Student Difficulties with Trigonometric Vector Components Persist in Multiple Populations

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Abstract

Student performance in physics is strongly influenced by difficulties understanding trigonometric vector components. This study investigated the persistence of these difficulties across multiple populations to determine if they are a universal issue. A total of 125 participants from four separate educational settings completed five trigonometry vector problems placed on a grid format. Performance was quantitatively assessed in terms of correctness and sign accuracy. Results indicated that trigonometry vector component problems are about 10% correct or worse in all populations. Common mistakes included using trigonometric functions on the incorrect side of a triangle, mislabeling the angle in question, and confusion over which functions to use. Correct use of trigonometric functions significantly depended on angle placements (F(4, 491) = 24.3, p < 0.001). Furthermore, a 2-way ANOVA revealed that correct use of trigonometric functions was also significantly dependent on the presence of specific response patterns and errors (F(2, 491) = 5.2, p = 0.005). The results suggest that trigonometry vector component problems are a universal issue and highlight the need for improved instruction in this critical area.

Key terms: trigonometric vector components, student performance, instruction, mathematics, vector graphing.

Introduction

An integral step in many physics problems is the application of trigonometric functions to analyze vectors in terms of perpendicular components. The task itself is frequently easier than gaining knowledge for introductory physics. As such, it is imperative that students are as close to 100% accurate as possible—especially when transforming from a vector decomposition. However, as we will show in this study, a significant number of students have specific difficulties even with the simplest of trigonometric vector problems.

Existing Research

Previous studies on student understanding of vector components and decomposition have found significant student difficulties. For example, Van Deventer and Wittmann found that only 34% of students in a large public U.S. university were able to select the correct magnitude of a trigonometric vector component when given multiple-choice options to choose from. In a related study about components of vectors on a grid, 87% and 80% of introductory physics students at a private Moscow university were able to correctly choose the x and y components of a vector presented on a grid. Apusae and Thanik found that only about 50% of the students understood the vector nature on velocity in inclined plane problems. More recently, Per et al. (2007) showed that less than half of students could calculate the magnitude of a vector given only the magnitude of a vector presented on a grid or in a table.

Participants and Design

Data was collected after relevant instruction over the aspects of trigonometric vectors to four different populations of students in different educational settings.

• Algebra-based mechanics (N = 63)
• Algebra-based E&M (N = 169)
• Calculus-based mechanics (N = 100)
• Calculus-based E&M (N = 188)

Data was presented for all four populations. In order to study the effects of angle placement on student performance, a common problem was constructed for all four populations. In addition, the question diagram was kept consistent across all populations, as was the list of descriptions for each coding category followed:

- **Sign Error**: Student responses differ from the correct response in terms of magnitude and/or sign.
- **Incorrect Angle Placement**: Inclined plane problems. The specific questions in each condition are shown in Table 1.

### Results (Simple Problems)

No differences were found between x and y components for the first two “simple” problems, so this category was collapsed.

Figure 2 presents the score for the four angle-configuration problems. The results indicated that the overall accuracy was about 40%, with no significant differences across any group or angle configuration. One-way ANOVA revealed no significant differences (F(2, 491) = 0.08, p = 0.92). This suggests that the student difficulties common to trigonometric vector components persist in multiple populations. Given the fundamental nature of such simple tasks, the distances from ceiling are troubling.

Specific student responses and errors also showed dependence on angle configuration (Fig. 3). The primary angle error was selecting an incorrect angle configuration and angle. Students used complementary trig functions more frequently when the angle was given from vertical, suggesting that students are more flexible with angles given from horizontal and attempt to transform problems into this familiar form.

Conclusions and Implications for Instruction

We found that, even post-instruction, student performance is nowhere the 90-100% accuracy needed for the essential skill of applying trigonometric functions to vector decomposition. Students often confuse sine with cosine when the angle is given from vertical and commit sign errors when the angle is given from the hypotenuse. On inclined plane problems, students very frequently need to either change signs or confuse sine and cosine, and answer incorrectly on directly drawn triangles and incorrectly placed angles. Overall, the results suggest that students often produce answers based on the common, canonical configurations seen in examples (e.g., angles taken from the horizontal axis). Consequently, the task of trigonometric vector components is the same as not all students draw triangles to aide in trigonometric vector break-down. The fact that students struggle with the basic application of trigonometric functions may be familiar to many instructors. This study documents the extent of the problem and carefully characterizes the nature of the student difficulties presented.

Using this data, we hope to draw triangles to aide in trigonometric vector break-down.

Acknowledgements

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References


Table 1 – Question makeup of the four conditions. H and V represent the horizontal or vertical line for both. X and Y designate the angle placement from either horizontal or vertical.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Angle Placement</th>
<th>Vector Component</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>H and V</td>
<td>θ</td>
<td>x</td>
<td>Given from vertical</td>
</tr>
<tr>
<td>H and V</td>
<td>θ</td>
<td>y</td>
<td>Given from vertical</td>
</tr>
<tr>
<td>H and X</td>
<td>θ</td>
<td>x</td>
<td>Given from horizontal</td>
</tr>
<tr>
<td>H and Y</td>
<td>θ</td>
<td>y</td>
<td>Given from horizontal</td>
</tr>
</tbody>
</table>

Table 2 – Summary of student performance on inclined plane problems. Students were presented with a vector triangle for each angle configuration, and asked to transform the vertical component into a vector presented on a grid or in a table.

<table>
<thead>
<tr>
<th>Angle</th>
<th>Aligned</th>
<th>Grid</th>
</tr>
</thead>
<tbody>
<tr>
<td>50%</td>
<td>65%</td>
<td>73%</td>
</tr>
<tr>
<td>79%</td>
<td>83%</td>
<td>44%</td>
</tr>
<tr>
<td>61%</td>
<td>71%</td>
<td>49%</td>
</tr>
</tbody>
</table>

Figure 3 shows the student response rate for each angle configuration. The results indicated that the overall accuracy was about 40%, with no significant differences across any group or angle configuration. One-way ANOVA revealed no significant differences (F(2, 491) = 0.08, p = 0.92). This suggests that the student difficulties common to trigonometric vector components persist in multiple populations. Given the fundamental nature of such simple tasks, the distances from ceiling are troubling. Specific student responses and errors also showed dependence on angle configuration (Fig. 3). The primary angle error was selecting an incorrect angle configuration and angle. Students used complementary trig functions more frequently when the angle was given from vertical, suggesting that students are more flexible with angles given from horizontal and attempt to transform problems into this familiar form.