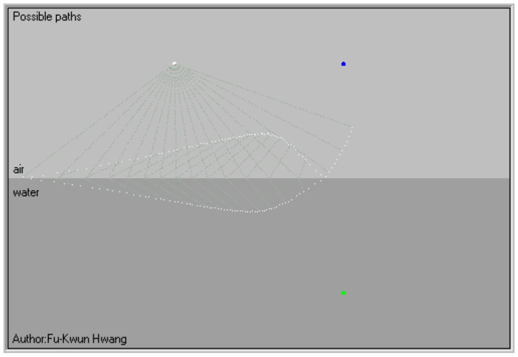
Worksheet for Exploration 34.4: Fermat's Principle and Snell's Law



This animation demonstrates Fermat's Principle: light travels along the path that takes the shortest time. You can click-drag the source (white dot) and the end-points (reflected light, blue, and refracted light, green). The animation will show you the possible paths for the light to take. The white path is the path that takes the shortest time. You can also click on the words at the interface ("air/water") to switch between a light source in air or one in water. Notice that for the reflected light, the angle of incidence equals the angle of reflection. Once the path is completed, you can click in the animation to show the angles for the angles of incidence and refraction for the refracted light.

a. Verify that the angles obey Snell's law.

n_1 =________

θ_1 =________

n_2 =________

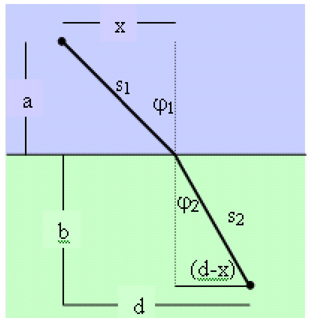
θ_2 =________

n_1 Sin θ_1 =________

n_2 Sinθ_2 =________

b. Click on the upper left-hand corner (on the words "Possible paths") to switch to "Real paths." (If you click on the words again, it will switch back to the "Possible paths".) What does the animation show in this mode and how is it different from the "Possible paths" mode?

(Calculus required): Using the diagram below (and the hints that follow), prove that you can derive Snell's law using Fermat's Principle.



v_1=speed in medium 1
v_2=speed in medium 2

c. Since the time for the light to travel through the two media (along any arbitrary path) is $t = s_1/v_1+s_2/v_2$, show that you can rewrite the time as

$$t = \frac{\sqrt{a^2 + x^2}}{v_1} + \frac{\sqrt{b^2 + (d-x)^2}}{v_2}$$

d. To find the path that minimizes the travel time between the two points, solve for $dt/dx = 0$. Why?

e. When you solve for dt/dx = 0, show that you get

$$\frac{x}{v_1\sqrt{a^2+x^2}} = \frac{d-x}{v_2\sqrt{b^2+(d-x)^2}}$$

f. Show (make the necessary substitutions) that this is the same as Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$.