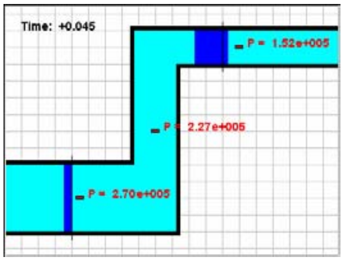
Worksheet for Exploration 15.2: Bernoulli's Equation



Bernoulli's equation describes the conservation of energy in an ideal fluid system. Assume an ideal fluid **(position is given in meters and pressure is given in pascals)**. The dark blue in the animation is a section of water as it flows into the region marked by the horizontal line and the corresponding water that must move out of the region in the top right. We will explore the connection between Bernoulli's equation and conservation of energy. Restart.

Note: The format of the pressure is written in short hand. For example, atmospheric pressure, 1.01 x 10^5 Pa, is written as 1.01e+005.

The relationship between the speed and dimensions of the water going in compared with the water leaving is governed by the continuity equation (what flows in must flow out unless there is a leak in the pipes!): Av = constant, where A is the cross-sectional area and v is the speed of the liquid. Assume the pipes are cylindrical.

a. What is the volume of both (darker) blue regions (should be the same)?

V_{left}=________

V_{right}=________

b. What is the speed of the water in the left pipe?

Speed left=_______

c. What is the cross-sectional area of the left pipe?

A_{left}=________

d. What is the speed of the water leaving the region (in the right pipe)?

Speed right=________

e. What is the cross-sectional area of the right pipe?

A_{right}=_________

f. Does the continuity equation hold?

As the water travels through this pipe system, work must be done on the fluid to raise it up and to increase its speed. The work done must be equal to the change in kinetic plus potential energy.

The upper blue region is effectively the same as the lower blue region but at a later time (after traveling up the pipe). So when you make measurements on that region, you are determining how the properties of that mass of water change.

g. Given the pressure (you can move the red pressure indicators), find the force (from the water behind it) on the lower left dark blue region.

P_{left}=_________

F_{left}=_________

h. What is the work done by that force for the duration of the animation?

W_{left}=_________

i. Similarly, find the force on the upper right dark blue region that opposes the motion.

P_{right}=_________ F_{right}=_________

j. What is the work done by that force for the duration of the animation (note that the displacement and force are in opposite directions, so this is negative work)?

W_{right}=_________

k. What is the net work done, then, during the duration of the animation on the water in the middle region?
   i. This really refers to the net work done to move the same volume of water that is in either dark blue region from the bottom to the top.

W_{net}=_________

l. Calculate the difference in kinetic energy of the dark blue regions. Note that since the volume is the same, the mass is the same. (The density of water is 1000 kg/m^3.)
   i. This is the difference in kinetic energy between a dark blue volume of water at the low left region and the kinetic energy of the same mass when it is up on the right.

$\Delta KE_{right-left}$=_________

m. Calculate the difference in potential energy of the center of mass of the dark blue regions. Does the net work equal the change in kinetic energy plus the change in potential energy?
   i. Again this change is really the difference between the right and left dark blue masses of water.

$\Delta PE_{right-left}$=_________

This is all described by Bernoulli's equation.

n. Show that the net work is equal to $(P_{left} - P_{right})\ Avt$.

o. Show that the net change in kinetic energy is $(1/2)\ \rho Avt\ (v_{right}^2 - v_{left}^2)$.

p. Show that the net change in potential energy is $\rho gAvt\ (y_{right} - y_{left})$.

P is the pressure, ρ is the density of the fluid, v is the speed of the fluid flow, A is the cross-sectional area, t is the time, and y is the height of the fluid. Combining these three terms, we get

$$(P_{left} - P_{right}) = (1/2)\rho(v_{right}^2 - v_{left}^2\ ) + \rho g(y_{right} - y_{left}),$$

or Bernoulli's equation, $P + 1/2\rho v^2 + \rho gy = \text{constant}$, so that Bernoulli's equation is simply another way to restate conservation of energy.