

## INVESTIGATION OF VORTEX LIGHT PRODUCED BY DIFFRACTION GRATINGS

### INTRODUCTION

In this experiment we will study light with orbital angular momentum. Circularly polarized light possesses *spin* angular momentum, analogous to Earth's rotation about its axis. *Orbital* angular momentum (OAM) of light is analogous to Earth's orbit around the Sun, with a structure explained below. We will generate beams with OAM by passing laser light through specially designed diffraction gratings. By constructing an interferometer and interfering the OAM beam with a reference plane wave, we can determine the magnitude and direction of the beam's orbital angular momentum.

### OPTICAL VORTICES

The propagation of ordinary light can be approximated by a plane wave. Light can also be manipulated to possess a spiraling azimuthal phase shift such that the phase  $\phi$  of the light corresponds to the angular position  $\Theta$ . In the simplest case:

$$\phi = \Theta$$

The wavefront of the light will spiral around the beam axis like the threads on a screw, and the light will possess orbital angular momentum (OAM) about the beam axis [1]. Laser beams with this structure are known as Laguerre-Gaussian modes. When projected on a screen, they appear as one or more concentric rings surrounding a central dark singularity or bright dot [2]. Light beams possessing OAM are termed optical vortices.

### CREATING VORTEX LIGHT

A Gaussian beam of light can be transformed into an optical vortex by numerous mechanisms. In this experiment, a Gaussian beam will be passed through a forked diffraction grating with a fringe defect (an

extra line in the grating, see below). The diffraction grating is essentially a hologram of the interference pattern between a plane wave and an optical vortex; passing the plane wave through the grating recovers the optical vortex. The interference pattern was calculated using the formula:

$$I(x, y) = \left| I_1 e^{ikx} + I_2 e^{il \arctan\left(\frac{y}{x}\right)} \right|^2$$

Where  $I_1$  and  $I_2$  are the intensities of the reference wave and the desired optical vortex, respectively;  $i$  is the imaginary unit,  $k$  is the wavenumber of the reference wave, and  $l$  ( $L$ ) represents the order of the grating (discussed below). This formula has been significantly simplified from the true mathematical description of the light wave [3]. The calculation produces a grayscale image of the diffraction grating, which yields the image below when converted to pure black-and-white. Note the fringe defect—the extra line partway through the grating. Fringe defects can involve more than one line (see next page). This image was photographed and thereby transposed onto 35 mm film. The developed film negatives are partially transparent and can be used as diffraction gratings.

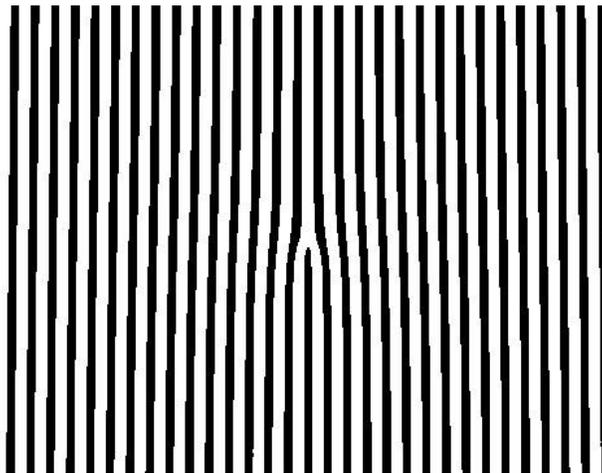


Fig 1: An example grating, exhibiting an order-1 fringe defect

A Gaussian laser mode passed through a pattern like the one above will diffract into several beams possessing OAM  $m$  per photon, given by:

$$m = \pm l\hbar, l = 0,1,2,3 \dots$$

This  $l$  is known as topological charge. The central bright spot possesses no OAM, while the magnitude of the OAM increases by  $l\hbar$  for the next bright spot outward. Bright spots with positive momenta are located to one side of the center, while negative momenta are located to the other side.

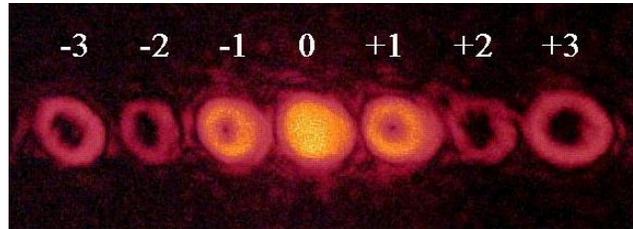


Fig 2. Vortices generated by a holographic diffraction grating [4].

#### PROCEDURE FOR PRODUCING DIFFRACTION GRATINGS

The forked diffraction gratings in this lab are holograms of the interference pattern between a plane wave and a Laguerre-Gaussian mode at a slight angle to each other, as indicated in the formula:

$$I(x, y) = \left| I_1 e^{ikx} + I_2 e^{il \arctan(\frac{y}{x})} \right|^2$$

The plane wave component is present as  $I_1 e^{ikx}$ , where  $I_1$  is the plane wave's intensity and  $k$  is its wavenumber. Similarly, the Laguerre-Gaussian mode is present as  $I_2 e^{il \arctan(\frac{y}{x})}$ , where  $I_2$  is its intensity and  $l$  is the charge of its OAM. To produce diffraction gratings from this formula:

- 1) Use a calculation program such as Matlab or Mathematica to evaluate  $I(x, y)$  over a broad range of  $(x, y)$ , storing the results in a matrix.
- 2) Map the range of values in the matrix to a grayscale, and display the matrix as an image.
- 3) Convert the image to pure black-and-white (color values of 255 or 0 only). If done correctly, the black and white lines should have roughly equal width.

- 4) Print the black-and-white image on white paper.
- 5) Use a traditional film camera to photograph the sheet of paper against a blank background (we hung ours against a whiteboard). Use black-and-white film with as high of resolution as possible (low ASA; ours was Ilford Pan-F Plus, with an ASA of 50).
- 6) Keep in mind that in diffraction,  $d\sin\theta = m\lambda$ , where  $d$  is the spacing between lines in the grating,  $\lambda$  is the wavelength of the diffracted light,  $m$  is an integer indicating a maximum in the diffraction pattern, and  $\theta$  is the angle to maximum  $m$ . The film negatives will be used as the diffraction gratings, so the line spacing on the film will determine the spread of the diffraction pattern.
- 7) Develop the film, and mount the developed negatives in projector slides.
- 8) A sample Matlab code is included at the end of this lab as a reference for producing images of the gratings (steps 1 and 2 in this procedure).

## ANGULAR MOMENTUM

It is important to distinguish light with OAM from more common polarized light. Polarized light is associated with the spin angular momentum of a photon. The spin angular momentum is specified by  $\sigma$ .

Circular polarization:  $\sigma = \pm\hbar$

Linear polarization:  $\sigma = 0$

Elliptical Polarization:  $|\sigma| < \hbar$

Question #1: Consider a HeNe(633nm) laser that produces a light beam which consists of a stream of photons.

- a) What is the energy of one such photon?
- b) What is the linear momentum of one such photon?
- c) What is the spin angular momentum of one such photon?

Question #2: Each photon's spin angular momentum is aligned either parallel or anti-parallel to the direction of propagation. Describe how circular polarization arises.

Orbital angular momentum is not related to the photon spin, but to the helical shape of the light's wavefront. For an electromagnetic wave, the angular momentum density is given by the following two equations [5]:

$$\mathbf{j} = \frac{1}{4\pi c} \mathbf{r} \times (\mathbf{E} \times \mathbf{B})$$

$$\mathbf{j} = \mathbf{r} \times \mathcal{P}$$

Where  $\mathbf{r}$  is the position vector,  $\mathcal{P}$  is the momentum density,  $c$  is the speed of light,  $\mathbf{E}$  is the electric field, and  $\mathbf{B}$  is the magnetic field. These equations show that if a beam's linear momentum has an azimuthal component it has angular momentum. The angular momentum density along the propagation direction  $z$  is then [5]:

$$j_z = r \mathcal{P}_\phi$$

$$j_z = \epsilon_0 \omega (l|u|^2 - \frac{1}{2} \sigma r \frac{\partial |u|^2}{\partial r})$$

where  $\omega$  is the angular frequency of the light, and  $u$  is a complex scalar function proportional to the electric field amplitude. We notice two independent contributions to the angular momentum of the light. The first term is proportional to  $l$  and the square of the field amplitude. The second term is our familiar polarization term. We find then the total angular momentum of the beam by dividing the angular momentum of the beam by the total energy of the beam [5].

$$\frac{J_z}{W} = \frac{l + \sigma}{\omega}$$

We finally find that one photon carries with it an angular momentum:

$$J = l\hbar + \sigma\hbar$$

So that the total angular momentum is given by two independent terms, the first due to orbital angular momentum, and the second due to the spin orbital angular momentum.

QUESTION #3: Which of the following properties of a photon are dependent on orbital angular momentum: polarization, spin and/or wavelength?

### **HERMITE-GAUSSIAN AND LAGURRE-GAUSSIAN MODES**

For a collimated beam of light Maxwell's wave equation reduces to the paraxial wave equation. This equation can be solved in either Cartesian or cylindrical coordinates [6]. The solution in Cartesian coordinates gives rise to the Hermite-Gaussian family of solutions. The solution in cylindrical coordinates gives rise to the Laguerre-Gaussian family of solutions.

The usual output of a laser is the zero order Hermite-Gaussian solution. For this mode, all points on a transverse plane have the same phase. This lab will investigate the lesser known Laguerre-Gaussian family of solutions. For this mode the phase winds as a function of angle. The phase structure of a Hermite-Gaussian mode corresponds to a planar phase surface. Dissimilarly the phase structure of a Laguerre-Gauss mode corresponds to a helical phase surface.

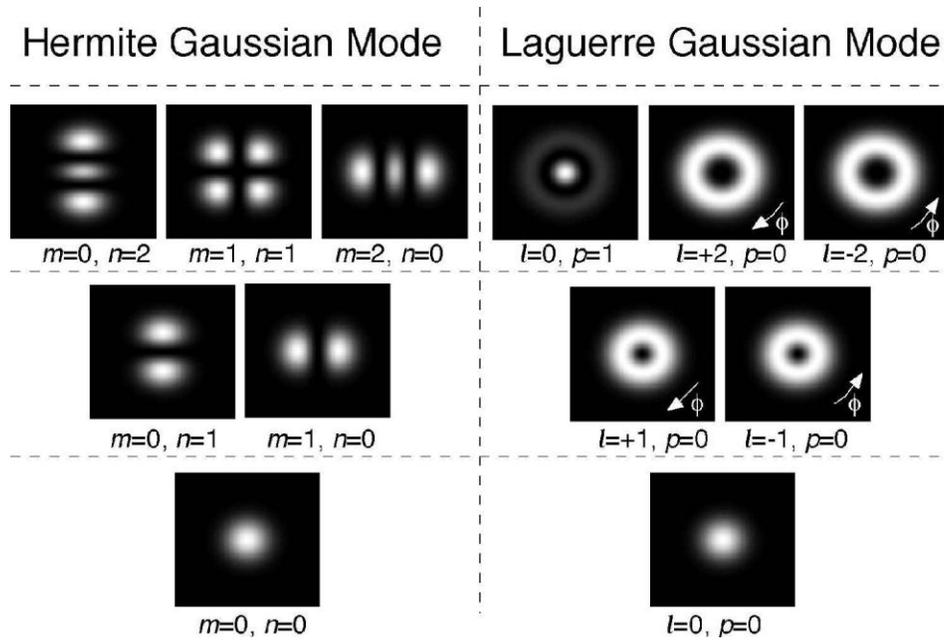


Fig 3. Laguerre-Gaussian modes have their distinct doughnut appearance because of the angular variance in phase. At the center, every possible phase cancels with its opposite, leaving a “hole” [7].

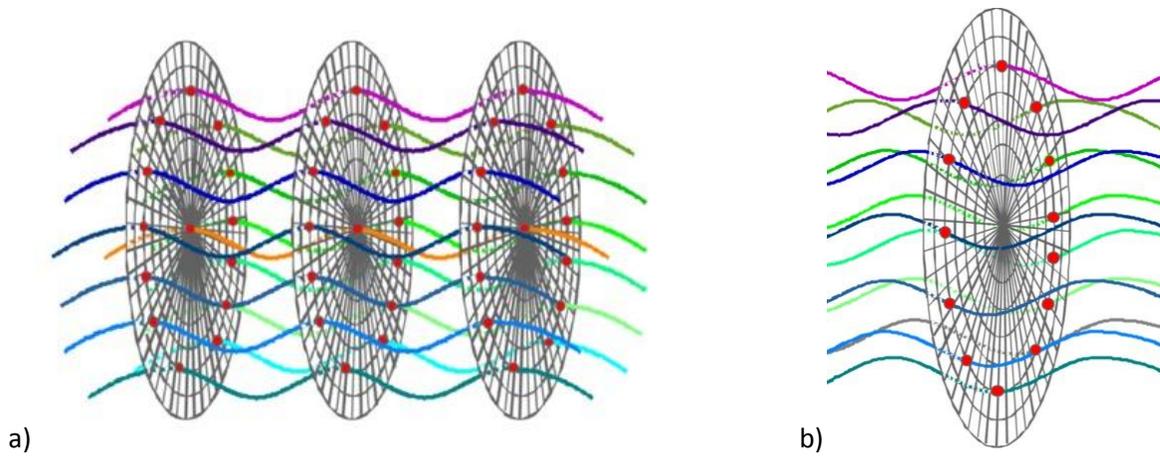


Fig 4. The red dots mark the intersection of the wave with the transverse plane. The phase of b) advances around the circle, whereas in a) it does not [8].

QUESTION #4: Which picture indicates a Hermit-Gaussian Mode and which picture indicates a Laguerre-Gauss Mode?

## IDENTIFICATION OF LAGURRE-GAUSSIAN MODES

It is well known that if two plane wavefronts are interfered they give a straight line fringe pattern, the spacing of which depends on the wavelength and the intersection angle. If we interfere a plane wavefront with a helical wavefront traveling parallel, we get a spiral pattern which has  $l$  spokes, this pattern is due to the azimuthal phase structure given by  $e^{il\varphi}$  where  $\varphi$  is the azimuthal angle and  $l$  is the mode index [7]. (If the wavefronts are not parallel, they form a straight-line diffraction pattern with a fringe defect, as described previously).

It is also possible to interfere a helical wavefront with its own mirror image. Because the phase term of one beam is  $e^{il\varphi}$  and the other  $e^{-il\varphi}$  the phase difference is  $e^{i2l\varphi}$  which results in  $2l$  dark spokes. These spiral patterns can be used to determine the charge  $l$  of any Laguerre-Gaussian mode [7].

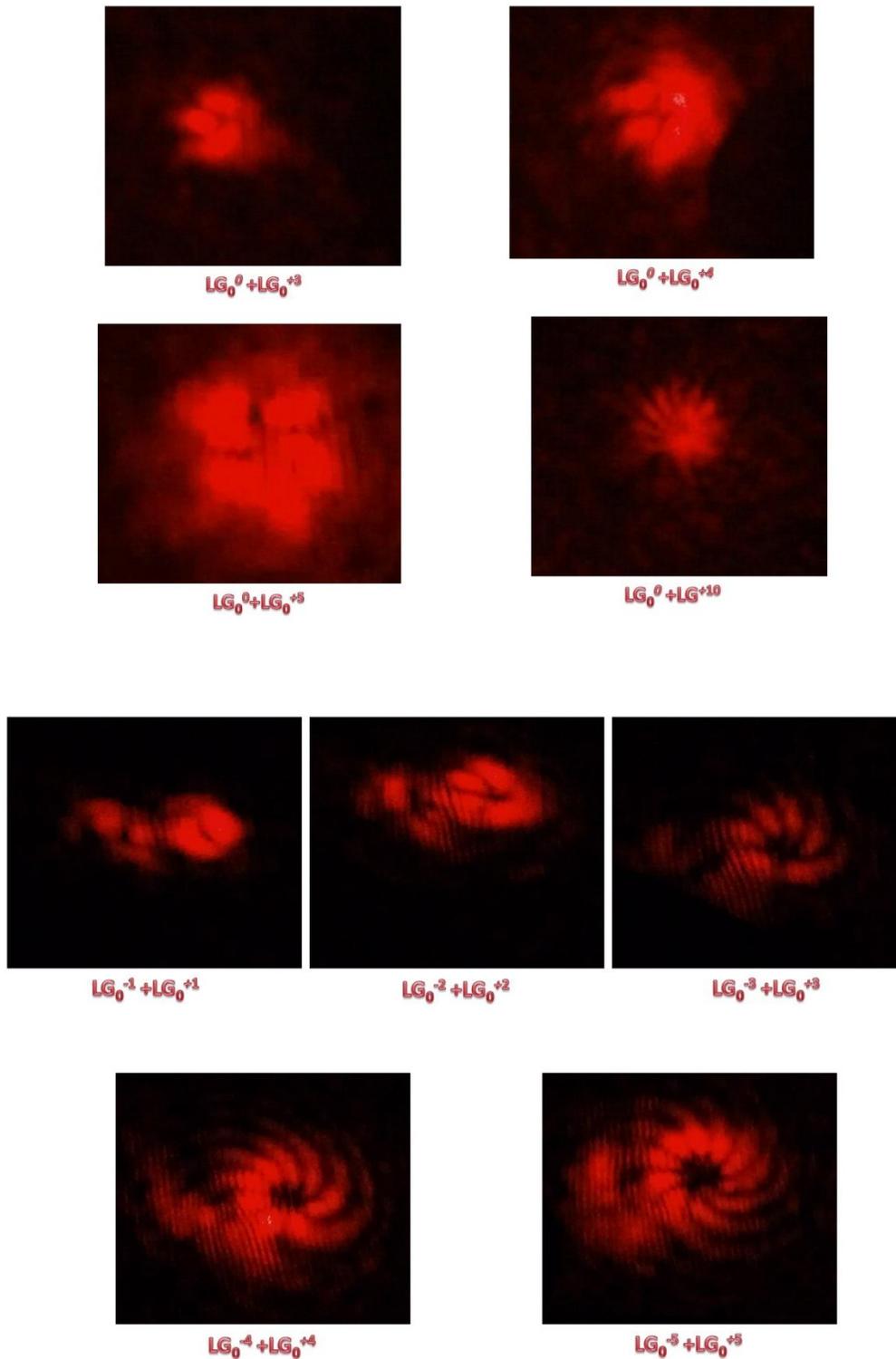


Fig 5. Left interference patterns of interfering two mirrored helical wavefronts. Right interference patterns of interfering a helical wavefront and a plane wave.

## INTERFEROMETRY

We will make use of the principle of superposition to combine separate waves together. If two beams with the same frequency combine the resulting pattern is governed by the phase difference between the two beams. Points of the same phase will undergo constructive interference while waves that are out of phase will undergo destructive interference.

A beam of light passed through the diffraction grating will separate into beams with the diffraction pattern seen in Fig 2. We will split this diffracted beam using a partial mirror. We will then select two *different* diffraction maxima, corresponding to two different Laguerre-Gaussian modes, which will be steered independently by mirrors. The two desired modes will be recombined at a second partial mirror. This combined beam will be projected onto a screen where an interference pattern will be seen. Neutral density filters and lenses may be needed so that the two modes have similar intensity and spatial size.

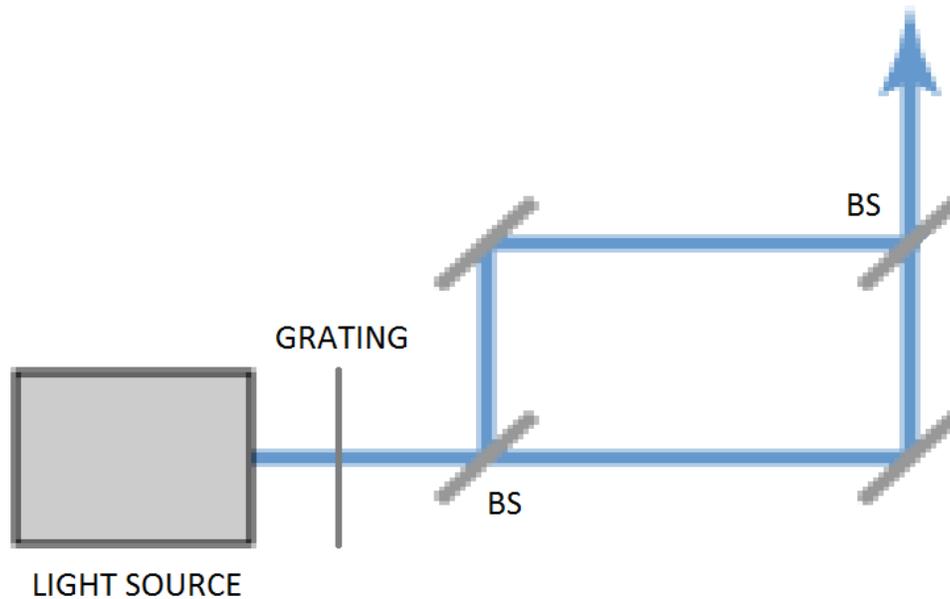


Fig 6. Example interferometer

### PROCEDURE FOR INTERFERING MODES

- 1) A random slide-mounted grating of unknown topological order will be assigned. Use a microscope to examine the diffraction grating. Sketch the diffraction grating, noting its orientation.
- 2) Build an interferometer which can be used to combine Laguerre-Gaussian modes.
- 3) Pass light through the grating and examine the diffraction pattern. Make a sketch.
- 4) Steer one beam through the interferometer.
- 5) Steer the other beam through the interferometer, making sure that the beams are collinear as they exit the second beamsplitter.
- 6) Lenses and Neutral density filters may be need, if so integrating them into you optical train.
- 7) Determine the topological charge of the grating. Photographing the diffraction pattern may be useful. It is recommended that this be done in a dark or at least dimly lit room, as the finely detailed interference patterns can be hard to see.
- 8) It is recommended that you interfere 2 LG modes instead of 1 LG and one HG mode, as we have found the spirals from 2 LG modes to be far more easily visible
- 9) Repeat procedure and catalog all assigned gratings.

## SUPPLIES

<b>Table 1.</b>
1x HeNe Laser
1x Optics Table
Camera
>2x Mirror
>2x Beam Splitters (non-polarizing)
>6x Optical Posts
>6x Optical Bases
>6x Optical Post Holders
Neutral Density Filter
Assorted Lenses
Assorted Optical Screws
Assorted Forked Diffraction Gratings

## SAMPLE MATLAB CODE

```

A=3200; %height (number of rows)
B=4000; %width (number of columns)
C=.66; %intensity of LG
D=.66; %intensity of plane wave
E=1; %charge of OAM
F=46.5; %Horizontal Adjustment factor
G=.1; %Plane wave wavenumber
M=zeros(A,B);
bwmap=[0:255]'/255;
colormap(bwmap)
for ro=1:A
    for co=1:B
        M(ro,co)=(abs(C*exp(i*E*atan2((ro-(A/2)),(co-(B/2)))))+D*exp(i*G*(co-(B/2+F))))^2);
    end
end
N=255*M;
image(N)
axis equal
    
```

Since the image will be resized during printing and photography, the choice of wavenumber (which, if size was fixed, would affect the line spacing of the grating) is arbitrary.

## REFERENCES

- [1] Allen, MW Beijersbergen, RJC, and JP Woerdman, 1992, *Phys. Rev. A* **45** 8185.
- [2] M Padgett, J Arlt, N Simpson, and L Allen, 1996, *Am. J. Phys.* **64** 77.
- [3] NR Heckenberg, R McDuff, CP Smith, H Rubinsztein-Dunlop, and MJ Wegener, 1992, *Opt. QuantumElectron.* **24** S951.
- [4] Azure Hansen, Stony Brook University, 2004.
- [5] E.J. Galvez and N. Zhelev. "Orbital Angular Momentum of Light in Optics Instruction," ETOP conference proceedings (2007).
- [6] T.T.A. Lummen. " *Optical tweezers.*" Paper 'Nanoscience', *Rijksuniversiteit Groningen*, 2004.
- [7] M. Padgett and L. Allen, 2000, *Contemp. Phys.* **41**, 275-285.
- [8] E.J. Galvez, P.R. Crawford, H.L. Sztul, M.J. Pysher, P.J. Haglin, and R.E. Williams, 2003, *Phys. Rev.* **90** 203910.

## FOR FURTHER READING

Optical Angular Momentum by L.Allen, Stephen M Barnett and Mile J Padgett (2004).