

COMPTON SCATTERING

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Purpose

The purpose of this experiment is to verify the energy dependence of gamma radiation upon scattering angle and to compare the differential cross section obtained from the data with those calculated using the Klein-Nishina formula and classical theory.

Introduction

Scattering of photons on atomic electrons, first measured by Arthur Compton in 1922, was one of the fundamental experiments that helped to establish the validity of quantum theory.

In Compton scattering the conservation of energy and momentum in the two-body interaction leads to the relation among the energy of the scattered photon, E'_ν , the energy of the incident photon, E_ν , and the scattering angle, θ , given by:

$$\frac{1}{E'_\nu} = \frac{1}{E_\nu} + \frac{1}{m_e c^2} (1 - \cos \theta),$$

where $m_e c^2 = 511.0$ KeV is the energy of the electron at rest.

The differential scattering cross section (the ratio of the scattered energy per unit solid angle to the incident energy per unit area) depends both on the photon energy and scattering angle. It is given by an expression, known as the Klein-Nishina formula,

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2} \frac{1}{(1 + \gamma(1 - \cos \theta))^2} \left(1 + \cos^2 \theta + \frac{\gamma^2 (1 - \cos \theta)^2}{1 + \gamma(1 - \cos \theta)} \right),$$

where $r_0 = (e^2/4\pi\epsilon_0 m_e c^2) = 2.82 \times 10^{-13}$ cm² is the classical electron radius and $\gamma = E_\nu/m_e c^2$. This formula takes into account the relativistic effects and additional quantum corrections, derived in quantum electrodynamics.

Fundamentals of Experiment

The experiment, shown in Fig. 1, consists of a γ -ray source, a scattering material (the PMMA scintillator) and a detector (the NaI scintillator) capable of measuring the energy and rate of scattering events as a function of angle between the incident and scattered photons. The source within its lead housing poses absolutely no health risk. Therefore, you are not required to wear radiation badges in the lab.

A scintillator is used as the scattering material to provide a signal that can be used to identify and select in the NaI detector only those events that are coincident with a scattering event in the PMMA. This coincident detection is important in rejecting background signals. In the absence of this additional background rejection the experiment would be much more difficult to perform.

Question: Explain, why different scintillating materials are used for scattering and detecting the γ -rays.

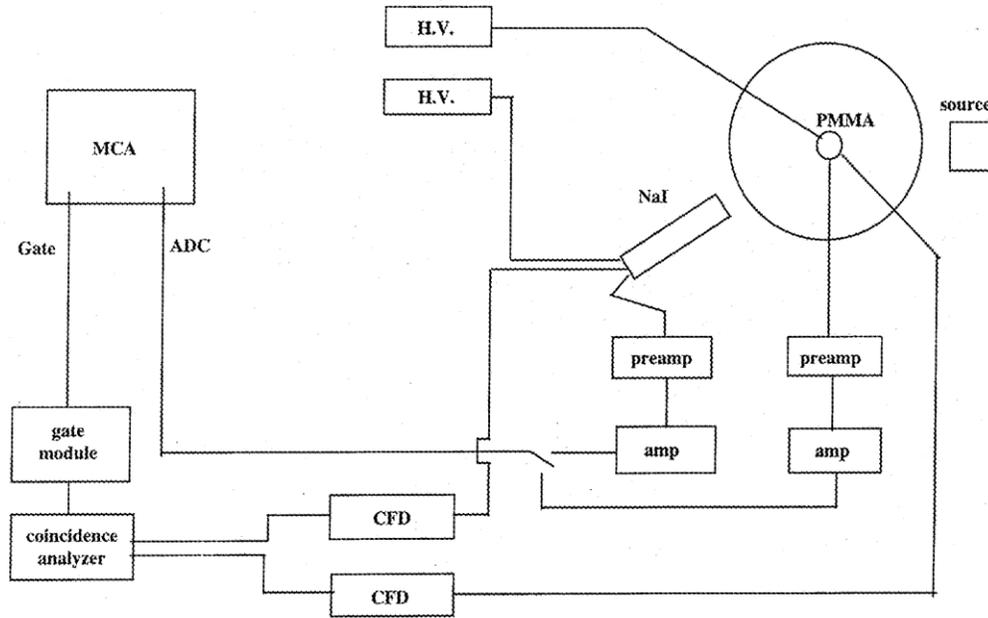


Figure 1: Block diagram of the scheme of the experiment.

Description of Electronics

A photomultiplier tube requires only a high DC voltage to operate. In this experiment the DC voltage is provided by a Canberra 3102D power supply. The tubes used in this experiment each have two outputs. The output pulse from the connector marked *DYNODE* is used to measure the energy deposited in the scintillator and that from the connector marked *ANODE* is used for timing and determining coincidence between events in the two scintillators.

A block diagram of the electronics is shown in Fig. 1. Pulses from the dynode are fed into a preamplifier/amplifier combination (Ortec model 113 and Canberra model 2012) and then into the analog-to-digital converter (ADC) input of the multi-channel analyzer (MCA), a board in the computer. The MCA consists of 4096 channels, the channels sequentially being associated with a increasing pulse height. The MCA is controlled by the computer using the software *MAESTRO*.

Pulses from the anodes of the PMT's are fed into constant fraction discriminators (CFD), Canberra model 2126. A CFD produces a logic output pulse if the input pulse exceeds a fixed value that can be set with the *THRESHOLD* dial. The logic output pulse starts a fixed time after the input pulse has risen to a certain fraction (for model 2126 this fraction is 0.4) of its peak value. If the pulses have the same shape but different amplitudes, as is the case in this experiment, then the time delay between the event that produced the input pulse and the logic output pulse from the CFD will be a constant, independent of pulse size. The magnitude of the time delay can be adjusted by the *WALK* control, a screw on the front panel.

Pulses from the positive outputs of the two CFD's are fed into a coincidence analyzer (Canberra model 2040), which produces a logic output pulse when the two input pulses are about 0.5μ 's or less of each other. The output pulses of the coincidence analyzer are then fed into the *START* input of a gate generator module (LeCroy model 222), which produces an output pulse suitable for controlling the MCA. The TTL (transistor-transistor logic) output of the gate generator is connected to the *GATE* input of the MCA. The MCA has a toggle switch with three positions. If the switch is in its middle position the MCA records all pulses into the ADC without reference to the gate pulse. If the switch is in the down position, the MCA only records a pulse into the ADC if there is a simultaneous gate pulse (coincidence). Conversely, when the switch is in the up position, a pulse into the ADC is recorded only when there is not a simultaneous gate pulse (anticoincidence).

Radioactive Calibration Sources

To adjust the electronics, prior to performing the Compton scattering measurements, you will need to use several standard radioactive sources. These sources are encapsulated in plastic and are all below 10μ Ci in activity. They can be handled with impunity, but when not in use they should be stored in a lead container. They should not be removed from Room 221. Among several provided γ -sources you might find the following most useful (check Melissinos for the γ -ray energies from these sources): ^{137}Cs , ^{133}Ba , ^{22}Na .

Using the calibration feature in *MAESTRO* one can either accumulate spectra from the calibration sources one at a time (each taken separately but in the same file) then calibrating after each spectra, or by taking spectra of several sources at the same time. Note: use KeV as the calibration unit or *MAESTRO* will not work properly.

Adjustments of Electronics

Check that the port hole in the lead shielding of the main ^{137}Cs source is covered.

a) Set up to measure energy spectrum

Turn on the power to the NIM (Nuclear Instrument Module) bin. Place ^{137}Cs calibration sources a few centimeters in front of the NaI scintillator. Apply a voltage of 800V to 1000V to the PMT and look at the dynode output with an oscilloscope. You should see pulses varying in amplitude with a group ranging from +400 to +800 mV and having widths in the range of 50-100 ns. The important criteria here is that the pulses have about the right amplitude and width. If the pulses do not have these characteristics try adjusting the voltage on the PMT. **DO NOT APPLY MORE THAN 1000V**. Note that the electronics has 50 Ω output impedance; therefore the electronics you use to look at the signal (i.e. the oscilloscope) must have 50 Ω input impedance. On some oscilloscopes you can select high input-impedance or 50 Ω input-impedance. If you cannot select the input impedance on the oscilloscope two 50 Ω input-impedance matchers are provided. What happens if you try to look at 50 Ω output-impedance signal with a high-impedance input device?

Connect the dynode output to the Ortec preamplifier (set the input capacitance to 500 pF for the NaI PMT and 1000 pF for the PMMA PMT) and the output of the preamplifier to the Canberra amplifier. Look at the output of the Canberra with the oscilloscope. The amplifier reshapes the pulses so that at the output they have a width of about 5 μs . You should adjust the gain of the amplifier so that the vast majority of the pulses have amplitude of less than 8 V. Note the change in pulse rate on moving the position of the source.

Turn on the computer and familiarize yourself with the operation of the Maestro software. Connect the output of the Canberra amplifier to the ADC input of the MCA (labeled IN on the card) and with the coincidence switch off (this switch is set in the software) observe the spectrum of the ^{137}Cs calibration source. Adjust the gain of the amplifier so that the main peak is about 80% of the maximum energy measurable with the MCA.

Question: What are the structures seen at the high and low ends of the spectrum?

Repeat the process outlined above with the PMMA scintillator and PMT. The average voltage from the dynode for the PMMA photomultiplier should be about +200 mV, and the pulses at the output of the amplifier should mostly be less than 5 V. Explain the results.

Calibrate the NaI detector using several calibration sources - the calibration sources can be used individually or simultaneously. Use the MCA in the non-coincidence mode and employ the features of Maestro such as region of interest (ROI), peak information, etc. to perform the calibration. The difference between "real time" and "live time" is called "dead time". Adjust the distance between the source and the PMT or the high-voltage on the PMT to keep the counting rate low enough to achieve a dead time of 2.5% or less.

Save your calibration data for future reference but do not close the file. If a file is closed and a new one created, the new file will not contain the calibration from the closed file. To preserve your calibration throughout the experiment, when an exercise has been

completed, “save” the file but do not close it, instead, do a “save as” and enter a name that describes the next exercise and clear the screen.

Make a plot of energy versus channel number and keep a record of the settings on the electronic components. You may have to redo the calibration if someone changes the settings. It is essential to check the calibration occasionally even if no one else uses the apparatus.

b) Set up of coincidence circuit

Move the NaI scintillator so that it is no more than a few centimeters from the PMMA. Place a ^{22}Na source (use source fabricated in 1988) directly between the two. The ^{22}Na decays via β^+ (positron) emission. The annihilation in the source of a positron with an electron, both at rest, occurs in about $0.1 \mu\text{s}$ producing two 511 KeV γ -rays traveling in opposite directions, as required to conserve energy and momentum. These simultaneous photons traveling in opposite directions can be used to test and adjust the coincidence circuits of the detection system.

Observe pulses from the anodes of the two PMT's on separate channels of an oscilloscope. The NaI pulses should have amplitudes of about -100 mV and widths of $\sim 100 \text{ ns}$. PMMA pulses should have amplitudes of about -300 mV and widths of $\sim 25 \text{ ns}$. Now connect the anodes of the PMT's to the inputs of the CFD's. Make sure that the CFD's are set in the constant fraction discrimination mode! Set the *THRESHOLD* dial of the CFD's to zero. Observe pulses from the positive outputs of the CFD's. The pulses should have amplitude of about 2 V and a width of $\sim 500 \text{ ns}$ for the NaI and $\sim 250 \text{ ns}$ for the PMMA scintillator. Trigger the scope on pulses in one channel but display both channels simultaneously (chop mode). With the ^{22}Na source in place there should be an observable number of pulses in the two channels that appear coincident. Note the pulse rate with the ^{22}Na source temporarily removed.

Now connect the positive outputs of the CFD's to the *A* and *B* inputs of the coincidence analyzer. Coincidence is determined by the arrival times of the leading edge of the pulses, not by overlap of the pulse envelopes. Set the range of resolving time to $0.1\text{-}1.0 \mu\text{s}$ and the associated variable dial to mid-range. Feed the output of the coincidence analyzer into the scaler (Ortec model 484) mounted in the NIM bin. The scaler counts the number of pulses it receives. With the ^{22}Na source removed the count rate should be 2 or 3 per second. With the source between the scintillators the pulse rate should increase to ~ 200 per second.

The pulses out of the coincidence analyzer should be $+4 \text{ V}$ in amplitude and $1 \mu\text{s}$ long. These are fed into the LeCroy gate generator that produces a pulse of 4 V at a selectable gate width. A gate width of at least $0.5 \mu\text{s}$ beyond the peak of the pulse you are measuring is required for gating the MCA (about $6 \mu\text{s}$). The width of the pulse produced by the gate generator can be selected with the Full Scale Width control. For fine adjustment use a screw driver to adjust the small screw under this control. If the gate pulse is not wide enough you will have trouble detecting coincidence at lower energies.

Measurement of the Total Cross Section for Compton Scattering

In the experiments that follow you will use the high activity ^{137}Cs source in the lead housing. The source is already aligned with the hole in the lead housing. With the port still covered, move the PMMA scintillator and its support out of the γ -ray beam. Place the NaI scintillator directly across the circle, i.e., about 120 cm) from the source. Remove the brick covering the hole to the source. Determine the beam profile by counting events in the photo-peak as a function of angle. Use 1° steps. Plot your results.

Place the NaI detector in the center of the beam. Measure the counting rate in the 662 KeV photo-peak with nothing in the beam path and then with varying thickness of PMMA plastic sheets. Use at least three different values of thickness.

Measurement of the Energy of the Scattered Photon and the Differential Scattering Cross Section

With the γ -rays from the source blocked by a brick, reposition the PMMA scintillator directly in the path of the beam. Reopen the beam port. Measure the energy and the rate of the scattered γ -rays detected by the NaI scintillator coincident with a scattering event in the PMMA scintillator. Do you need to measure the total number of events in both structures in the spectrum, or just the main peak to obtain the correct results?

This measurement should be performed at a number of angles between 10° and 160° . Count for sufficiently long at each angle, perhaps three hours, to obtain good statistics.

Analysis

a) Total Cross Section for Compton Scattering

Determine linear attenuation coefficient in the PMMA from the results of the measurements with different thickness of the scintillator. Assuming that this attenuation is entirely due to the Compton scattering, calculate total Compton scattering cross section, given the density of the PMMA of 1.18 g/cm^3 . Compare the results of your measurement with the theoretical value:

$$\begin{aligned}\sigma_{comp} &= \int \frac{d\sigma}{d\Omega} d\Omega = 2\pi \int \frac{d\sigma}{d\Omega} \sin \theta d\theta = \\ &= 2\pi r_0^2 \left\{ \frac{1+\gamma}{\gamma^2} \left[\frac{2(1+\gamma)}{1+2\gamma} - \frac{1}{\gamma} \ln(1+2\gamma) \right] + \frac{1}{2\gamma} \ln(1+2\gamma) - \frac{1+3\gamma}{(1+2\gamma)^2} \right\}.\end{aligned}$$

b) Energy of Scattered Photon

With an appropriate plot compare the measured dependence of the energy of the scattered photon to that given by the Compton formula. What is the value of the rest mass of the electron you obtain from the experimental results?

c) Differential Scattering Cross Section

Compare your angular dependence measurements of the count rate, $C(\theta)$, with the differential cross section predicted by the Klein-Nishina formula. The difference between the two is due to a number of corrections that one needs to apply to the data. Consider the following:

- 1) The efficiency of detecting a photon in the NaI scintillator is dependent upon the photon energy and hence on the scattering angle. This can have a significant influence on the angular dependence of the counting rate and must be addressed.
- 2) There is a certain probability that a photon scattered in the PMMA will scatter a second time before exiting the plastic, a process of multiple Compton scattering. Given the size of the PMMA, the effect of multiple scattering cannot be dismissed as unimportant. To handle this problem accurately would require going to numerical methods and employing Monte Carlo simulations. This is beyond the scope of our laboratory experiment. Instead we will describe the phenomenon below and make a number of simplifying approximations.
- 3) For a fixed position of the NaI detector there is a range of scattering angles for the photon. The PMMA target and the NaI detector both have a finite size. Show that the effect of the resulting distribution of scattering angles is small in this experiment and could be neglected.
- 4) Not all events in the PMMA are the result of Compton scattering. How important is this correction?

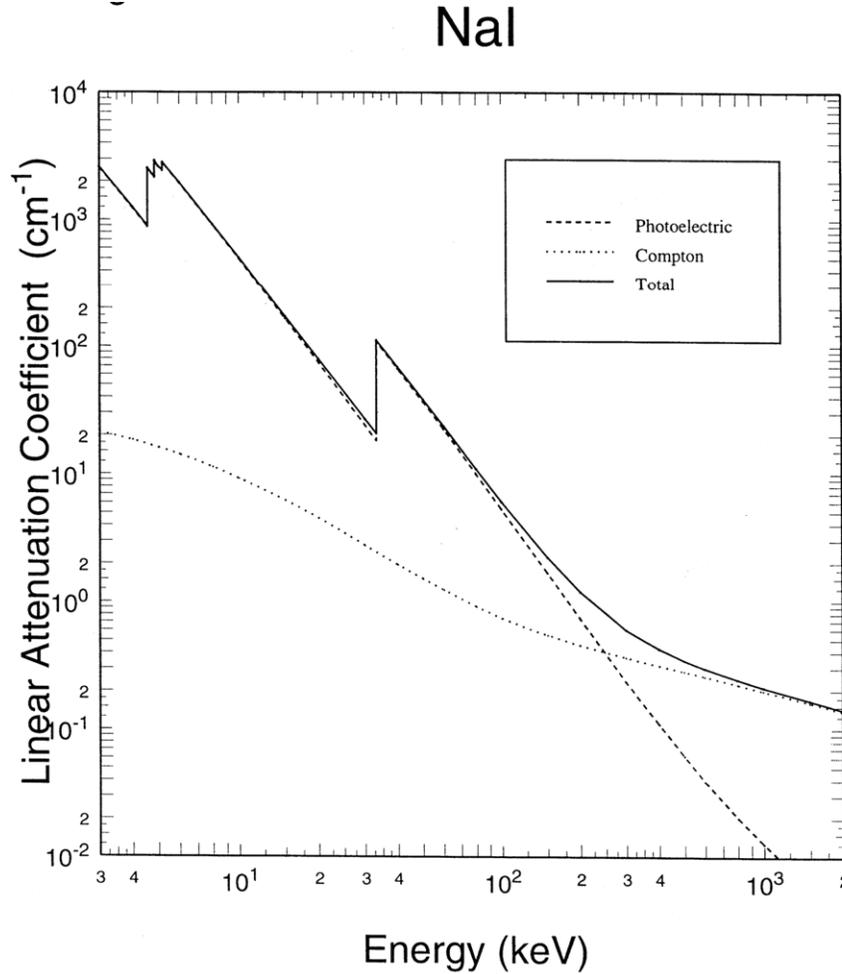


Figure 1. Attenuation coefficient in NaI scintillator.

d) Correction for the efficiency of the NaI scintillator

We consider, first, the energy dependence of the efficiency of the NaI. The linear attenuation coefficient of NaI, shown in Fig.1, can be expressed empirically, between 100 and 700 keV, as

$$\mu(E) = 5.18 E^{-2.85} + 0.539 E^{-0.45} \text{ cm}^{-1},$$

where E , the photon energy, is expressed in units of 100keV. The first term on the right is the contribution of the photoelectric absorption and the second term is the Compton component. Then the probability that a scattered photon of energy E will be recorded by the NaI is given by:

$$\alpha(\theta) = 1 - e^{-\mu t},$$

where t is the length of the NaI crystal (5.1cm). The quantity α is a function of θ since μ is dependent upon E , which in turn depends upon θ .

e) Correction for multiple scattering in the PMMA scintillator

The phenomenon of multiple Compton scattering in the target PMMA has the property of both increasing and decreasing the counting rate in the NaI. Consider a photon that is first scattered at an angle such that it would be incident on the NaI. If it is scattered a second time and consequently does not arrive at the NaI, then the event rate measured by the detector is decreased. The situation is reversed for a photon that is scattered initially at an angle such that it is not incident upon the NaI but on being scattered again and possesses a scattering angle to reach the detector, in this case the measured rate is increased.

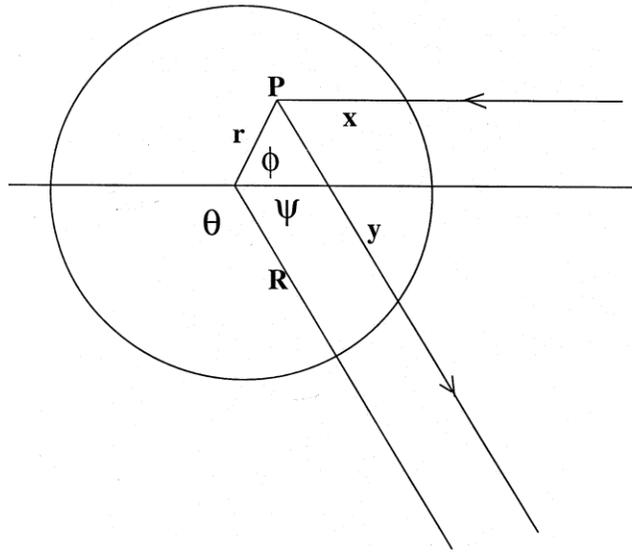


Figure 3: Scintillator geometry.

The decrease in count rate due to the scattering of photons that would otherwise be incident upon the NaI can be approximated as follows. A uniform, parallel beam of photons is incident on the PMMA cylinder perpendicular to its axis of symmetry. (Check that this is a reasonable assumption based on the measured beam profile.) Suppose an incident photon scatters at point P in the material, and the angle the scattered photon makes with the direction of the incident beam is θ ; see Fig. 3. The probability that the incident photon will reach point P having traversed a distance x in the PMMA is $\exp(-\mu_i x)$ where μ_i is the linear attenuation coefficient of PMMA at the incident photon energy, $E_V = 661\text{keV}$. The photon scattered through the angle θ must travel a distance y to emerge from the PMMA. The probability that it will do so without further scattering is $\exp(-\mu_s y)$ where μ_s is the linear attenuation coefficient at the scattered photon energy, E_v . The distances x and y can be expressed in terms of variables in cylindrical coordinates.

$$x = R((1 - q^2 \sin^2 \phi)^{1/2} - q \cos \phi),$$

$$y = R((1 - q^2 \sin^2 \psi)^{1/2} - q \cos \psi),$$

where $q = r/R$ and $\psi = \pi - \theta + \phi$.

The probability that an incident photon, Compton scattered by the angle θ will arrive at the detector, is obtained by taking the integral over the volume of the PMMA target:

$$\beta(\theta) = \frac{\int e^{-\mu_i x} e^{-\mu_s y} dV}{\int e^{-\mu_i x} dV}.$$

The enhancement of the detector signal as a consequence two sequential Compton scattering events yielding a final photon propagating at the angle θ is much more difficult to estimate. It involves, at the least, a double integral over the target volume, which we will not pursue here. Instead we make a very crude approximation. Since Compton scattering is peaked in the forward direction ($d\sigma/d\Omega$ is largest for small θ), the linear attenuation coefficient reflects primarily small angle scattering. (The fact that $d\sigma/d\Omega$ must be multiplied by $\sin\theta$ in the integral to obtain the number propagating with angle θ weakens the argument.) The decrease in intensity of the incident beam with distance x in the PMMA is compensated by a comparable increase in number of photons with somewhat lower energies propagating in close to the same direction. The crude approximation is to remove the $\exp(-\mu_i x)$ terms from the expression for $\beta(\theta)$ and write:

$$\beta(\theta) = \frac{1}{V} \int e^{-\mu_s y} dV.$$

It is straightforward to obtain $\beta(\theta)$ by numerical integration using for the energy dependence of μ in PMMA the expression $\mu(E) = 0.205E^{-0.365} \text{ cm}^{-1}$, where E is expressed in units of 100 KeV.

f) Calculations

With these corrections in mind we now return to the relation between $C(\theta)$ and $d\sigma/d\Omega$. The corrected count rate is:

$$C_{corr}(\theta) = \frac{C(\theta)}{\alpha(\theta)\beta(\theta)}.$$

This corrected count rate is proportional to $d\sigma/d\Omega$:

$$A \cdot C_{corr}(\theta) = \frac{d\sigma}{d\Omega}.$$

The proportionality constant A can be obtained from:

$$A \int C_{corr}(\theta) d\Omega = \int \frac{d\sigma}{d\Omega} d\Omega = \sigma_{comp}$$

Compute $\alpha(\theta)$ and plot its angular dependence. Calculate $\beta(\theta)$ by numerical integration and graph it as well. Calculate $C_{corr}(\theta)$ and numerically integrate to obtain the coefficient A . Plot your result for $A \cdot C_{corr}$ and compare with $d\sigma/d\Omega$ obtained from the Klein-Nishina formula.

References

1. R. A. Dunlap, *Experimental Physics: Modern Methods*, New York: Oxford University Press, 1988. ([QC33 .D86 1988](#)) Chapters 11 and 12.

2. A. Melissinos, *Experiments in Modern Physics*, New York: Academic Press, 1966. ([QC33 .M45](#)) Chapters 5 and 6.
3. W. R. Leo, *Techniques for Nuclear and Particle Physics Experiments*, 2nd rev ed., New York: Springer, 1994. ([QC793.46 .L46 1994](#)) Parts of chapters 2, 3, 7, 8, 9, 12, 14 and 15.
4. W. Heitler, *The Quantum Theory of Radiation*, 3rd ed, Oxford: Clarendon Press, 1960. ([QC475 .H43 1960](#)) P. 219.