μA.
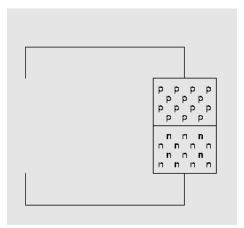
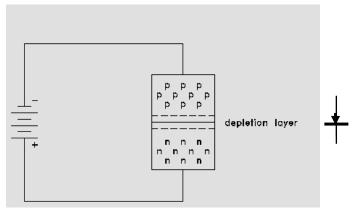Figure 2.1: Schematic view of a silicon *pn* diode.

Figure 2.2: By reverse-biasing a *pn* diode, a charge-free region is created at the *pn* boundary called the depletion layer. This is sometimes denoted as a *pin* diode, where "*i*" denotes the intrinsic charge-free depletion layer. The circuit symbol for a diode is shown on the right.

Measuring the photocurrent is fairly easy if the current is large enough relative to the "dark" current that may be present in the diode when it is reversed biased. The Hewlett-Packard 974A multi-meters will operate well as ammeters down to about 1μA (10^{-6} A). This is not sufficient for measurements at low light levels, but there is a fairly simple alternative for measuring currents below this range: Attach one of the provided banana plugs, which have a 1 kΩ to 100 kΩ load resistor soldered across the terminals, to the photo-diode box. Connect a multi-meter in voltmeter mode and measure the *potential drop* across that resistor, as indicated in Figure 2.4. Since the multi-meter can detect voltages as low as 3 microvolts (μV), the minimum measurable current through a 100 kΩ resistor is 30 pA (3 x 10^{-11} amperes). Note that the input impedance of the meter is about 10 MΩ, so very little of the photo-current is shunted inside the meter.

In all measurements for this experiment, ***choose a load resistor that is small enough that the measured potentials are substantially less than 0.5 Volts***. Under these conditions, the voltage across the known resistor is a *linear function* of the light intensity.

A note of caution: Although the voltage output of a photodiode is related to the input light intensity, the relationship can be far from linear. In an open circuit (*i.e.*, no current flow), the

voltage is practically constant, independent of intensity, as long as the light is bright enough to overwhelm the intrinsic electron-hole recombination rate. The terminal voltage under these circumstances approximately equals the band gap potential, which is 1.12 volts for silicon. That is why we measure the *current* through a load resistance of order 10 kΩ, not the voltage across an "infinite" (*i.e.* large) load resistance. The photo-current through the load resistor is proportional to the light power, provided that the voltage drop on the load resistor is small compared to the band gap.

A note of historical interest: The explanation for light-induced electron emission from metal surfaces (the *photo-electric effect*) was the scientific contribution for which Einstein was awarded the Nobel Prize in 1921. Contrary to what you might think, he did *not* receive a Nobel prize for either special or general relativity, as it was not common to award more than one Nobel prize in one field to a single person. Marie Curie over the same period did receive two Nobel prizes, but one was in physics and the other was in chemistry.

2.3 The Light Source

The light source used in this experiment is a tungsten filament lamp that operates at low voltage but at fairly high current. Using a regulated power supply, the filament temperature is held constant; this will be important throughout this experiment. The lamp was originally manufactured for automobile use so the electrical resistance must be low to generate sufficient light with the 12-volt power usually available. In place of the lead-acid battery in a car, we have a DC power supply to provide the necessary DC current. A diagram is shown in Figure 2.4. For these experiments, the power consumption of the lamp must be carefully measured. This requires knowing both the *electric current* passing through the tungsten filament and the *voltage* across it. The resistance of the lamp can then be used to estimate the *filament temperature* since the resistivity increases in a well-known way (more or less linear) as the metal gets hotter. This is shown and tabulated in Table 2.1.

The lamp and detector mount on rails so that the positions can be easily changed. The whole apparatus is placed in an enclosure so that light from outside sources can be minimized. The Hewlett-Packard E3632A power supplies are convenient for these measurements since the output voltage and current are monitored and displayed. *Do not exceed 13 volts for the lamp.* Keep in mind that the resistance of the wire leads to the lamp causes a *significant* drop in voltage to the lamp. Therefore, the voltage directly across the lamp must be measured with the small hand-held multi-meter, while the lamp current can be read accurately from the readout on the HP power supply.

2.4   Light Intensity and Geometry

The intensity of a light source or any source radiating electromagnetic (EM) waves varies in a predictable way as a function of distance to the source and angle with respect to the direction of incidence. These dependences are strictly a consequence of geometry. A light source that is not collimated by a lens and that is not intrinsically beam-like (such as a laser) will emit light (or

other EM radiation) uniformly in all directions. Further, the source may be regarded as a point-like source if the observation distance is much greater than the size of the source.

In the following sections, you will measure the intensity of light from a small, hot tungsten filament using the photodiode circuit described earlier. It is important to keep stray scattered light from affecting these measurements so "dark" boxes using black curtains have been provided to reduce the light from room lights and windows. A light baffle with a 1" diameter hole also should be used to reduce scattered light within the enclosure itself. Use your judgment in positioning the equipment to reduce reflected and scattered rays inside the box. (A scattered ray is one that does not follow a straight path from the tungsten filament to the detector.) Any reflected or scattered light many not preserve the blackbody radiation spectrum form the lamp. Finally, any remaining current in the detector can be estimated by interposing an opaque card in front of the hole in the light baffle and measuring the residual photocurrent present. *Subtract the background reading from all measurements.* Such background subtractions are standard practice in the physical sciences. Keep these procedures in mind throughout this course – as accurate light intensity measurements using a photo-diode will be required in several of the subsequent experiments.
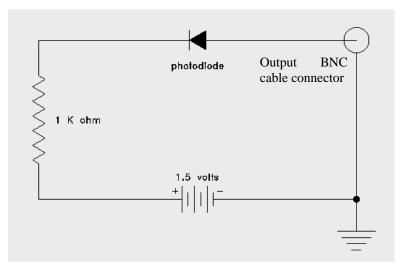

Figure 2.3: The internal wiring diagram for the Physics 341 photodiode boxes (see the opened box showing the circuit).

1) Set up the tungsten lamp, power supply and HP Digital Voltmeter (DVM) as shown in Figure 2.4. Use the photodiode box with the smaller sensor (3 mm x 3 mm) for measuring light intensity. Operate the light bulb at 10 volts, *as measured across the bulb electrodes.* Measure the light intensity (which is proportional to the photodiode current), as a function of distance from the filament for *at least eight points*, suitably chosen, over a 2-meter range. One or more light baffles, suitably positioned (note their location), can be used to minimize stray light. The photodiode should be set with its surface set at the optimum angle to maximize the photocurrent. Use a load resistor of 1 to 10 kΩ on your DVM and set the HP-DVM to DC voltage mode. ***Remember that the DVM voltage should be <0.5 V at the smallest distance to ensure a linear photodiode response***. (If you want to get in closer, you can use a smaller resistor or switch the meter to current mode.)

2) Assuming a uniform point source, how would you expect the net photodiode current to vary with distance? Graph your results using Excel®, Vernier *Logger Pro*® or another suitable program and fit the data to a power law to test your hypothesis. How does the program indicate the goodness of the fit? You need to cite that criterion when displaying a fit to your data. *Note:* rather than blindly extracting a best-fit value of the exponent, it is better to test whether the expected dependence on distance is obeyed *within the experimental uncertainties*. These can be estimated or otherwise determined (*e.g.* by measuring a known reference voltage source). Explain your results in terms of geometry and such fundamental quantities as the total power emitted by the incandescent lamp. See Appendix 2.A for hints on how to explore the functional behavior.
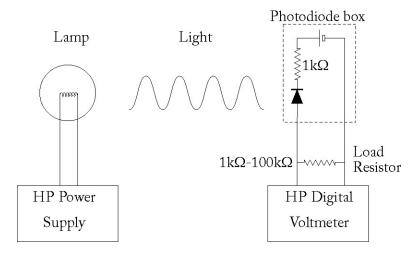

Figure 2.4: Schematic of the setup for light intensity *vs.* distance measurements.

3) In the previous measurements, it was assumed that the light rays struck the photodiode at normal incidence. Determine how the observed intensity changes as the incidence angle is varied. For these measurements, use the larger photodiodes (10.0 mm x 10.0 mm) that are mounted on rotating platforms. The 0° incidence angle can be found by looking for the light reflected back from the diode and aligning the back reflection to overlap with the incoming beam, a technique we use often in optics. Use light baffles to again minimize stray light. Take a minimum of five measurements at different angles. What is the expected functional relationship between intensity and angle? Explain why this occurs. Plot your data in such a way that you would expect a straight line, so that you can test if the behavior is as expected. Include uncertainties (error bars) on your plots. These can be added by hand on the printed plots if needed, although most graphing programs will allow you to add error bars.

4) The dependence of illumination on angle is the chief cause of seasonal variations in temperature on the Earth's surface. The Earth's spin axis is tilted by 23.4^o with respect to the Earth's orbital plane around the Sun. Ann Arbor is located 42.4^o north of the Equator. Calculate the angle of the Sun's rays with respect to normal incidence at noon in mid-summer and mid-winter. From this, determine the corresponding ratio of average light intensity between the two seasons. Note: The light intensity at noon as a function of the

time of the year approximately follows a sinusoidal dependence. The corresponding curve for the average daily temperature high lags that curve by about a month or so. For example, the intensity minimum occurs on Dec. 21st, while the coldest days typically occur about a month later.

5) A majority of scientists believe that the current level of fossil fuel burning is leading to global climate changes that will have very unpleasant impacts on our lives. The present alternative sources of power are either solar energy (a form of nuclear fusion energy), used directly or via secondary effects such as wind or waves, or nuclear fission using nuclear reactors. Direct conversion of solar to electrical energy and power using silicon photovoltaic devices similar to the sensors used in this experiment eventually might become an economical solution, at least in areas with lots of sunshine. To estimate the area of silicon cells required to satisfy our energy needs, we first determine the optimum load resistance to extract electrical power. The maximum power is extracted by a load resistance R_L when the product of the voltage across the load, in this case the load resistor, times the current through it ($V_L I_L$) is at a maximum. Use the available solar-cell samples, connect them to various load resistors and measure V_L *vs.* the load resistance. A resistor switch box is available for this purpose. Plot the power V_L^2/R_L *vs.* R_L at about 12 values of R_L between 1 and 50 Ω or more. ***Make sure that the power maximum occurs near the middle of your graph, not at one of the edges***. Note that the resistor box readings may not be accurate, especially on the low end, so it is a good idea to check the resistance values with your handheld DVM. From the graph, estimate the maximum power that the solar cell can deliver to a load, and the value of the load resistance for this. Compare the load resistance for maximum power with the internal resistance of the cell and discuss. You can use Equation 2.3 below to determine the internal resistance of the solar cell. Note that solar cells essentially are photo-diodes. Hence, the internal resistance when illuminated is *not fixed*; it is a strong function of the illumination and current.

The following equations can be used to find the internal resistance of a power source such as a solar cell. From *Kirchhoff's* loop rule, the voltage across the load resistance is

$$V_L = V_0 - I_L R_S \tag{2.1}$$

where I_L is the current through the load resistance R_L; R_S is the internal resistance of the power supply; and V_0 is the open circuit voltage ("EMF") of the power source. I_L can be determined from

$$V_L = I_L R_L \tag{2.2}$$

Eliminating I_L from (2.1) and (2.2) gives:

$$V_L = V_0 R_L / (R_L + R_S) \tag{2.3}$$

This says that if the internal resistance is much smaller than the load resistance, the voltage output will not be that much different from the open circuit voltage. In any case

you can deduce R_S using the above equations together with your measured values of V_L for the various R_L values you used.

6) The average American consumes about 1 kilowatt of power during the daytime. The sun can provide 1 kW/m^2 in the form of incident sunlight. Using the efficiency noted below for the photodiodes used in this experiment, how large an area covered by solar cells like those used here would be required per person? The US Census Bureau claims that the US population was 292,287,454 on January 1, 2004 (http://www.census.gov/).   How many square meters of solar cells would be required to satisfy the estimated daytime (personal) power use in the US? What fraction of the area of the sun-rich state of Nevada would that be? (The area of Nevada is 286367 km^2.)

2.5 Blackbody Radiation and the Stefan-Boltzmann Law

The origins of quantum mechanics arose from the failure of classical methods to explain the spectral distribution of light from hot objects. The experimental model system is a hot cavity such as an oven with walls at a uniform temperature T. A small hole is cut into an oven wall to allow a small fraction of the electromagnetic radiation to escape. Such light is called *blackbody radiation*. Surprisingly, the spectra of hot bodies such as the Sun or a lamp filament are very close to that of an idealized black body. Even the cosmic microwave background radiation has a blackbody spectrum corresponding to a temperature of 2.725 K. (See, for example, http://en.wikipedia.org/wiki/Cosmic_microwave_background_radiation) .

Statistical mechanics plus classical electromagnetism predicted a *spectral distribution* called the *Rayleigh-Jeans Law*, which can be written as:

$$\frac{dI}{df} = \frac{2\pi f^2}{c^2}kT = \frac{2\pi}{\lambda^2}kT \qquad (2.4a)$$

where *dI/df* is the *power emitted per unit surface area and per unit frequency at frequency f (or wavelength λ) by an object at temperature T*. Although the derivation of this formula is beyond the scope of this course, the form can be easily explained. In classical statistical mechanics, every possible degree of freedom should acquire an average energy of $\frac{1}{2}kT$. For example, a monatomic gas molecule has energy $\frac{3}{2}kT$ because it can move in three independent directions. The number of independent modes of oscillation in a cavity scales like f^2, for reasons similar to the relationship of the area of a sphere to its radius. The total intensity per unit frequency at a given frequency is the product of the number of modes available and the average energy in each. This is the physical content of Equation 2.4a.

We may re-express Eqn. 2.4a in terms of wavelength. Using $\lambda = c/f$ and $\left(\frac{dI}{df}\right)|df| = \left(\frac{dI_\lambda}{d\lambda}\right)|d\lambda|$,

we find

$$\frac{dI_\lambda}{d\lambda} = \frac{2\pi c}{\lambda^4}kT = \frac{2\pi f^4}{c^3}kT \qquad (2.4b)$$

where $dI_\lambda / d\lambda$ is the *power emitted per unit surface area and per unit wavelength at wavelength λ (or frequency f) by an object at temperature T.* **Note of caution:** $dI_\lambda / d\lambda$ in Equation 2.4b is often written as S_λ, I_λ, $u(\lambda)$ or other forms, *e.g.* as in the figure next to Equation 2.6c. The quantities *dI/df* and $dI_\lambda / d\lambda$ are *quite different*: they differ in numerical values, in units and in their dependence on *f* or λ.

From a theoretical point of view, Equations 2.4a and 2.4b are not satisfactory. As the frequency *f* becomes large (hence λ becomes small, as in the ultraviolet region), the predicted intensity increases without limit, even for objects at modest temperature. This is called "*the ultraviolet catastrophe*". On the other hand, at low frequencies (hence large λ (*i.e.,* at the red and infra-red wavelengths), the formula gave accurate predictions of the experimental results, indicating that at least some aspects were basically correct.

The problem was solved in two steps. Since Equations 2.4a and 2.4b lead to an infinite radiation rate, another approach must be found to compute the total intensity emitted by a hot object. Ludwig Boltzmann found a thermodynamic argument to show that the total radiation intensity, *i.e. dI/df* integrated over all frequencies, is given by

$$I = \sigma T^4$$
(2.5)

where σ is a constant of nature and *T* is the absolute temperature (in K). (See derivations in your textbooks or that given in *Wikipedia*.) This agreed with earlier experimental measurements by Josef Stefan. Equation 2.5 is called the *Stefan-Boltzmann Law*. The unusually rapid increase in radiation with temperature is a consequence of the mass-less nature of photons, the carriers of electromagnetic energy. (The same kind of behavior governs *phonons*, the quanta of *acoustic energy* in solids and liquids.)

Although the Stefan-Boltzmann law correctly describes the total power emitted per area, it does not predict the actual form of the spectral distribution. That step was taken by Max Planck who postulated that electromagnetic energy was emitted in discrete units or quanta, each with an energy given by *hf*, where *h is the Planck constant*, 6.6256 x 10^{-34} joule-second (see Table 2.4). For photon energies with *hf >kT*, it would no longer be possible to populate each mode with *kT* average energy since a fraction of *hf* is no longer allowed. The consequence is an additional factor of
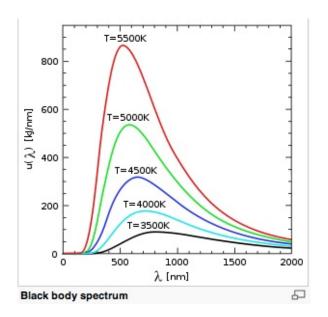
$$\frac{hf/kT}{e^{hf/kT}-1}$$

that reduces the spectral distribution given by Equation 2.4. This factor approaches unity for small *f*, preserving the long wavelength Rayleigh-Jeans behavior, while avoiding the ultra-violet divergence. The result is the *Planck spectral distribution*:

$$\frac{dI}{df} = \frac{2\pi hf^3}{c^2} \frac{1}{e^{hf/kT}-1}$$
(2.6a)

If we rewrite this in terms of power per area and wavelength interval at a given wavelength, we obtain for the Planck distribution in terms of λ (recall $c = \lambda f$):

$$\frac{dI_\lambda}{d\lambda} = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} \tag{2.6b}$$

This distribution (shown below) has a maximum at the wavelength given by *Wien's Law*:

$$\lambda_{max} = \frac{hc}{4.965} \frac{1}{kT} = \frac{2.898 \times 10^6 \, nm \cdot K}{T} \tag{2.6c}$$

By integrating the above equations over all frequencies or wavelengths, one can recover Equation 2.5 and, in addition, find that the constant, σ, is explicitly given by:

$$\sigma = \frac{2\pi^5 k^4}{15 h^3 c^2} = 5.67 \times 10^{-8} W / m^2 \cdot K^4 \tag{2.7}$$

It turns out that hot bodies such as the sun or a lamp filament behave much like a canonical "blackbody". In the first part of this experiment we use an ordinary tungsten wire lamp filament as a surrogate blackbody.

7) Verify the Stefan-Boltzmann Law by determining the total intensity radiated as a function of filament temperature. The filament temperature can be inferred from the lamp resistance and the resistivity data tabulated in Table 2.1. For these purposes, the room temperature (cold) resistance of the filament can most accurately be obtained with the small Hewlett-Packard multi-meter operating in ohmmeter mode. The filament resistance is very small, so *be sure to subtract the resistance of the ohmmeter leads*. The tabular values contained in Table 2.1 have been entered into an Excel® spreadsheet file loaded on the lab PCs for Expt.2W09. With these data you can use Excel® (or another program) to interpolate temperature automatically from your current and voltage measurements.

The intensity emitted can be calculated from the total electric power supplied to the lamp, if we assume all of it goes into electromagnetic radiation. You will need to find the surface area of the filament responsible for the blackbody emission. Two quantities must be determined: the wire diameter (or radius) and its length. The length is hard to measure because the filament is tightly coiled, but it can be computed from the room temperature (cold) resistance, R, the wire radius, r, using the formula you may recall from introductory physics courses:

$$R = \rho \frac{\ell}{\pi r^2} \qquad (2.8)$$

where ρ is the resistivity given in Table 2.1 and ℓ is the wire length. You will need to measure the filament wire radius of a similar lamp that has been dissected to expose the tungsten wire. A precision micrometer is available for this purpose (per usual with any device, check the zero offset and adjust or compensate for this). The emitting area is $A = 2\pi r \ell$. The total intensity, I, at the surface of the tungsten equals the total electrical power iV divided by the area,

$$I = \frac{iV}{A} \qquad (2.9)$$

where i is the current through the lamp, V is the potential across the lamp and A is the area you have derived from the above. Note that V should be measured *directly at the lamp terminals*, to eliminate the voltage drop in the power leads.
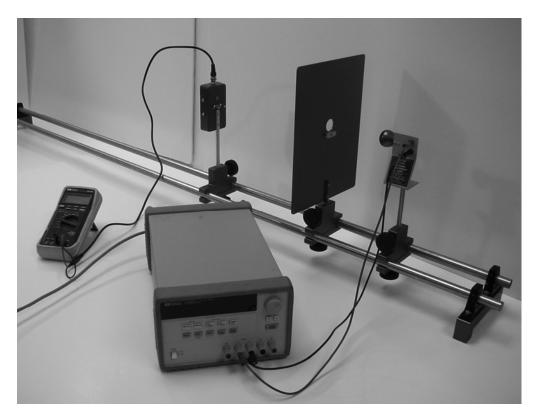
Figure 2.5: Typical setup for the Stefan-Boltzmann measurements (set baffle as needed to minimize stray light). Usually we use LL convention (light from left).

8) Take data with power supply voltages from about 4 V to 13 V in steps of 1 V. Plot the intensity radiated as a function of the filament temperature. Include estimated uncertainties (error bars) in your plot. Fit your data to $I = \sigma T^4$ using Excel® or Vernier *Logger Pro*® and determine the constant σ and power *n and estimate and justify their uncertainty.* You can use the Trend line function of Excel® for this or the equivalent in the *Logger Pro*® (or other) programs. You will find that the fit is poor at lower filament temperatures due to the temperature non-uniformity in the filament, which you can see by inspection. Thus you may want to drop one or two low temperature points to see if that improves the fit, but note this and any improvement found.   Compare your results to the expected values for an ideal blackbody. Use your value for σ to estimate the *Boltzmann constant k* (and its uncertainty*)* assuming known values of *c* and *h* (Equation 2.7).

9) Next investigate the Planck spectral distribution (Equation 2.6) for one or more of the higher filament temperatures. Use the photodiode boxes with the large sensors (10 mm x 10 mm) for this part of the measurement and a resistor of 10K or greater on the HP DVM to measure the photodiode current. Set the photodiode at a known distance ~20 cm from the source with one or more light baffles as necessary.   To measure the light intensity at different frequencies and hence wavelengths, sets of interference filters are available with pass bands at 450 nm (blue), 550 nm (green) , 650 nm (red) and other wavelengths through the near infra-red region . The bandwidth and transmission for each band are provided in Table 2.2. Handle the filters with care and avoid getting fingerprints on the glass surfaces. Only a small amount of light passes through the filters, especially the blue and red and infrared filters. It is best to mount the filters right on the photodiode box; this together with the light baffles minimizes the background light that can get around the filter. This is necessary as the intensity away from the maximum will be low and hard to measure accurately if there is stray light.   Be sure to subtract the background photo-current reading you observe when the hole in the baffle is covered with an opaque card. Refer to Appendix 2.B to make sure you obtain all the data you need. Take photodiode current data at *one or more of the higher filament voltages*, *e.g.,* 11 V to   13 V with the full set of filters. (Some of the filters will need to be shared with the other groups).

10) Consult Appendix 2.B to determine how to calculate $[dI_\lambda / d\lambda](\lambda)$ from the photodiode currents you have measured and the data provided in Tables 2.2 and 2.3. Plot $[dI_\lambda / d\lambda](\lambda)$ *vs*. λ for each filament temperature along with the Planck and Rayleigh-Jeans predictions for $[dI_\lambda / d\lambda](\lambda)$. [Note: You may have to arbitrarily scale the Planck and classical predictions by appropriate factors to get them on the same graph.]

11) The Planck spectral distributions (Equations 2.6) give distinctly different predictions than the classical Rayleigh-Jeans law (Equations 2.4).   In one case, the temperature dependence is given by $1/(e^{hf/kT} - 1)$, whereas the latter predicts a simple linear behavior with *T.* You should compare the *shape* of the photodiode current *vs.* temperature curve with the quantum-mechanical and classical predictions normalized as needed, ignoring

the various normalization factors that don't depend on temperature. See *Additional questions* in Section 2.7 below (to be answered in your lab report summary).

2.6 Blackbody Temperature of an Unknown Source

Carefully install one of the 120 V halogen lamps on the optical bench and plug it into a 120 V outlet. Make sure it has the UV filter glass on the front of the lamp. Using the techniques outlined above with the filter sets provided, measure the emission spectrum of this source.

12) Fit the spectrum with a blackbody spectral distribution (Equation 2.6b) and obtain its blackbody temperature. You will likely need to normalize the calculation to fit the data once you have the correct shape i.e. temperature determined (Hint: You can use Wien's Law as a guide). Graph your data and the best fit, renormalized as needed. Indicate the goodness of fit criterion and provide a reasonable estimate of the uncertainty in *T*.

13) How does this temperature compare with the surface temperature of the Sun (which is a main sequence star)? A red giant star?

2.7 Additional Questions and Analysis

14) You probably found that in the "inverse square law" test the exponent was somewhat less than expected. Estimate (reasonably quantitatively) how much the finite size of the light source can affect your "inverse square law" test for small separations. (Suggestion: Try adding a constant amount to all your distances to see if you can get a power closer to -2.00)

15) What *trig function* best describes the behavior of light intensity with angle of incidence on the photodiode? Explain why this would be expected.

16) In the test of the Stefan-Boltzmann Law, what uncertainty causes the largest error? (*e.g.*, lamp current, filament temperature, filament diameter, room temperature filament resistance, . . .). *Defend your answer with some specific numbers.*

17) Estimate the *uncertainty in the coefficient* σ that you obtained in testing the Stefan-Boltzmann law by making reasonable variations on some of your data points. Discuss *systematic effects and assumptions* that would cause the measured value to deviate significantly from the expected value. Would you expect the tightly coiled lamp filament to act like an ideal "black body" radiator?

18) Do the same for your value of the exponent of *T* in Equation 2.5. Is it consistent with an exponent of 4.00? Discuss.

19) What are the appropriate *mks* units for $[dI_\lambda / d\lambda](\lambda)$ (You may use references to answer this. **However**, note that different symbols are used in literature for what we denote as $[dI_\lambda / d\lambda](\lambda)$ in Equation 2.6b.)

20) We assumed that all the electrical power supplied to the lamp filament is transformed into radiation. Discuss the validity of this assumption.　(*Where else might it go*?)

Temp		Resistivity	Temp		Resistivity
K	R/R_{293K}	$\mu\Omega \cdot cm$	K	R/R_{293K}	$\mu\Omega \cdot cm$
293	1	5.48	2000	10.34	56.67
300	1.03	5.65	2100	10.96	60.06
400	1.47	8.06	2200	11.58	63.48
500	1.93	10.56	2300	12.21	66.91
600	2.41	13.23	2400	12.84	70.39
700	2.94	16.09	2500	13.49	73.91
800	3.47	19	2600	14.14	77.49
900	4	21.94	2700	14.79	81.04
1000	4.55	24.93	2800	15.46	84.7
1100	5.1	27.94	2900	16.12	88.33
1200	5.65	30.98	3000	16.8	92.04
1300	6.22	34.08	3100	17.47	95.76
1400	6.79	37.19	3200	18.16	99.54
1500	7.36	40.36	3300	18.85	103.3
1600	7.95	43.55	3400	19.56	107.2
1700	8.54	46.78	3500	20.27	111.1
1800	9.13	50.05	3600	20.99	115
1900	9.74	53.35			

Table 2.1:　Resistivity for tungsten as a function of temperature.

Color	λ (nm)	$\Delta\lambda$ (nm)	Trans-mittance, η	f (Hertz)	Δf (Hertz)	$E=hf$ (eV)
blue	450	40.1	0.69	6.662×10^{14}	5.945×10^{13}	2.755
green	550	40.1	0.748	5.451×10^{14}	3.982×10^{13}	2.254
red	650	35.8	0.828	4.612×10^{14}	2.542×10^{13}	1.907
red	700	50.0	0.60	4.286×10^{14}	3.06×10^{13}	1.771
IR1	880	50.0	0.55	3.409×10^{14}	1.94×10^{13}	1.409
IR2	950	50.0	0.55	3.158×10^{14}	1.66×10^{13}	1.305
IR3	1064	10.0	0.50	2.820×10^{14}	0.26×10^{13}	1.165

Table 2.2:　Interference filter parameters.

λ (nm)	300	350	400	450	500	550	600	650	700
η_{diode}	0.413	0.504	0.547	0.646	0.748	0.824	0.846	0.852	0.855

λ (nm)	750	800	850	900	950	1000	1050	1100	
η_{diode}	0.852	0.843	0.832	0.814	0.783	0.709	0.45	0.18	

Table 2.3:   Hamamatsu S2386-5K photodiode quantum efficiency, η_{diode}, versus wavelength, λ.

c	2.99792458 x 10^{8}m/s	speed of light
e	1.60217733 x 10^{-19} C	electron charge
h	6.6260755 x 10^{-34} J · sec	Planck constant
k	1.380658 x 10^{-23} J/°K	Boltzmann constant

Table 2.4: Fundamental physical constants.

2.7   Additional Tips for Keeping Lab Notebooks and Writing Reports

Both the raw data and *most* of the numerical analysis belong in your notebook. The formal summary of the results along with relevant graphs are then attached separately. Enter information either by writing some text and/or by affixing Excel$^®$ or other printouts, graphs, *etc.*

A common mistake that students make is to wait till after the second week of an experiment to start analyzing the data. Often they find they missed some crucial piece of data and they can't finish their analysis.   You are strongly encouraged to complete as much of the data taking measurements as possible in the first week of a given experiment ***and start analyzing your data right away***. *Then if you find something strange, you can re-measure or otherwise obtain the necessary data or information the second week.*

As in Experiment 1, the numbered items describe the measurements you should perform and questions to answer. Try to describe your measurements in enough detail so that you could reproduce the experiment. Simple diagrams or photographs are often very helpful. You should find that the mere act of writing down what you are doing will help you to think! Answers to questions should refer to the appropriate numbered section in the manual.

Appendix A: Extracting Power Law Behavior from Data

Power law behavior frequently occurs in physics. In this experiment, one expects to find that light intensity varies as R^{-2} and the radiated power from a hot filament varies as T^4. It would be nice to have a systematic way of testing your data to see what exponents are actually measured.

Assume we have a power law relationship:

$$y = x^p$$

The logarithmic derivative is defined by:

$$\frac{d\log y}{d\log x} = \frac{x}{y}\frac{dy}{dx} = \frac{x}{x^p} \cdot px^{p-1} = p \qquad (2.10)$$

This is a handy tool for finding the power law behavior at any point along a curve. To apply it to the data for this experiment, fit your data to suitable smooth curves. The slope of the curve can then be evaluated and inserted into Equation 2.10 to find the local effective power law exponent.

Appendix B: Estimating the Measured Photocurrent

The actual photodiode current measured in Section 2.5 is a somewhat messy product of a number of factors. The steps to finding this relationship are given below.

The power radiated per filament area and unit frequency or unit wavelength is given by Equations 2.6a and 2.6b, respectively. Thus, the power emitted into the filter pass band is:

$$P_{\Delta f} = \frac{dI}{df} \cdot \Delta f \cdot A_{fil} \qquad (2.11a)$$

or,

$$P_{\Delta\lambda} = \frac{dI_\lambda}{d\lambda} \cdot \Delta\lambda \cdot A_{fil} \qquad (2.11b)$$

where Δf or $\Delta\lambda$ is the bandwidth transmitted by the interference filter (Table 2.2) and A_{fil} is the lamp filament area. However, this power is radiated isotropically over 4π steradians. Thus, the fraction of this power intercepted by the photodiode is $A_{\text{diode}}/(4\pi R^2_{\text{diode}})$ where A_{diode} is the active area of the photodiode and R_{diode} is the distance of the diode from the tungsten lamp. Furthermore, the interference filter does not perfectly transmit all the light within the stated bandwidth. The actual transmittance, η_{filter}, is given in Table 2.2.

With these corrections, the power incident on the photodiode is

$$P_{diode} = \frac{dI}{df} \cdot \Delta f \cdot A_{fil} \cdot \left(\frac{A_{diode}}{4\pi R_{diode}^2} \right) \cdot \eta_{filter} \tag{2.12a}$$

or,

$$P_{diode} = \frac{dI_\lambda}{d\lambda} \cdot \Delta \lambda \cdot A_{fil} \cdot \left(\frac{A_{diode}}{4\pi R_{diode}^2} \right) \cdot \eta_{filter} \tag{2.12b}$$

Of course, what you measure with the photodiode is current, not power. Each photon absorbed allows one electron to flow. The photon arrival rate can be found by dividing the power by the energy of a single photon, *hf*. One more efficiency factor is required: the photodiode produces an electron-hole pair for each incident photon with a probability <1. This *quantum efficiency*, η_{diode}, is provided in Table 2.3. The final result is:

$$i_{diode} = \frac{dI}{df} \cdot \Delta f \cdot A_{fil} \cdot \left(\frac{A_{diode}}{4\pi R_{diode}^2} \right) \cdot \eta_{filter} \cdot \eta_{diode} \cdot \left(\frac{e}{hf} \right) \tag{2.13a}$$

or,

$$i_{diode} = \frac{dI_\lambda}{d\lambda} \cdot \Delta \lambda \cdot A_{fil} \cdot \left(\frac{A_{diode}}{4\pi R_{diode}^2} \right) \cdot \eta_{filter} \cdot \eta_{diode} \cdot \left(\frac{e\lambda}{hc} \right) \tag{2.13b}$$

Use the above equations to extract $\frac{dI_\lambda}{d\lambda}$ from the measured diode currents for each filter used at one of the known filament lamp temperatures and then for the unknown source.